16.90 Computational Methods in Aerospace Engineering

Spring 2012

March 5th at 2:30pm

Due: Thursday ~~March 1st, 2012 at 4 pm~~

Project #1: ODEs and Aeroelastic Limit Cycle Oscillations

Note: *Projects are meant to be open-ended and to allow some flexibility and creativity. Therefore it is important for you to show all relevant steps, numerical plots, and justifications for the choices made in your work.*

Background

A next generation airplane design uses very light-weight structures, and relies on a computerized feedback controller to stabilize aeroelastic oscillations of its wing. In order to pass a certification test, the new design must demonstrate that no structural damage could occur if the aeroelastic control computer is forced to reboot in-flight, e.g., caused by a lightning strike. This project uses computational simulation to predict whether the current design is able to pass this certification test.

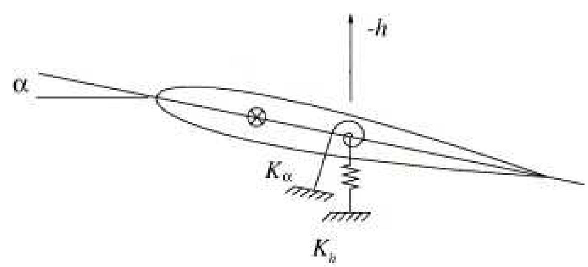

Figure 1: The 2-degree-of-freedom aeroelastic model

We model the aeroelastic vibration of the wing in the absence of the controller with two physical variables: the pitch, $\alpha(t)$, and plunge, $h(t)$. The pitch is in radians and the plunge is in nondimensional units of chord lengths. Figure 1 describes these two variables. The plunging degree of freedom is governed by a linear stiffness and damping forces, but the pitch degree of freedom includes both the linear components and a nonlinear coupling term between the pitch and plunge motions. Specifically, the motion is governed the following system of ODEs

$$\begin{aligned} M_{hh}\frac{d^2h}{dt^2} + M_{h\alpha}\frac{d^2\alpha}{dt^2} + D_h\frac{dh}{dt} + K_h h + L = 0 \\ M_{\alpha\alpha}\frac{d^2\alpha}{dt^2} + M_{\alpha h}\frac{d^2h}{dt^2} + D_\alpha\frac{d\alpha}{dt} + K_\alpha(1 + k_{NL}h^2)\alpha + M = 0 \end{aligned} \tag{1}$$

where the parameters for our airplanes are

$$
\begin{aligned}
M_{hh} &= 1 \\
M_{h\alpha} &= 0.625 \\
M_{\alpha\alpha} &= 1.25 \\
M_{\alpha h} &= 0.25 \\
D_h &= 0.1\ s^{-1} \\
D_\alpha &= 0.25\ s^{-1} \\
K_h &= 0.2\ s^{-2} \\
K_\alpha &= 1.25\ s^{-2} \\
k_{NL} &= 10
\end{aligned} \tag{2}
$$

In the right hand side of Equation (1), L and M are the aerodynamic lift and moment, respectively. We model both the aerodynamic lift and moment as linear with respect to the angle of attack:

$$
\begin{aligned}
L &= 1s^{-2}\, Q\, \alpha \\
M &= -0.7s^{-2}\ Q\, \alpha
\end{aligned} \tag{3}
$$

Here Q is the aerodynamic pressure, which depends on the airspeed. It is nondimensionalized so that $Q = 1$ at a design airspeed V_{NO}. At the moment of the controller failure $t = 0$, the initial conditions

$$
\begin{aligned}
&h(0) = 0 \\
&\frac{dh}{dt}(0) = 0 \\
&\frac{d\alpha}{dt}(0) = 0 \\
&0 < \alpha(0) < 0.08 \text{ radians}
\end{aligned} \tag{4}
$$

Tasks

1. Consider two design airspeeds, V_{NO} and V_{NE}, corresponding to $Q = 1$ and $Q = 1.5$, respectively. For the maximum initial pitch $\alpha = 0.08$ radians, solve the equations of motion for 60 seconds using (i) classical four-stage Runge Kutta (RK4), and (ii) a fourth-order backwards differentiation (BD4) scheme.

   Comment on the accuracy and efficiency of the ODE solution using each of these two schemes. Which scheme is the best one to use for this system? Also comment on the behavior of the oscillation at the two different airspeeds.

2. To determine the possibility of structural failure, the designer of the airplane is concerned about the maximum plunge and pitch motions experience during $0 < t < 60$ seconds, for a range of possible initial pitch angles $0 < \alpha < 0.08$ radians. Estimate the maximum values of $|h(t)|$ and $|\alpha(t)|$ during that time period, for both $Q = 1$ and $Q = 1.5$.

   How confident are you in the results you obtain? Why?

3. The controls engineer has just determined that the down time of the aeroelastic control computer would not exceed 20 seconds. The structural engineer is now given the task of ensuring that no structural damage could occur during a gap of 20 seconds. In particular, at $Q = 1.5$, $|h(t)|$ should not exceed one chord length, and $|\alpha(t)|$ should not exceed 0.2 radians.

   Both the pitch and plunge degrees of freedom can be stiffened; damping can also be increased through structural modification. But all of these changes would increase the structural weight of the wing. Stiffening the plunge degree of freedom increases K_h; each $0.01s^{-2}$ increase in K_h requires 1% increase in the empty weight of the airplane. Stiffening the pitch degree of freedom is more difficult; each $0.01s^{-2}$ increase in K_α requires 2% increase in the empty weight of the airplane. Increasing the plunge damping D_h by each $0.01s^{-1}$ requires 1% increase in the empty weight; and increasing the ~~plunge~~ pitch damping by each $0.01s^{-1}$ requires 2% increase in the empty weight. A combination of any amount of these four types of modification can be used.

   How would you stiffen the wing to meet the design requirement? Discuss how confident you are that your modification results in a safe (and yet light-weight) airplane.

In answering the questions above, please include informative plots of all your numerical results, accompanied by clear written arguments and explanations. If you're making an assertion about how well a method performs or why the system behaves in a certain way, please support this assertion with numerical results and plots.

Please turn in a hard copy of your project writeup in room 37-461, but upload your codes to Stellar.