MAST30013 – Techniques in Operations Research

Semester 1, 2020

Group Project

Submission: Reports (typeset in LaTex) to be submitted before the (strict) due date: **4pm, 4 June**.

Groups: must be composed of a minimum of 3 and a maximum of 4 members. It is your responsibility to talk to your colleagues and create/find a group, as early as possible in the semester.

Evaluation:

Reporting: You should present a coherent and self-contained report presenting your algorithms and results for the questions below.

Presentation: In the last week of class, groups will present their work in 12 minutes + 3 minutes for questions. You are not expected to explain the problem in detail during the presentation. Instead, focus on the innovative aspects that your group has found: Did you find any nice application for the problem? Which solution algorithm did you implement? What conclusions did you make?

Project Description: Traffic Assignment

Background

Traffic assignment (a.k.a. route assignment, route choice) concerns the selection of paths (alternatively called routes) between origins and destinations in transport networks.

You are tasked to solve the following problems:

Problem 1: Non-linear regression

You are given a set of traffic data (x, y), $x, y \in \mathbb{R}^+$, where x is the traffic demand and y is the travel cost/time (per unit of traffic). See Figure 1. Fit a non-linear regression to the data provided. You can download the data set from Canvas.

Details:

1. Provide an interpretation of the demand-cost relation.
2. Fit a curve of cubic form $y = \beta_1 x^3 + \beta_2 x^2 + \beta_3 x + \beta_4$. This will create predictions for the travel costs of each of the data point that we already have, and these predictions will have errors. You find the coefficient of the regression curve by minimising the sum of the squares of all the errors. Model this problem as an *unconstrained convex* problem. Show the problem is unconstrained and convex. Define all variables.
3. Implement an algorithm for solving your model. Do a computational study (different initial solutions, parameter settings, etc.).
4. Can the problem be solved with a pre-existing Matlab (or other software) function? What is the package/function doing? Do a computational study (different initial solutions, parameter settings, etc.).

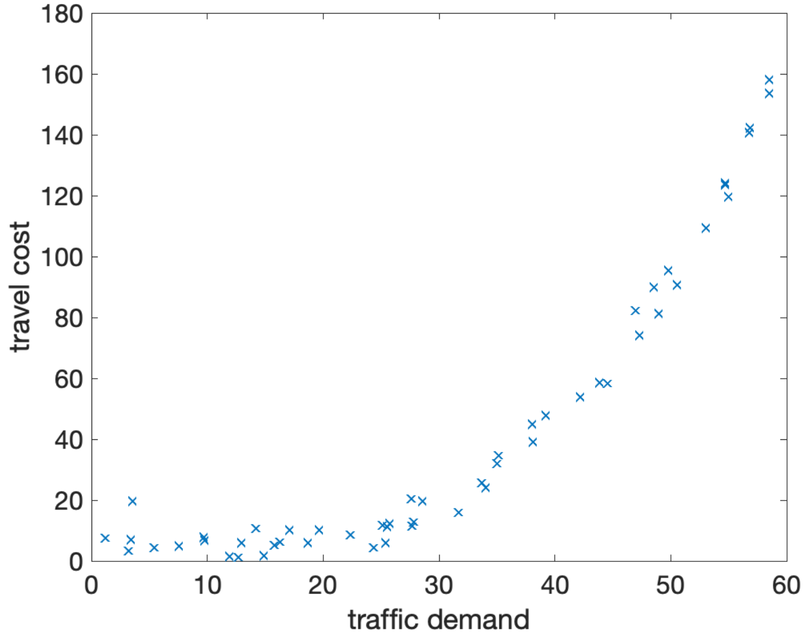

Figure 1: Traffic data: travel cost (per unit of traffic) against travel demand

Problem 2: System optimal

In 1952, Wardrop proposed two fundamental equilibrium principles of travel choices. One of them is the system optimal principle, which assumes all drivers cooperate with one another in choosing their paths to minimise the total travel cost.

Now you are given a traffic network as depicted in Figure 2. There are three types of nodes (i.e. intersections): source nodes O = $\{o_1, o_2\}$, transit nodes V = $\{v_1, v_2, \ldots, v_4\}$, and destination node $\{d\}$. Travel cost on each link ij, a directional edge connecting two nodes i and j, is defined by the demand-cost relation discussed in Problem 1 with parameters specified in Figure 2.

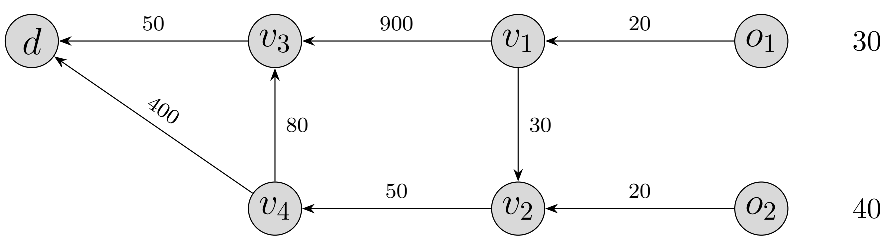

Figure 2: A simple traffic network. For all links, $(\beta_1, \beta_2, \beta_3) = (0.002, -0.01, 0.1)$ and β_4 is displayed next to the link. Travel demands are displayed next to origin nodes.

Details:

1. Model the problem as a constrained optimisation.
   - Define all variables.
   - List all constraints and explain each of them.
2. Derive KKT conditions.

3. Implement an algorithm for solving your model.

4. Discuss your findings.

Reference

Wardrop, J. G. (1952). Some Theoretical Aspects of Road Traffic Research. Proceedings of the Institution of Civil Engineers. 1 (3): 325–362.

Important Notes

- You will be marked on the quality of your report. Make sure it is self contained and follows the structure of a scientific report.
- It is the group's responsibility to ensure that everyone contributes their fair share to the project. All members of the group will receive the same mark.
- It is recommended that members of the group share a Dropbox folder for their work or use Overleaf. This is so that all group members can have access to all parts of the project at any time.
- This project contains an extensive programming component. It is assumed that all students are able to program in Matlab (or a software of your choice). The lecturer will **not** assist in issues relating to the debugging of code.
- Matlab code is strongly encouraged. Other software code is accepted.
- Students may research ideas on the web, but full credit to the relevant authors must be given if the students use any of these ideas. Code may NOT be copied from existing sources. Collaboration between groups is NOT allowed.

Report Layout

- Title
- Authors (student numbers)
- Date
- Summary (a brief summary of the objectives, findings, conclusions and recommendations) < 1 page
- Introduction/Background (an overview of the project and what to expect in the following sections)
  - What is the problem you are looking at?
  - Motivation, applications
  - Summary of what the report aims to achieve, methods used, etc.
- Theory/Methods
  - Answer Q2 of Problem 1 and Q1, Q2 of Problem 2
  - Any theory that you would like to add in order to justify an algorithm that you will use in Q3, Q4 of Problem 1 and/or Q3, Q4 of Problem 2.
  - Description of the algorithms you will be testing (pseudo code).
- Experimental results
  - This section covers Q1, Q3, Q4 of Problem 1 and Q3, Q4 of Problem 2
  - Experimental setup
    - Describe the experiments that you will be performing
    - Which algorithms will you be comparing and how will you measure their performance?
    - What software and hardware did you use for testing?
    - What parameters did you use and why?
  - Results
    - Present the results of your experiments using tables and graphs.
    - Discuss each table and graph in the caption.
    - At the end of the results subsection, discuss the conclusions you have made from the graphs and tables.
- Conclusion/Discussion
  - Summarise the report
  - Related problems; Related literature; Alternative approaches not explored (optional)
- Appendix (optional)
  - Flow charts
  - Computer programs, outputs
  - ...