16.90 Pset #5

Alan Wang

Problem 1

$$\frac{dU}{dt} + \frac{\partial F}{\partial x} = 0 \qquad F = vU \quad \Rightarrow \quad \Delta x \frac{dU_i}{dt} + F(U_{i+1}, U_i) - F(U_i, U_{i-1}) = 0$$

$$F = (U_R, U_L) = \frac{1}{2} v (U_R + U_L) - \frac{1}{2} |v| (U_R - U_L)$$

a.) $v > 0$

$$= -\frac{1}{2} v (U_{i+1} + U_i) - \frac{1}{2} v (U_{i+1} - U_i)$$

$$= -U_{i+1}$$

FV: $\Delta x \frac{dU_i}{dt} + vU_i - vU_{i-1} = 0$

FD: $\frac{dU_i}{dt} + v \delta_x^- U_i = 0$

b) $v < 0$

FV: $\Delta x \frac{dU_i}{dt} - vU_{i+1} + vU_i = 0$

FD: $\frac{dU_i}{dt} - \delta_x^+ U_i = 0$

c.) $\Delta x \frac{dU_i}{dt} + \frac{1}{2} v (U_{i+1} + U_i) - \frac{1}{2} v (U_i + U_{i-1}) = 0$

FV: $\Delta x \frac{dU_i}{dt} + \frac{1}{2} v U_{i+1} - \frac{1}{2} v U_{i-1} = 0$

FD: $\frac{dU_i}{dt} + v \delta_{2x} U_i = 0$