MIT 16.90: Problem Set 6

Spring 2016

Due April 6th, 2016

1. **Fourier Analysis and the CFL Condition**

   Consider the following PDE

   $$\frac{dU}{dt} + v\frac{dU}{dx} = cU$$

   where v and c are constants, and $v > 0$.

   (a) Derive an ODE that describes the time evolution of the m^{th} Fourier mode $\hat{U}_m(t)$ by applying Fourier analysis. For what values of c is the solution bounded?

   For the remainder of this problem, assume that the condition on c such that the solution is bounded is met. In other words, the analytic behavior of the solution is stable.

   (b) Now, consider the following numerical scheme for the PDE

   $$\frac{U_j^{n+1} - U_j^n}{\Delta t} + v\frac{U_j^n - U_{j-1}^n}{\Delta x} = cU_j^n$$

   Determine the timestep restriction that satisfies the CFL condition.

   (c) Apply matrix eigenvalue stability analysis (assuming periodic boundary conditions) to determine the maximum stable time step for this discretization.

   (d) Is it possible for the scheme to be unstable while satisfying the CFL condition?

   (e) Is it possible for the scheme to be stable but not satisfy the CFL condition?