MIT 16.90 Spring 2013: Problem Set 2

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Due: Friday Feb 22, in class

Problem 2. 1 *Reading Assignment*

- Unit I: 4. Multi-Step Methods.
- Unit I: 5. Consistency, Zero Stability, and the Dahlquist Equivalence Theorem.
- Unit I: 6. Nonlinear Systems of ODEs.
- Unit I: 7. Eigenvalue Stability.
- Unit I: 8. Stiffness.
- Unit I: 9. Implicit Methods.
- Unit I. 10. Runge-Kutta Methods.

Problem 2. 2 *Accuracy and stability analysis*

Consider the following numerical integration stencils:

1.
$$v^{n+1} = \alpha_1 v^n + \alpha_2 v^{n-1} + \beta_1 \Delta t F(v^n)$$

2.
$$v^{n+1} = \alpha_1 v^n + \beta_1 \Delta t F(v^n) + \beta_2 \Delta t F(v^{n-1})$$

Recall in the first problem set, we had $\alpha_1 = \frac{3}{2}$, $\alpha_2 = -\frac{1}{2}$, and $\beta_1 = \frac{1}{2}$ for the first scheme, and $\alpha_1 = 1$, $\beta_1 = \frac{4}{3}$ and $\beta_2 = -\frac{1}{3}$ for the second scheme.

- Are either of these schemes zero stable for the choice of α_i and β_i given in the first problem set?
- For each scheme, is it possible to make them more accurate by modifying the coefficients α_i and β_i? If it is, find the coefficients that produce the most accurate scheme for each stencil.
- Are the resulting schemes zero stable?

- Consider the ODE

$$u_t = -u \qquad u(0) = 1$$

  Use the schemes you derived above to integrate this ODE numerically from $0 \leq t \leq 1$. Produce plots comparing the error between the exact and numerical solutions at $t = 1$ for $\Delta t \in \{0.1, 0.01, 0.001, 0.0001, 0.00001\}$. Does the global order of accuracy observed in the simulation results agree with the global order predicted using truncation error analysis?