# MIT 16.90 Spring 2013: Solution Set 1

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Due: Friday Feb 15, in class

**Solution 1. 1** *Reading Assignment*

- See the notes for solutions.

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**Solution 1. 2** *Truncation error analysis*

1. The truncation error is computed as

$$\tau = \frac{3}{2}u^n - \frac{1}{2}u^{n-1} + \frac{1}{2}\Delta t u_t^n - u^{n+1}$$

Inserting the appropriate Taylor series approximations into the expression for truncation error, we have

$$\begin{aligned} \tau &= \frac{3}{2}u^n - \frac{1}{2}[u^n - \Delta t u_t^n + \frac{1}{2}\Delta t^2 u_{tt}^n + \mathcal{O}(\Delta t^3)] \\ &+ \frac{1}{2}\Delta t u_t^n - [u^n + \Delta t u_t^n + \frac{1}{2}\Delta t^2 u_{tt}^n + \mathcal{O}(\Delta t^3)] \\ &= -\frac{3}{4}\Delta t^2 u_{tt}^n + \mathcal{O}(\Delta t^3) \end{aligned}$$

The leading term of the truncation error is thus $-\frac{3}{4}\Delta t^2 u_{tt}^n$. Since the leading term of the truncation error is $\mathcal{O}(\Delta t^2)$, the local order of accuracy is $p = 2 - 1 = 1$. The global order of accuracy is the same.

2. The truncation error is now

$$\tau = u^n + \frac{4}{3}\Delta t u_t^n - \frac{1}{3}\Delta t u_t^{n-1} - u^{n+1}$$

We again use Taylor series approximations to simplify the truncation error:

$$\begin{aligned} \tau &= u^n + \frac{4}{3}\Delta t u_t^n - \frac{1}{3}\Delta t[u_t^n - \Delta t u_{tt}^n + \mathcal{O}(\Delta t^2)] \\ &- [u^n + \Delta t u_t^n + \frac{1}{2}\Delta t^2 u_{tt}^n + \mathcal{O}(\Delta t^3)] \\ &= -\frac{1}{6}\Delta t^2 u_{tt}^n + \mathcal{O}(\Delta t^3) \end{aligned}$$

The leading term of the truncation error is thus $-\frac{1}{6}\Delta t^2 u_{tt}^n$. Since the leading term of the truncation error is $\mathcal{O}(\Delta t^2)$, the local order of accuracy is $p = 2 - 1 = 1$. The global order of accuracy is the same.

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