MIT 16.90 Problem Set 2

Spring 2016

Due February 24, 2016

1. **Accuracy and Stability analysis**

   Consider the following numerical integration scheme given by

   $$v^{n+1} = \alpha_1 v^n + \alpha_2 v^{n-1} + \beta_1 \Delta t f(v^n)$$

   (a) Let $\alpha_1 = \frac{3}{2}$, $\alpha_2 = -\frac{1}{2}$, and $\beta_1 = \frac{1}{2}$. For this choice of the coefficients, is the scheme zero stable? Show why or why not.

   (b) Find the coefficients α_1, α_2, and β_1 that result in the most accurate integration scheme by finding an expression for the truncation error and substituting taylor series into appropriate terms. What familiar integration scheme do you end up with?

   (c) Is the resulting scheme derived in part (b) zero stable? Show why or why not.

2. **Adams-Bashforth**

   Consider the explicit 2-step scheme

   $$v^{n+1} - v^n = \Delta t \left(\frac{3}{2} f(v^n) - \frac{1}{2} f(v^{n-1}) \right)$$

   (a) Is this scheme zero stable? Show why or why not.

   (b) Determine the global order of accuracy of this scheme. Under this scheme, as $\Delta t \to 0$, will the global error (i.e. maximum difference between v^n and $u(n\Delta t)$) approach zero?
   (Hint: Consider what the Dahlquist Equivalence Theorem states about Convergence)

3. **Stiff linear ODE**

   Consider the ODE

   $$\frac{d\mathbf{u}}{dt} = \begin{bmatrix} -100 & 99 \\ 99 & -100 \end{bmatrix} \mathbf{u}, \tag{1}$$

   where $\mathbf{u} \in \mathbb{R}^2$.

(a) Determine the range of Δt for which the forward Euler method,

$$v^{n+1} = v^n + \Delta t f(v^n),$$

is eigenvalue stable in solving Equation 1.

(b) Determine the range of Δt for which Adams-Bashforth,

$$v^{n+1} = v^n + \Delta t \left(\frac{3}{2} f(v^n) - \frac{1}{2} f(v^{n-1}) \right)$$

is eigenvalue stable in solving Equation 1. (Note: This is much more involved.)