MIT 16.90: Problem Set 5

Spring 2016

Due March 30, 2016

1. **Relating Finite Volume and Finite Difference Methods**

   In this problem, we will look at the relationship between finite difference and finite volume methods for the one-dimensional convection problem,

   $$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0, \qquad F = vU$$

   where v is a constant. Assume a mesh with spacing Δx. The finite volume discretization for this problem reduces to,

   $$\Delta x \frac{dU_i}{dt} + F(U_{i+1}, U_i) - F(U_i, U_{i-1}) = 0,$$

   where the flux is approximated from,

   $$F(U_R, U_L) = \frac{1}{2} v \left(U_R + U_L\right) - \frac{1}{2} |v| \left(U_R - U_L\right). \tag{1}$$

   Recall that the idea behind this choice of flux is that the flux is calculated using the value of the state that will convect to the interface as time moves forward. **Note: in the following problem, we will not discretize the time derivative but just leave it as dU_i/dt. The problem focuses on the spatial derivative approximation only.**

   (a) Consider the case where $v > 0$. Write out the finite volume discretization in terms of U_{i-1}, U_i, and U_{i+1}. What is the equivalent finite difference discretization for this finite volume discretization?

   (b) Consider the case where $v < 0$. Write out the finite volume discretization in terms of U_{i-1}, U_i, and U_{i+1}. What is the equivalent finite difference discretization for this finite volume discretization?

   (c) Now, instead of the flux given in Equation (1), consider using a flux which simply averages the states, i.e.

   $$F(U_R, U_L) = \frac{1}{2} v \left(U_R + U_L\right)$$

   Write out the finite volume discretization in terms of U_{i-1}, U_i, and U_{i+1}. What is the equivalent finite difference discretization for this finite volume discretization?