MIT 16.90 Spring 2014: Problem Set 2

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Due: Monday Feb 24, in class

Problem 2. 1 *Reading Assignment*

- 1.7 Stiffness and implicit methods
- 1.8 Multi-step methods
- 1.9 Runge-kutta methods

Problem 2. 2 *Accuracy and stability analysis*

Consider the following numerical integration methods:

1. $$v^{n+1} = -4v^n + 5v^{n-1} + 4\Delta t F(v^n) + 2\Delta t F(v^{n-1})$$
2. $$v^{n+1} = v^n + \frac{3}{2}\Delta t F(v^{n+1}) - \frac{1}{2}\Delta t F(v^{n-1})$$

- For each scheme, determine whether it is zero stable.
- For each scheme, determine its global order of accuracy.
- For each scheme, determine whether it is eigenvalue stable in solving the equation

  $$\frac{du}{dt} = -\lambda u$$

  for a real, nonnegative λ. If your answer depends on Δt, state the range of Δt for which the scheme is eigenvalue stable.

Problem 2. 3 *Backward Euler for a nonlinear equation*

Write down the backward Euler scheme for solving the nonlinear equation

$$\frac{du}{dt} = -u^2 \, .$$

From it, how do you compute v^{n+1} from v^n?
