MIT 16.90: Problem Set 9

Spring 2016

Due May 4th, 2016

1. **Monte Carlo for Turbine Blade Temperature with Dependent Parameters**

   Consider the turbine blade temperature problem discussed in class and in the notes. In class, as well as in Project 3, we perform Monte Carlo simulations to obtain the hot metal temparature T_{mh} by assuming that all of the input parameters are independent and have their own distribution. However, in reality, such parameters may not necessarily be independent of one another. For example, the temperature of the flow T_{gas} may have an effect on the temperature of the cooling fluid T_{cool}. In this problem, we consider a dependent relationship between these two input parameters and perform Monte Carlo analysis with this dependency.

   In this problem, we consider only T_{gas} and T_{cool} to have any variability. All other parameters have the following nominal values

$$h_{gas} = 3000\ W/m^2K$$
$$k_{tbc} = 1\ W/mK$$
$$L_{tbc} = 0.0005\ m$$
$$k_m = 20\ W/mK$$
$$L_m = 0.003\ m$$
$$h_{cool} = 1000\ W/m^2K$$

   while T_{gas} has a uniform distribution between 1500 K and 2500 K with mean value of $\bar{T}_{gas} = 2000\ K$. We take the value of T_{cool} to vary according to

$$T_{cool} = k\left(\frac{T_{gas} - \bar{T}_{gas}}{\bar{T}_{gas}}\right)\bar{T}_{cool} + \bar{T}_{cool} + \tilde{T}_{cool}$$

   In the above equation, $\bar{T}_{cool} = 600\ K$. Additionally, k is some sensitivity parameter dictating the variability of T_{cool} based on the variability of T_{gas}, while $\tilde{T}_{cool}$ represents some inherent variability in T_{cool} itself. Consider the following three cases:

   - **Case I:** $k = 0$ (no dependence of T_{gas} on T_{cool}), $\tilde{T}_{cool}$ is a uniform distribution between [-150 K,150 K]
   - **Case II:** $k = 0.5$ (some dependence of T_{gas} on T_{cool}), $\tilde{T}_{cool}$ is a uniform distribution between [-75 K,75 K]
   - **Case III:** $k = 1.0$ (high dependence of T_{gas} on T_{cool}), $\tilde{T}_{cool} = 0$ so there is no inherent variability

   Modify the script bladeTgas.m in the 16.90 Spring 2016 Working Folder, under MCIntro, and perform Monte Carlo simulations with N = 10,000 for each case. Also provide the following (for each case):

   - The 99 percent confidence interval for each of T_{gas}, T_{cool}, and T_{mh}
   - Scatterplot of T_{gas} vs. T_{cool} from Monte Carlo simulations. Let $T_{limit} = 1500\ K$, and indicate which points correspond to a failed blade (where $T_{mh} > T_{limit}$) by marking it red on the scatterplot during your plotting routine.

   What do you notice about the behavior of the scatterplot as k increases and the variability of $\tilde{T}_{cool}$ decreases?