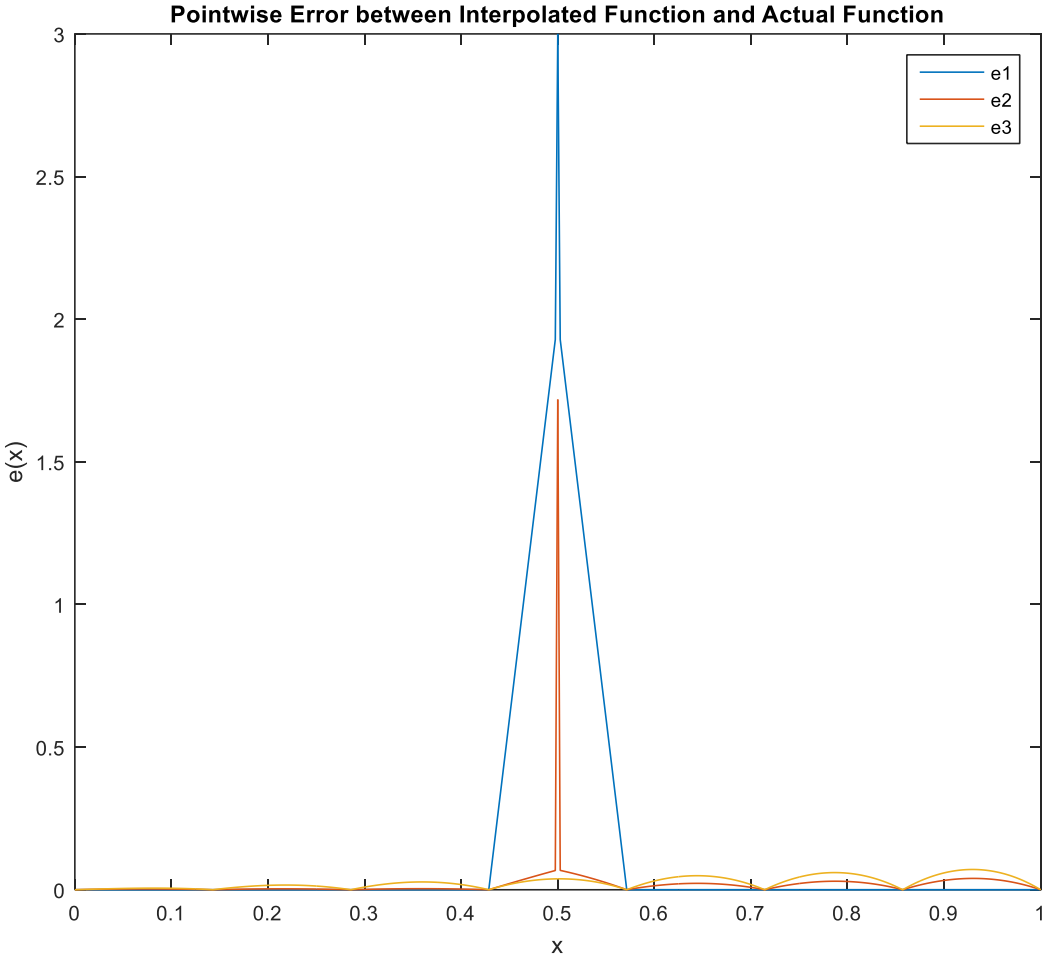
Alan Wang

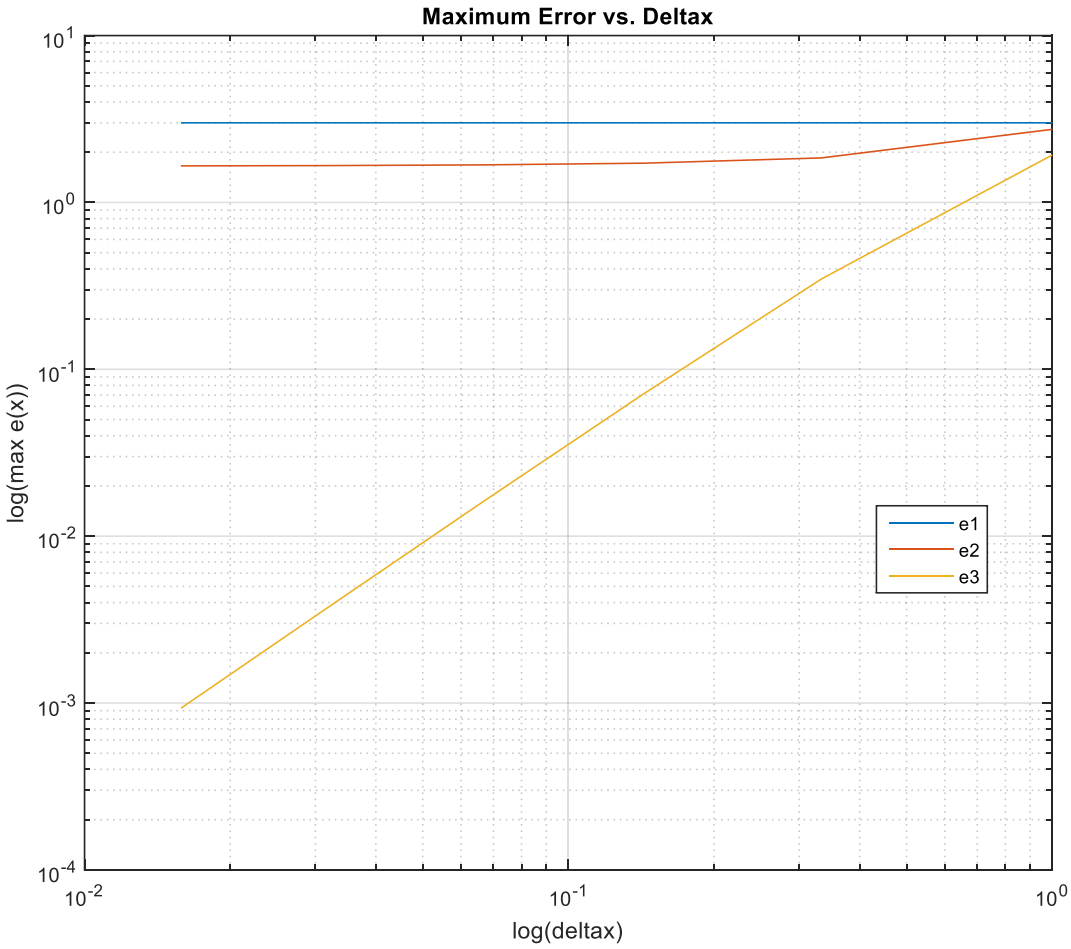
16.90 Pset #7

Plots and Code

a)



b)


Maximum Error vs. Deltax
log(max e(x))
log(deltax)
e1
e2
e3
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-2
10^-1
10^0

Script:

```
%% Alan Wang
%16.90 Pset 7
%Nodal Point Interpolation
%% Setup
n = 8;
xi = 0;
xf = 1;
xj = linspace(xi,xf,n);
u1 = @u1fun;
u2 = @u2fun;
u3 = @u3fun;


%% Part a
points = 49*n+1;
x = linspace(xi,xf,points);
u1_values = u1(x);
u2_values = u2(x);
u3_values = u3(x);
v1 = nodalApprox(u1,xi,xf,n);
v2 = nodalApprox(u2,xi,xf,n);
v3 = nodalApprox(u3,xi,xf,n);
e1 = abs(u1_values-v1);
e2 = abs(u2_values-v2);
e3 = abs(u3_values-v3);
e1max8 = max(e1);
e2max8 = max(e2);
e3max8 = max(e3);

%plots
figure(1)
plot(x,e1,x,e2,x,e3)
legend('e1','e2','e3')
xlabel('x')
ylabel('e(x)')
title('Pointwise Error between Interpolated Function and Actual Function')

%% Part b
N = [2,4,8,16,32,64];
deltax = zeros(1,length(N));
e1max = zeros(1,length(N));
e2max = zeros(1,length(N));
e3max = zeros(1,length(N));
for i = 1:length(N)
    n = N(i);
    deltax(i) = 1/(N(i)-1);
    points = 49*n+1;
    x = linspace(xi,xf,points);
    u1_values = u1(x);
    u2_values = u2(x);
    u3_values = u3(x);
    v1 = nodalApprox(u1,xi,xf,n);
    v2 = nodalApprox(u2,xi,xf,n);
    v3 = nodalApprox(u3,xi,xf,n);
```

```
    e1 = abs(u1_values-v1);
    e2 = abs(u2_values-v2);
    e3 = abs(u3_values-v3);
    e1max(i) = max(e1);
    e2max(i) = max(e2);
    e3max(i) = max(e3);
end

%plots
figure(2)
loglog(deltax,e1max,deltax,e2max,deltax,e3max)
legend('e1','e2','e3')
xlabel('log(deltax)')
ylabel('log(max e(x))')
grid on
title('Maximum Error vs. Deltax')
```

Functions:

```
function vx = nodalApprox(fx,xi,xf,n)
%interpolate1D - approximates a function as a linear combination of linear
%nodal basis functions
%   fx - function handle for function interpolating
%   phi- function handle for basis functions: phifun
%   xi - initial x
%   xf - final x
%   n  - number of nodal points

xj = linspace(xi,xf,n);
dx = xj(2)-xj(1);
phi = @phifun;
phiCell = phi(xj,dx);
a = zeros(1,n);
points = 49*n+1;
vx = zeros(1,points);

for i = 1:n
    a(i) = fx(xj(i));
    vx = vx + a(i)*phiCell{i};
end
end


function u1 = u1fun(x)
%u1 piecewise function from 16.90 pset 7
%   Detailed explanation goes here

n = length(x);
u1 = zeros(1,n);
for i = 1:n
    if x(i)>=0 && x(i)<0.5
        u1(i) = 5;
    elseif x(i)>0.5 && x(i)<=1
        u1(i) = 1;
    end
```

```
end
end


function u2 = u2fun(x)
%u2 piecewise function from 16.90 pset 7
%   Detailed explanation goes here
n = length(x);
u2 = zeros(1,n);
for i = 1:n
    if x(i)>=0 && x(i)<0.5
        u2(i) = exp(x(i));
    elseif x(i)>0.5 && x(i)<=1
        u2(i) = exp(2*(x(i)-.25));
    end
end
end


function u3 = u3fun(x)
%u3 piecewise function from 16.90 pset 7
%   Detailed explanation goes here

u3 = 5*(1-x.^3);
end


function phi = phifun(xj,dx)
%basis function
%   xj - xj
%   dx - change in nodal x

n = length(xj);
phi = cell(1,n);
points = n*49+1;
x=linspace(0,1,points);

for i = 1:n
    phi{i} = zeros(1,length(x));
    if i == 1
        ind = (x <= xj(i+1));
        phi{i} = (xj(i+1)-x)/dx;
        phi{i} = phi{i}.*ind;
    elseif i == n
        ind = (x >= xj(i-1));
        phi{i} = (x-xj(i-1))/dx;
        phi{i} = phi{i}.*ind;
    else
        for k = 1:length(x)
            if x(k)>=xj(i-1) && x(k)<=xj(i)
                phi{i}(k) = (x(k) - xj(i-1))/dx;
            elseif x(k)>xj(i) && x(k)<=xj(i+1)
                phi{i}(k) = (xj(i+1)-x(k))/dx;
            else
                phi{i}(k) = 0;
```

```
            end
        end
    end
end


end
```