16.90 Pset #7 Alan Wang

Problem 1 - Analysis of the 1D Nodal Basis Functions for Interpolation.

$$u_1 = \begin{cases} 5 & 0<x<0.5 \\ 1 & 0.5<x<1 \end{cases} \qquad u_2 = \begin{cases} e^x & 0<x<0.5 \\ e^{2(x-0.25)} & 0.5<x<1 \end{cases} \qquad u_3 = \begin{cases} 5(1-x^3) & 0<x<1 \end{cases}$$

Approx each function using $v(x) = \sum_{j=1}^{N} a_j \phi_j(x)$ w/ N equally spaced nodes

$$\phi_j(x) = \begin{cases} 0 & x < x_{j-1} \\ \frac{x - x_{j-1}}{\Delta x} & x_{j-1} < x < x_j \\ \frac{x_{j+1} - x}{\Delta x} & x_j < x < x_{j+1} \\ 0 & x > x_{j+1} \end{cases}$$

$a_j = u(x_j)$

$e(x) = |u(x) - v(x)|$

Within each element use 50 pts to determine error

$e_{max} = \max_x |u(x) - v(x)|$ store max pointwise error for each element, take the max of those

$u_1(x) \approx v_1(x) = a_1\phi_1(x) + a_2\phi_2(x) \ldots + a_8\phi_8(x)$ $\qquad a_1 = u_1(x_1) = 5$

a.) N=8 See Attached Plots

$$\begin{bmatrix} 0 & .1429 & .2857 & \cdots & .8571 & 1 \end{bmatrix}$$
x_1 x_2 x_3 x_7 x_8

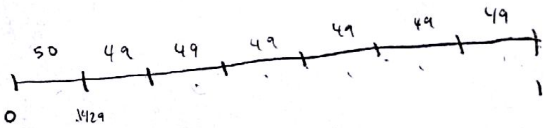
50 50

$$\phi_1(x) = \begin{cases} 0 & x \le 0 \\ \frac{x}{\Delta x} & x \ne 0 \\ \frac{.1429 - x}{\Delta x} & 0 \le x \le .1429 \\ 0 & x > .1429 \end{cases} \qquad \phi_2(x) = \begin{cases} 0 & x \le 0 \\ \frac{x}{\Delta x} & 0 < x \le .1429 \\ \frac{.2857 - x}{\Delta x} & .1429 < x < .2857 \\ 0 & x \ge .2857 \end{cases}$$

[0, .1429] $\qquad$ [0, .2857]

$$\phi_3(x) = \begin{cases} 0 & x < .1429 \\ \frac{x - .1429}{\Delta x} & .1429 \le x \le .2857 \\ \frac{.428 - x}{\Delta x} & .2857 < x \le .428 \\ 0 & x > .428 \end{cases}$$

[.1429, .428]



n·49 + 1

$$\phi_8 = \begin{cases} 0 & x < .8571 \\ x - .8571 & .8571 \le x \le 1 \\ 0 & x > 1 \end{cases}$$

$$\phi = \{ [\phi_1] \; [\phi_2] \; [\phi_3] \; \cdots \; [\phi_8] \}$$

$$\boxed{\begin{aligned} e_{1\,max} &= 3 \\ e_{2\,max} &= 1.7185 \\ e_{3\,max} &= .0711 \end{aligned}}$$

b.) For e_1 max : The order of convergence is $p=0$. Max error scales as $O(\Delta x^0)$ or 1

e_2 max: The order flattens out but as approaches $\Delta x = 10^0$ scales as $p=1$. or $O(\Delta x^1)$

e_3 max: The order of convergence is $p=2$. scales w/ $O(\Delta x^2)$

See Attached Plot.