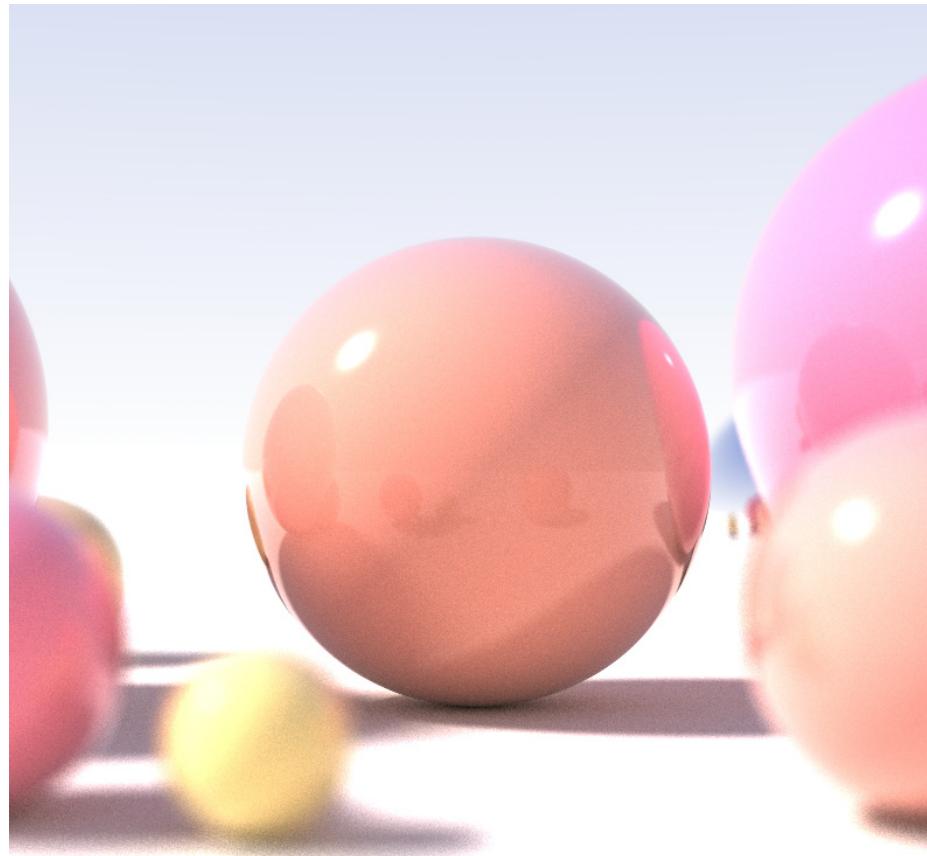


# Differentiable Rendering



Evangelos Kalogerakis  
574/674

# What is rendering?

The process of generating a 2D image from a 3D representation

# What is rendering?

The process of generating a 2D image from a 3D representation

**Rasterization**

**Ray tracing**

# What is rendering?

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**Rasterization**

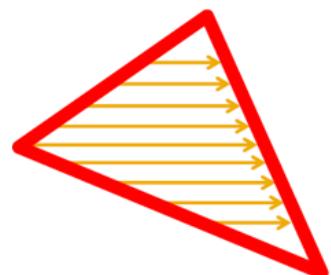
for each polygon

project it onto 2D plane

for each pixel in polygon

shade the pixel

**Ray tracing**



# What is rendering?

The process of generating a 2D image from a 3D representation

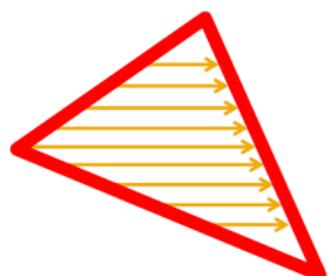
## Rasterization

for each polygon

project it onto 2D plane

for each pixel in polygon

shade the pixel



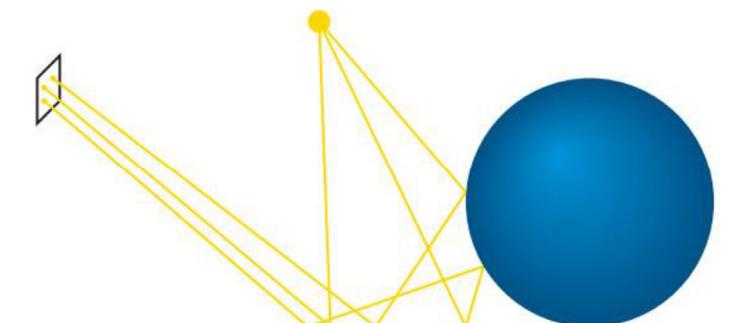
## Ray tracing

for each pixel throw ray

for each object

calculate intersection

shade the pixel



# What is rendering?

Both are heavily parallelized in GPU hardware!

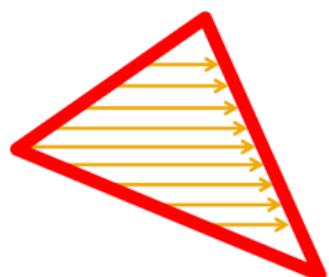
## Rasterization

for each polygon

project it onto 2D plane

for each pixel in polygon

shade the pixel



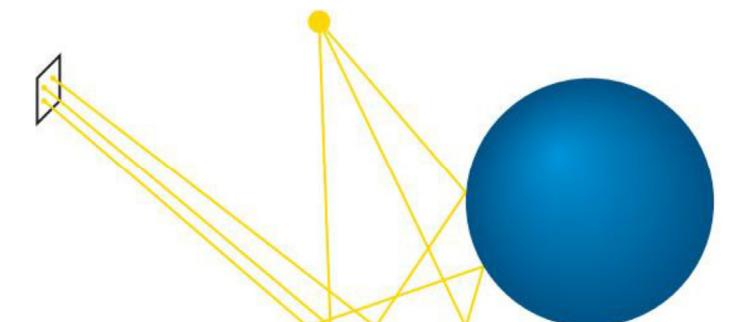
## Ray tracing

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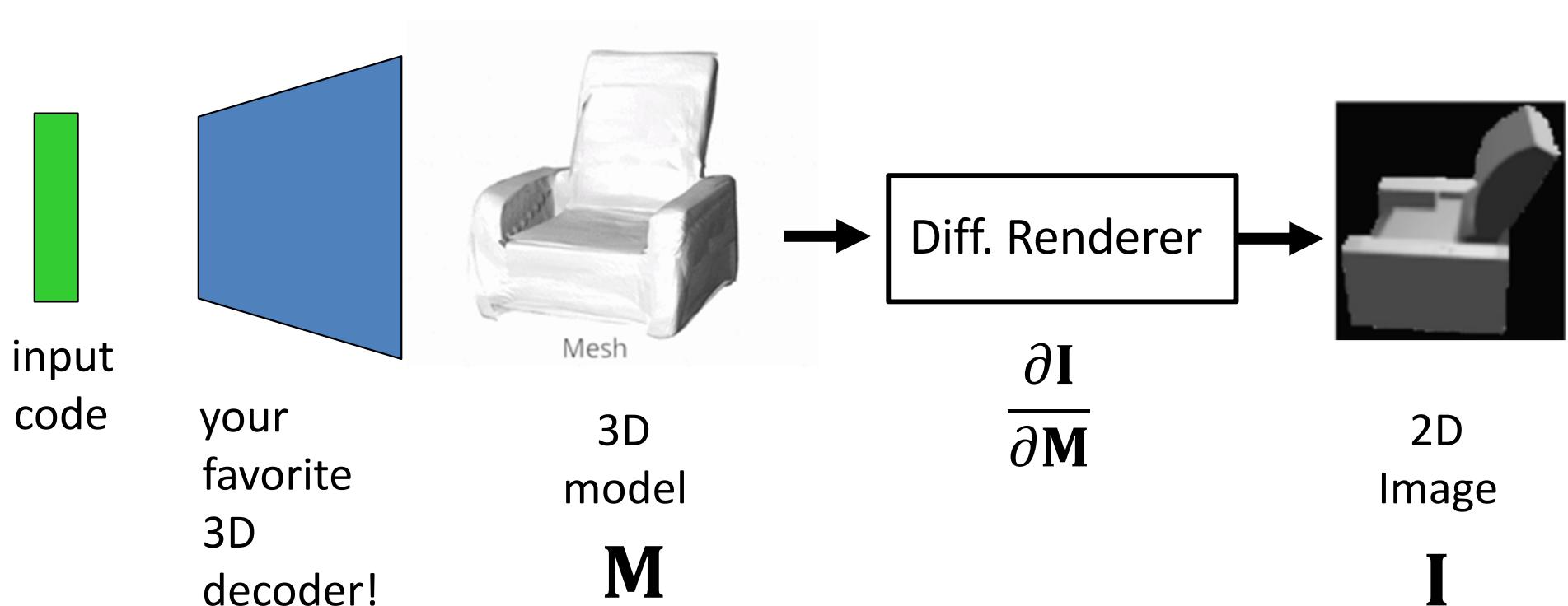
calculate intersection

shade the pixel



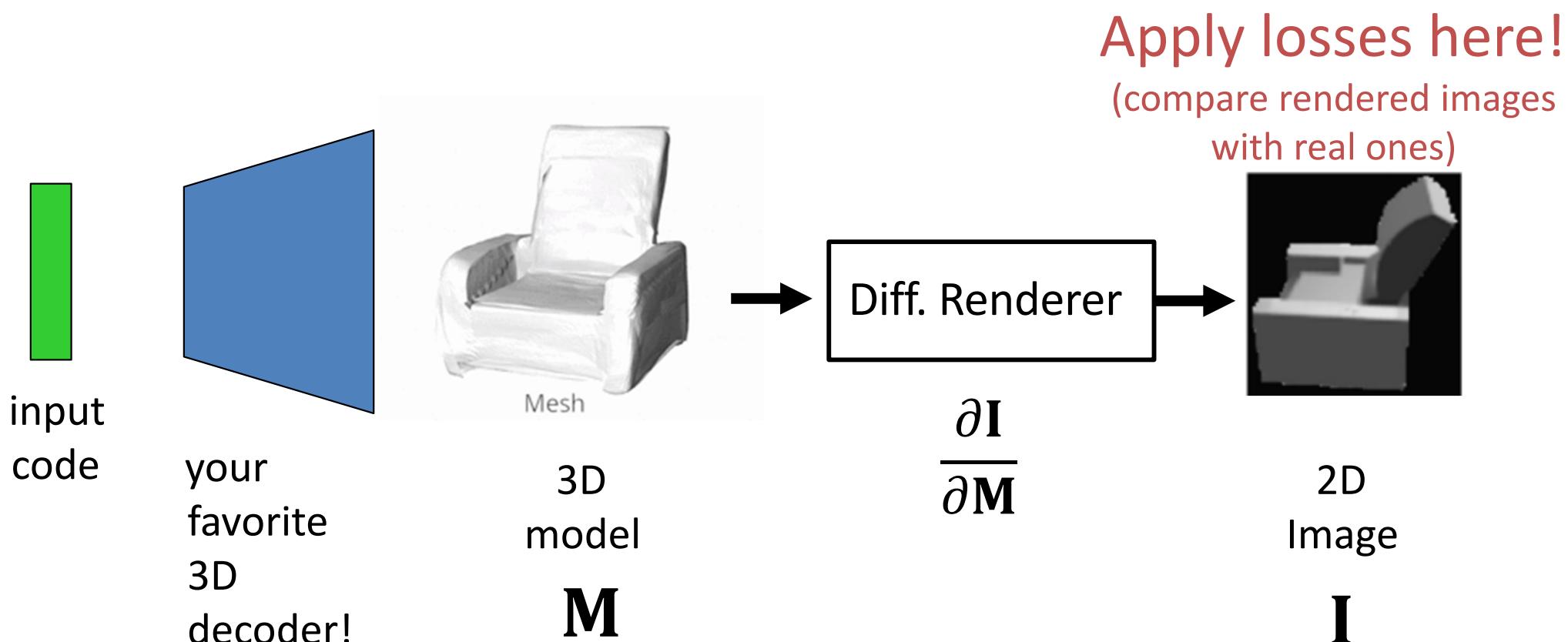
# What is **differentiable** rendering?

Allows incorporating rendering as a module in a NN!



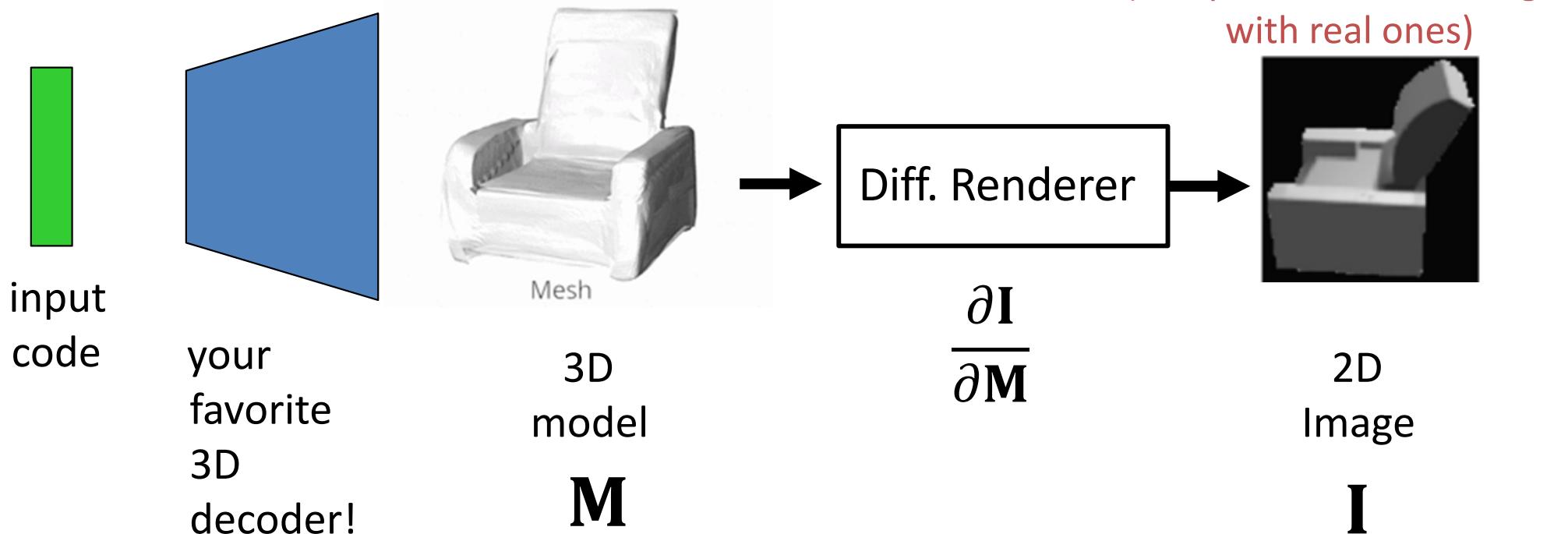
# What is **differentiable** rendering?

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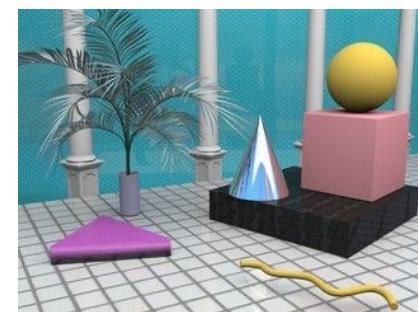
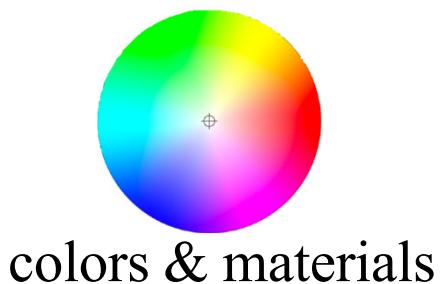
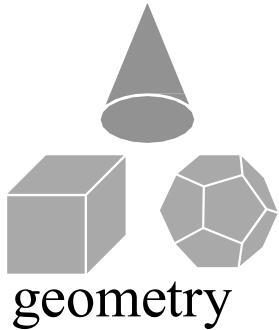


# What is **differentiable** rendering?

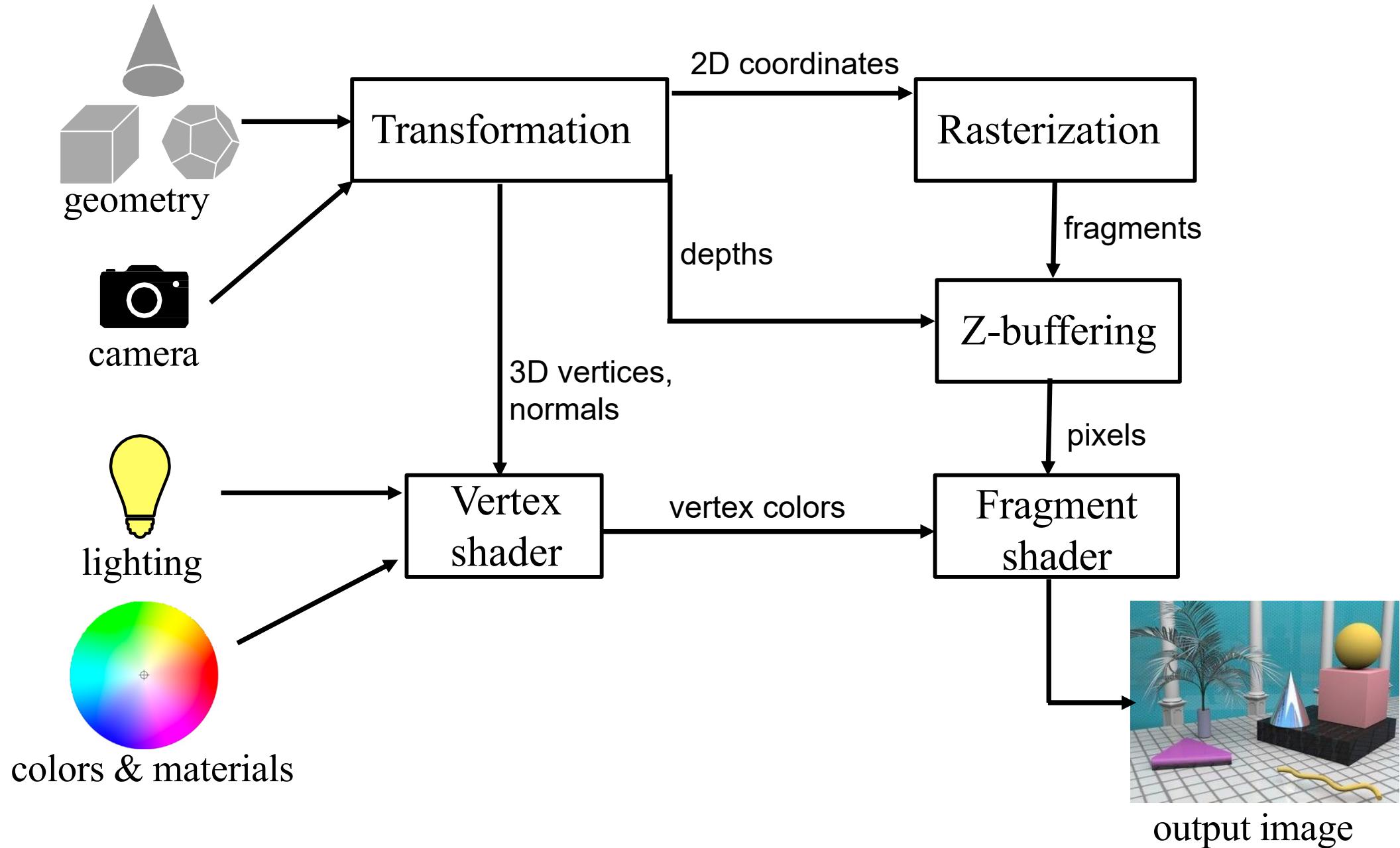
Allows training 3D DL nets with losses coming from 2D image datasets (tons of them available!) **Apply losses here!**  
(compare rendered images with real ones)



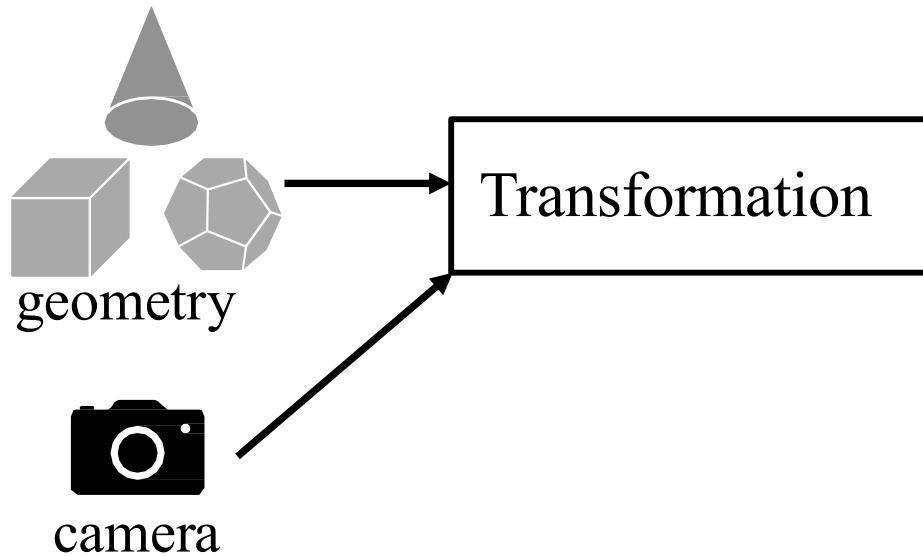
# Rasterization pipeline



# Rasterization pipeline



# Rasterization pipeline



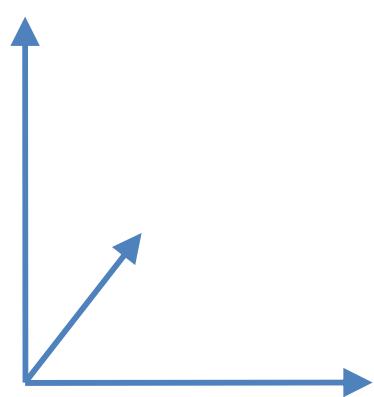
# View transformation

Position and orient a camera in your scene...



# Change of coordinate systems

Suggest that an object is represented with respect to the world coordinate system (in blue)

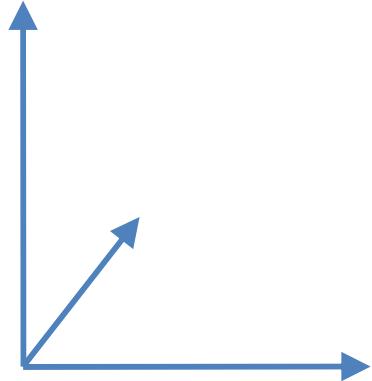


$$\bullet \quad p = [x, y, z]^T$$

Original coordinate system

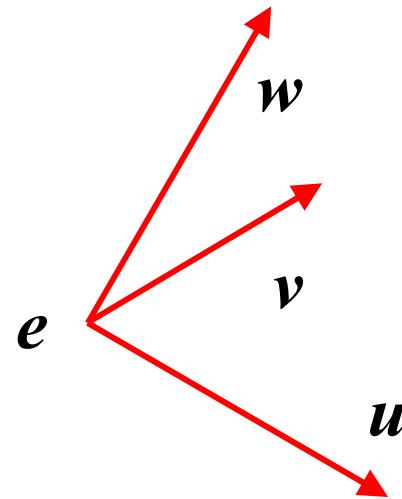
# Change of coordinate systems

We want to change the coordinate system so that the camera is the center of our world...



Original coordinate system

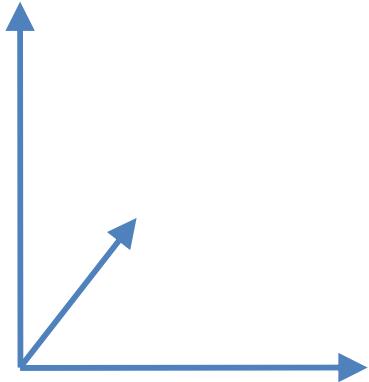
$$\bullet \quad \mathbf{p} = [x, y, z]^T$$



New coordinate system  
( $u, v, w$  are orthonormal vectors,  
 $u = [u_1, u_2, u_3]^T$   
 $v = [v_1, v_2, v_3]^T$   
 $w = [w_1, w_2, w_3]^T$   
and  $e = [e_1, e_2, e_3]^T$  is the origin)

# Change of coordinate systems

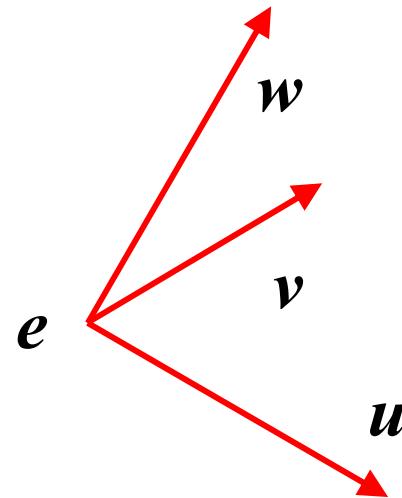
How do we compute the position of the point according to the **new coordinate system**?



Original coordinate system

$$\bullet \quad p = [x, y, z]^T$$

$$p' = ?$$



New coordinate system

( $u, v, w$  are orthonormal vectors,

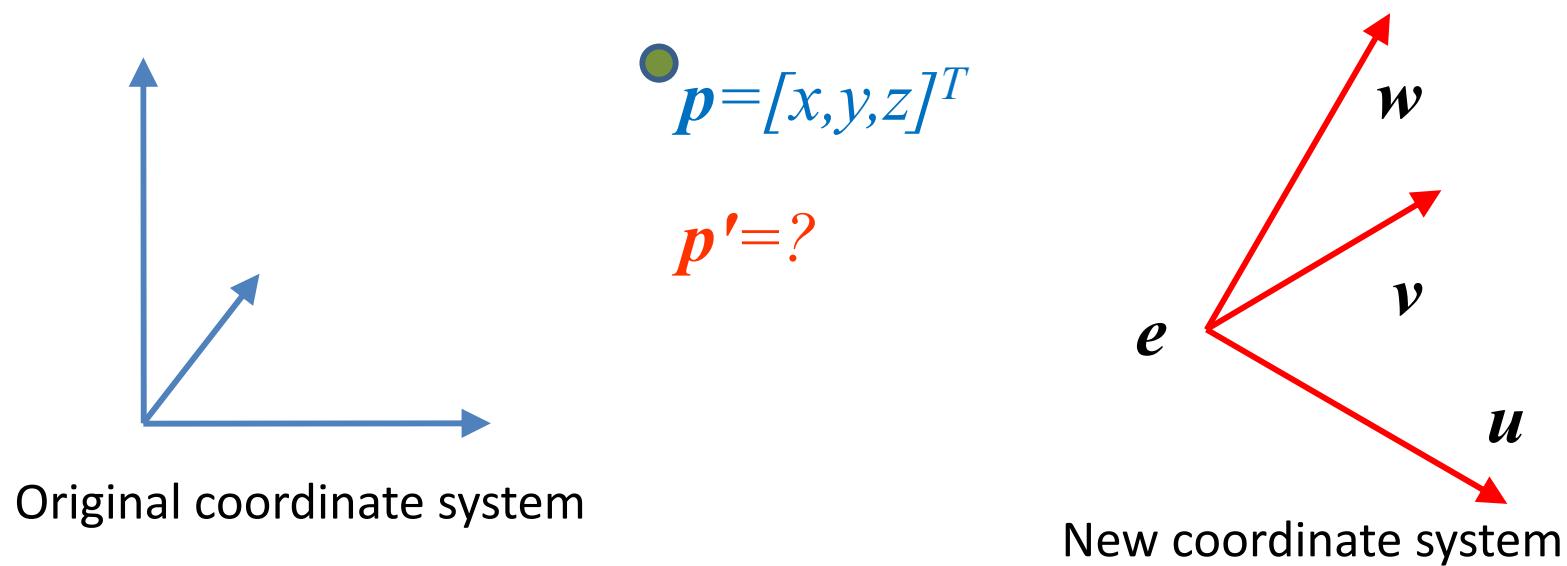
$$u = [u_1, u_2, u_3]^T$$

$$v = [v_1, v_2, v_3]^T$$

$$w = [w_1, w_2, w_3]^T$$

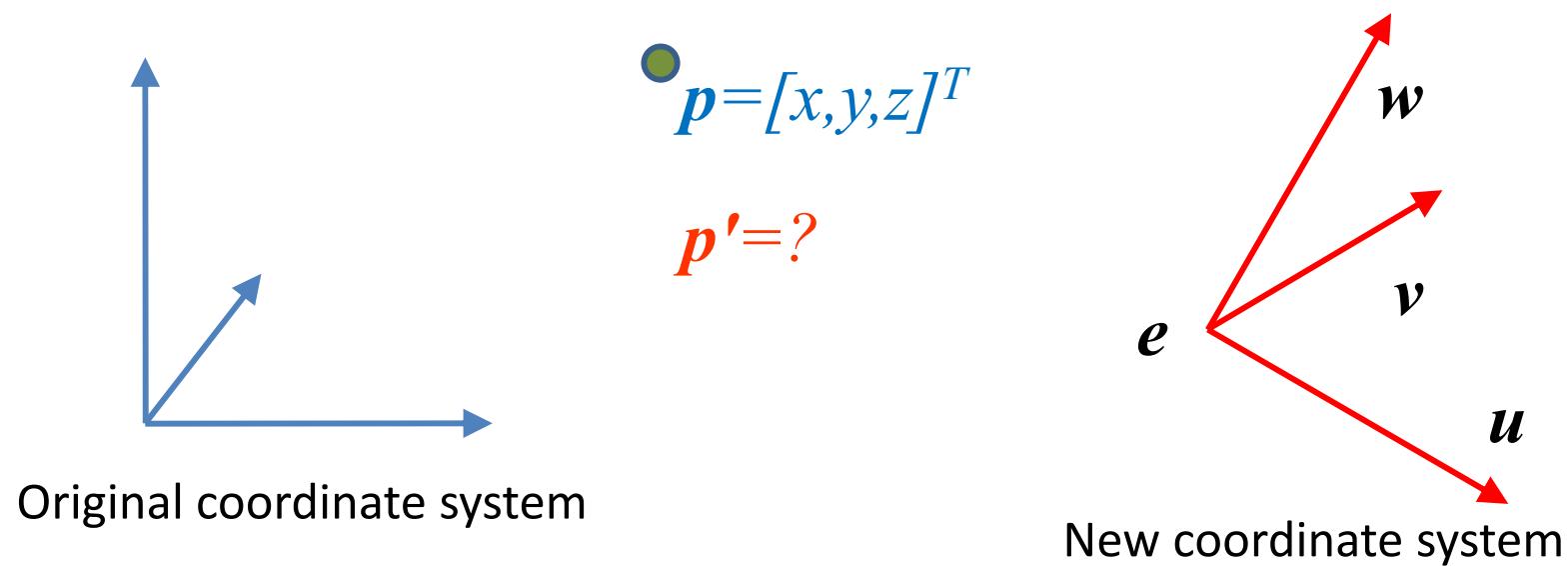
and  $e = [e_1, e_2, e_3]^T$  is the origin)

# Change of coordinate systems



$$\mathbf{p}' = \begin{bmatrix} u_1 & u_2 & u_3 & 0 \\ v_1 & v_2 & v_3 & 0 \\ w_1 & w_2 & w_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_1 \\ 0 & 1 & 0 & -e_2 \\ 0 & 0 & 1 & -e_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{p}$$

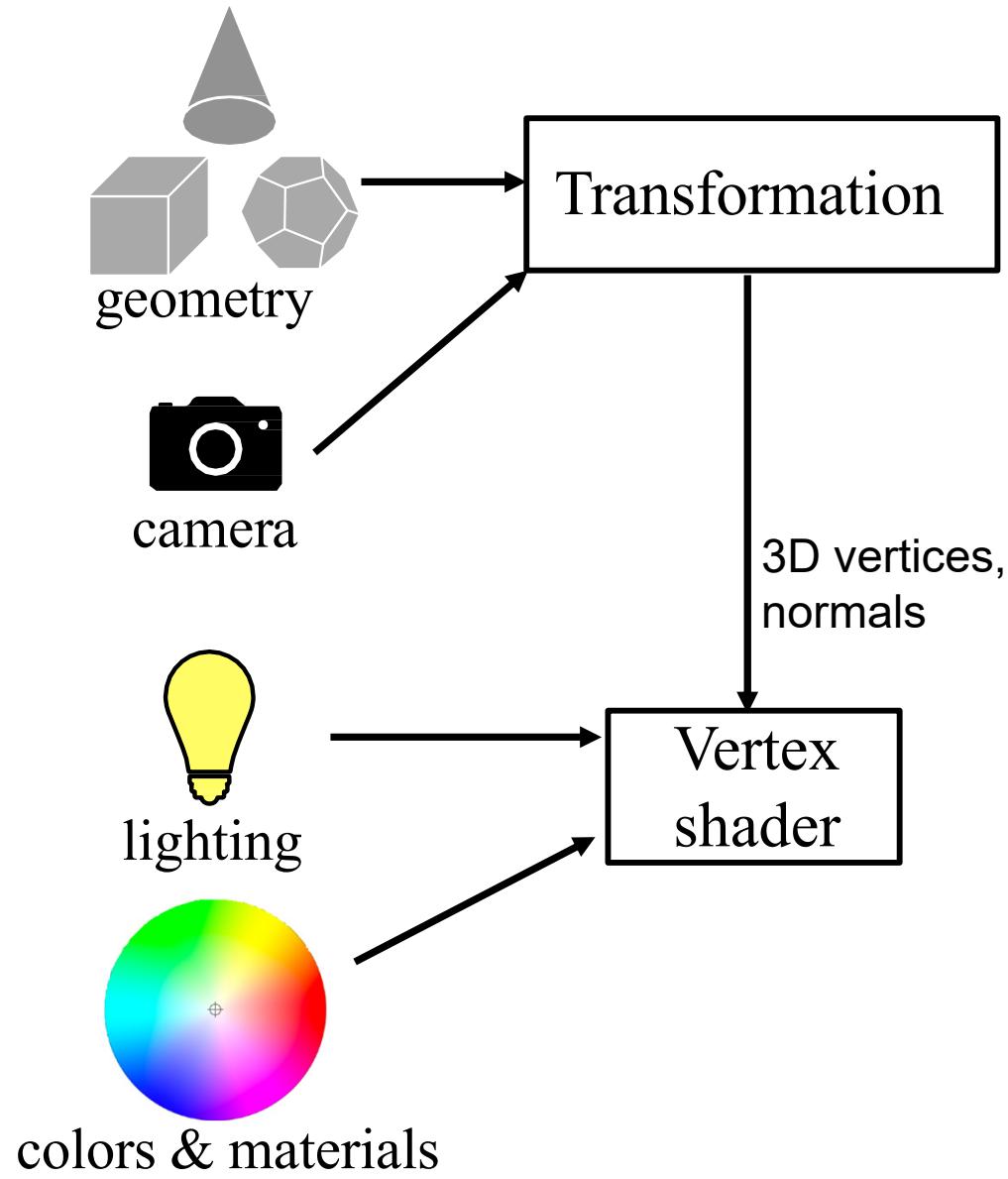
# Change of coordinate systems



$$\mathbf{p}' = \begin{bmatrix} u_1 & u_2 & u_3 & 0 \\ v_1 & v_2 & v_3 & 0 \\ w_1 & w_2 & w_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_1 \\ 0 & 1 & 0 & -e_2 \\ 0 & 0 & 1 & -e_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{p}$$

DIFFERENTIABLE

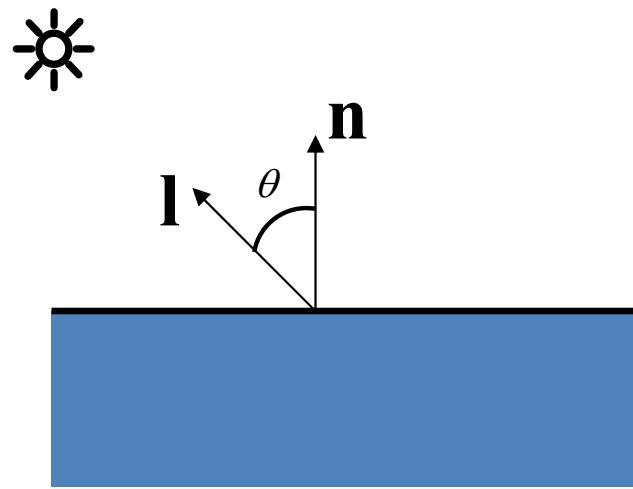
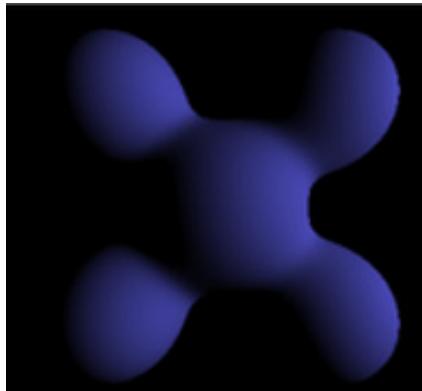
# Rasterization pipeline



# Diffuse / Lambertian Shading

Surface reflects light with equal intensity in all directions.

Obeys **Lambert's cosine law**: "*the amount of light the surface receives at a point depends on the angle between its normal and the vector from the surface point to the light*"

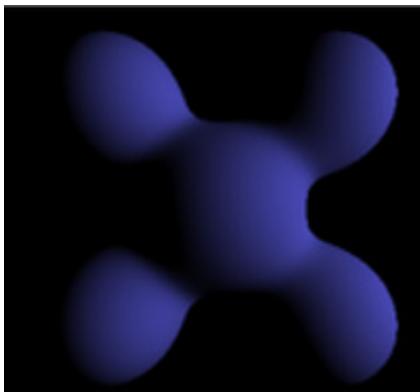


# Diffuse / Lambertian Shading

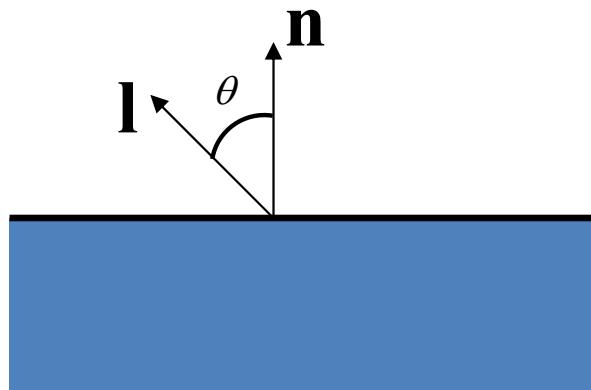
Surface reflects light with equal intensity in all directions.

Obeys **Lambert's cosine law**: "*the amount of light the surface receives at a point depends on the angle between its normal and the vector from the surface point to the light*"

$$C_{\text{diffuse}} = C \cdot \max(\mathbf{n} \bullet \mathbf{l}, 0)$$

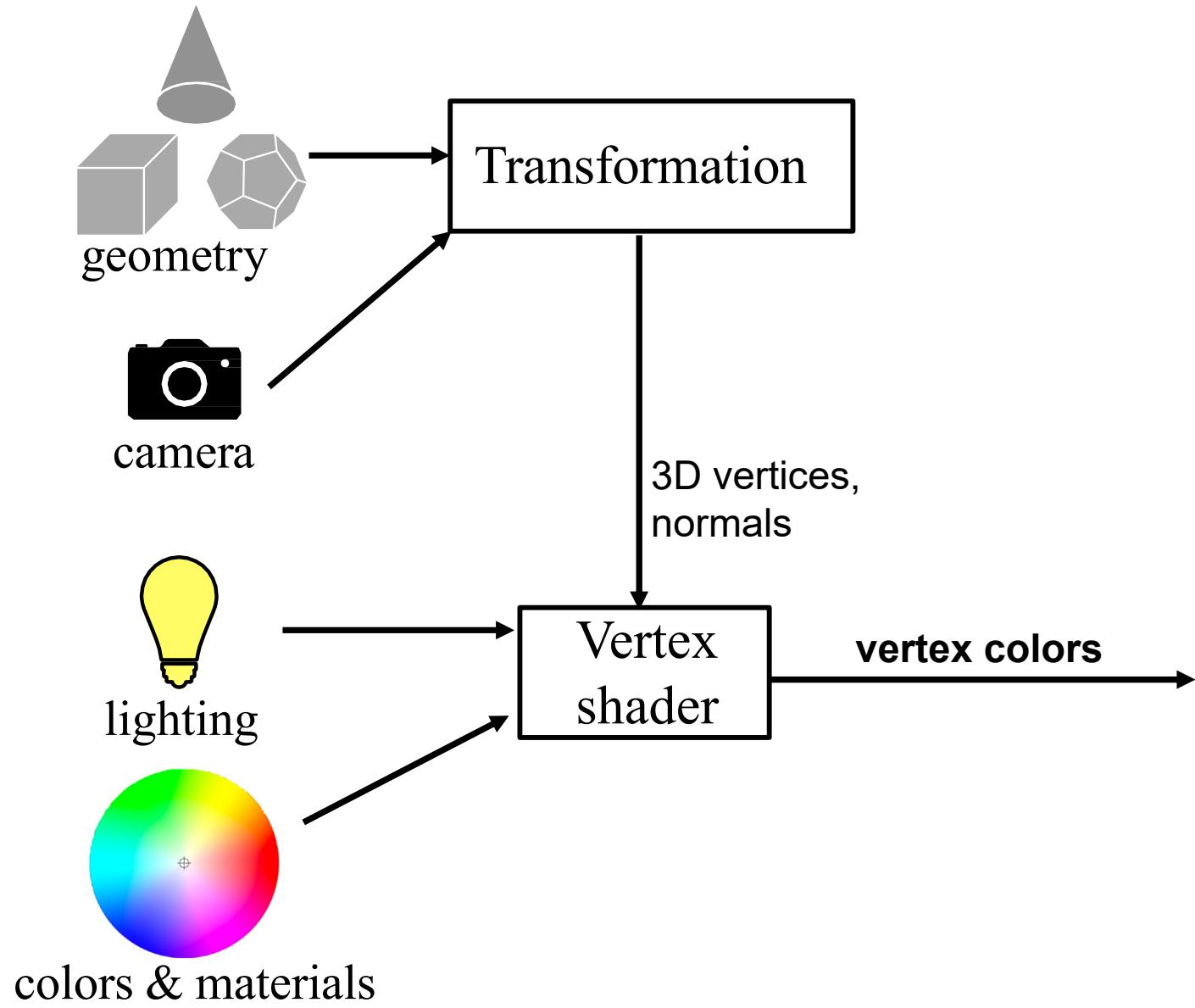


**C: surface color**

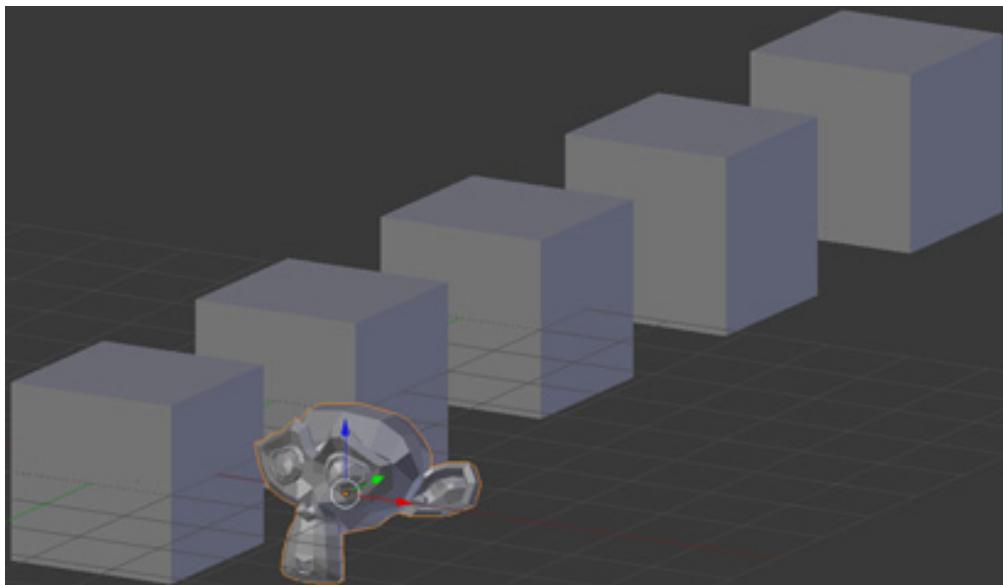


**DIFFERENTIABLE**

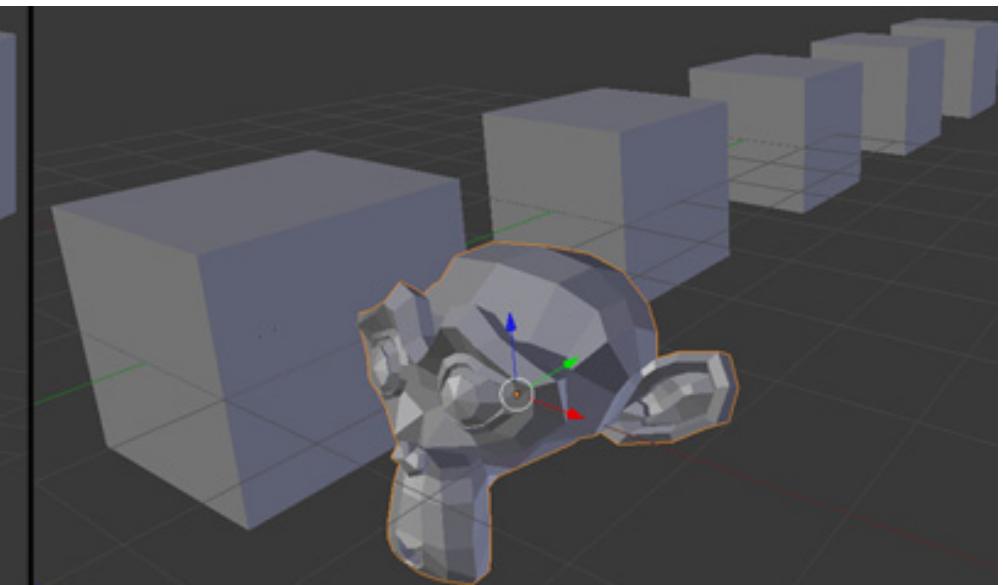
# Rasterization pipeline



# Two main types of projection



orthographic projection

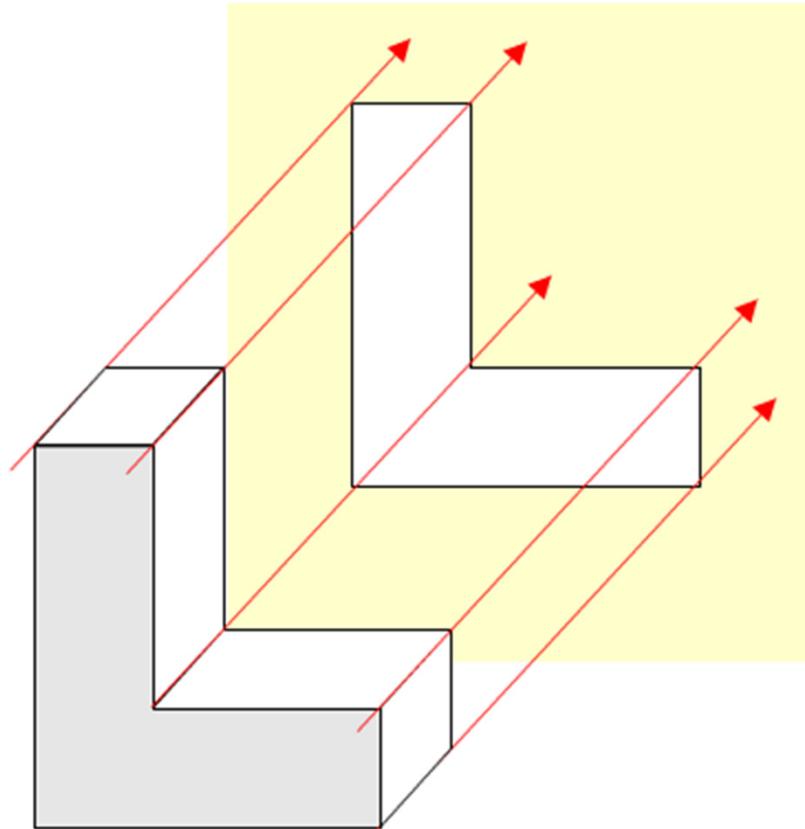


perspective projection

The **perspective projection** makes distant objects look smaller.

# Orthographic Projection

Project everything along the Z axis to the  $z=0$  plane.



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

# Orthographic Projection

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Orthographic Projection

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \\ 1 \end{bmatrix}$$

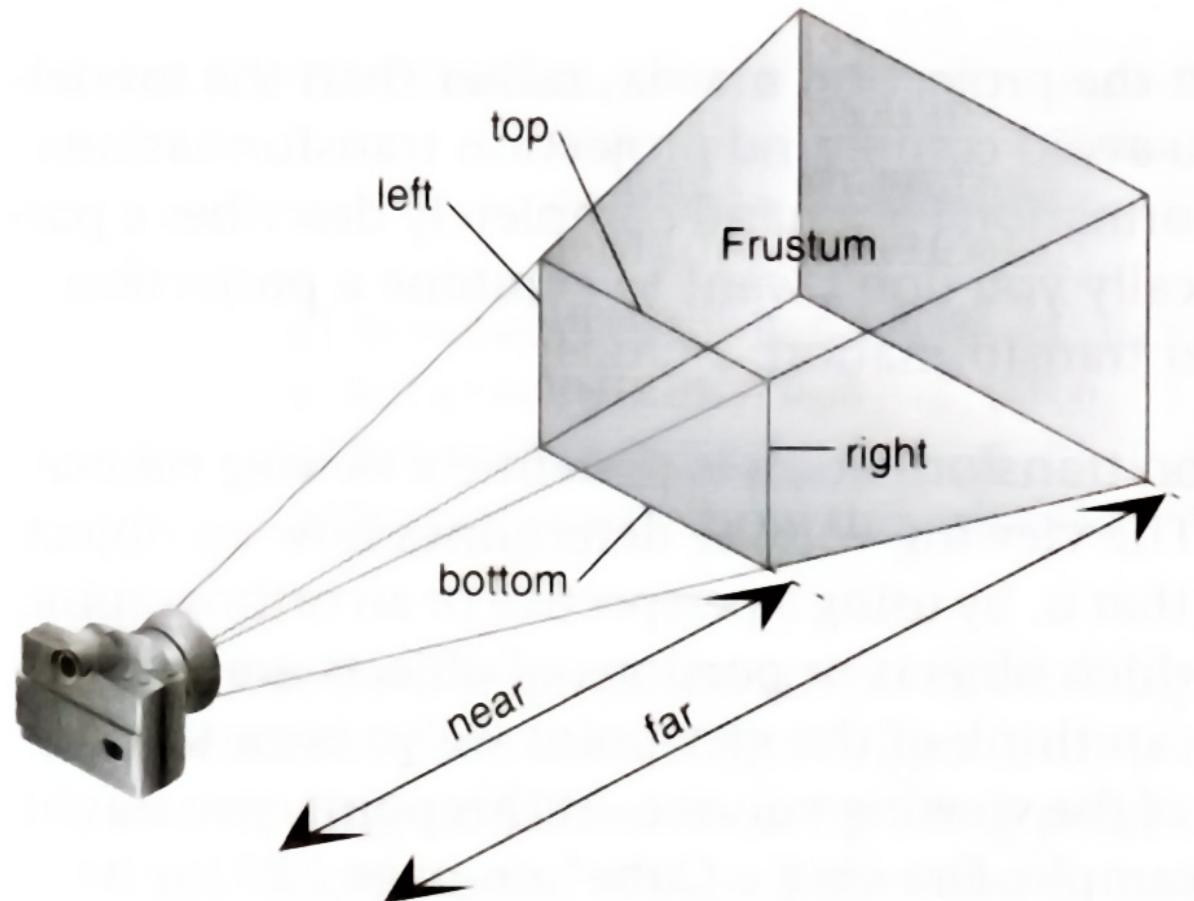
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$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \\ 1 \end{bmatrix}$$

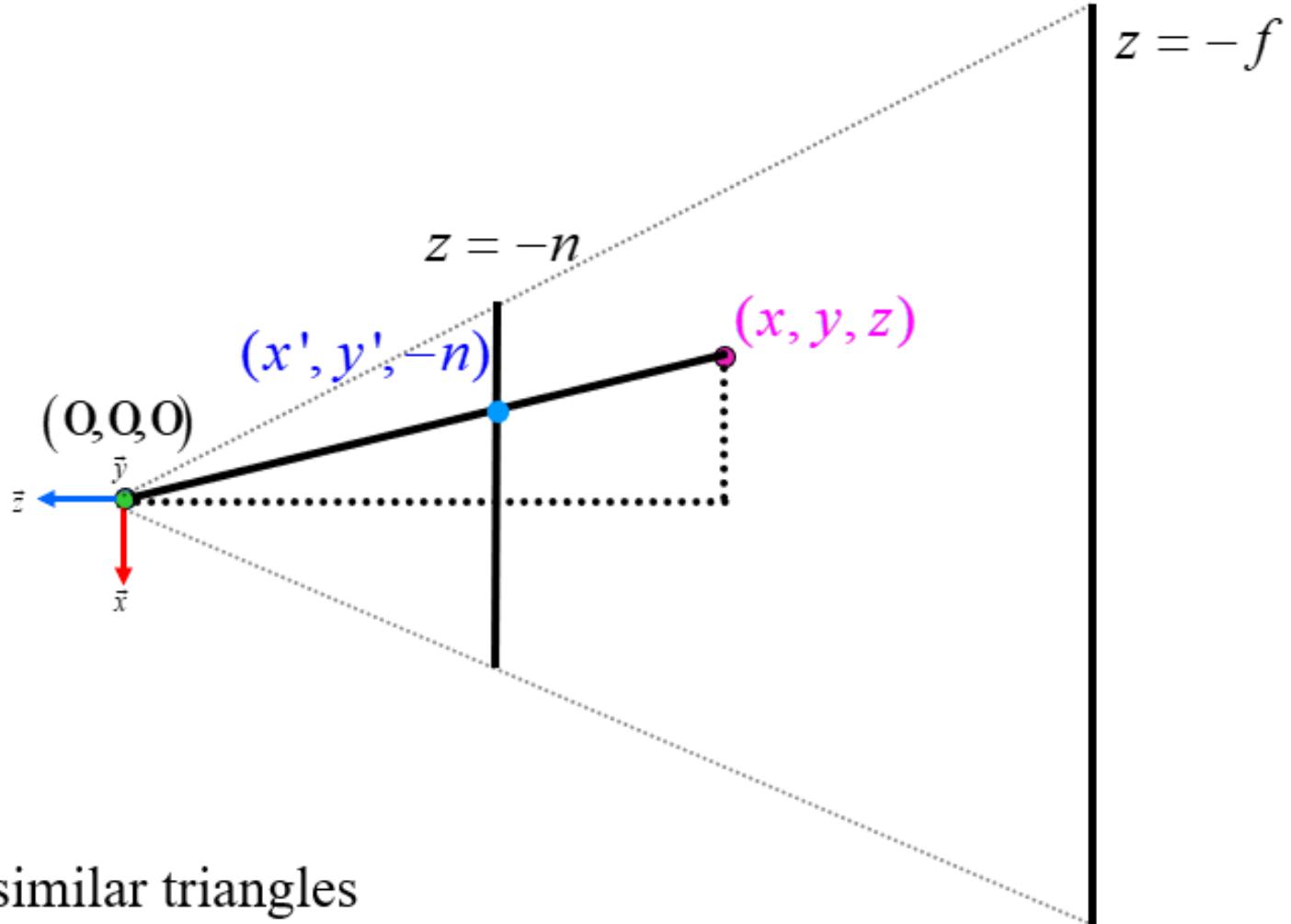
DIFFERENTIABLE

# Perspective Projection

Viewing volume has the shape of a **frustum**.



# Overhead View of Frustum - Perspective Projection



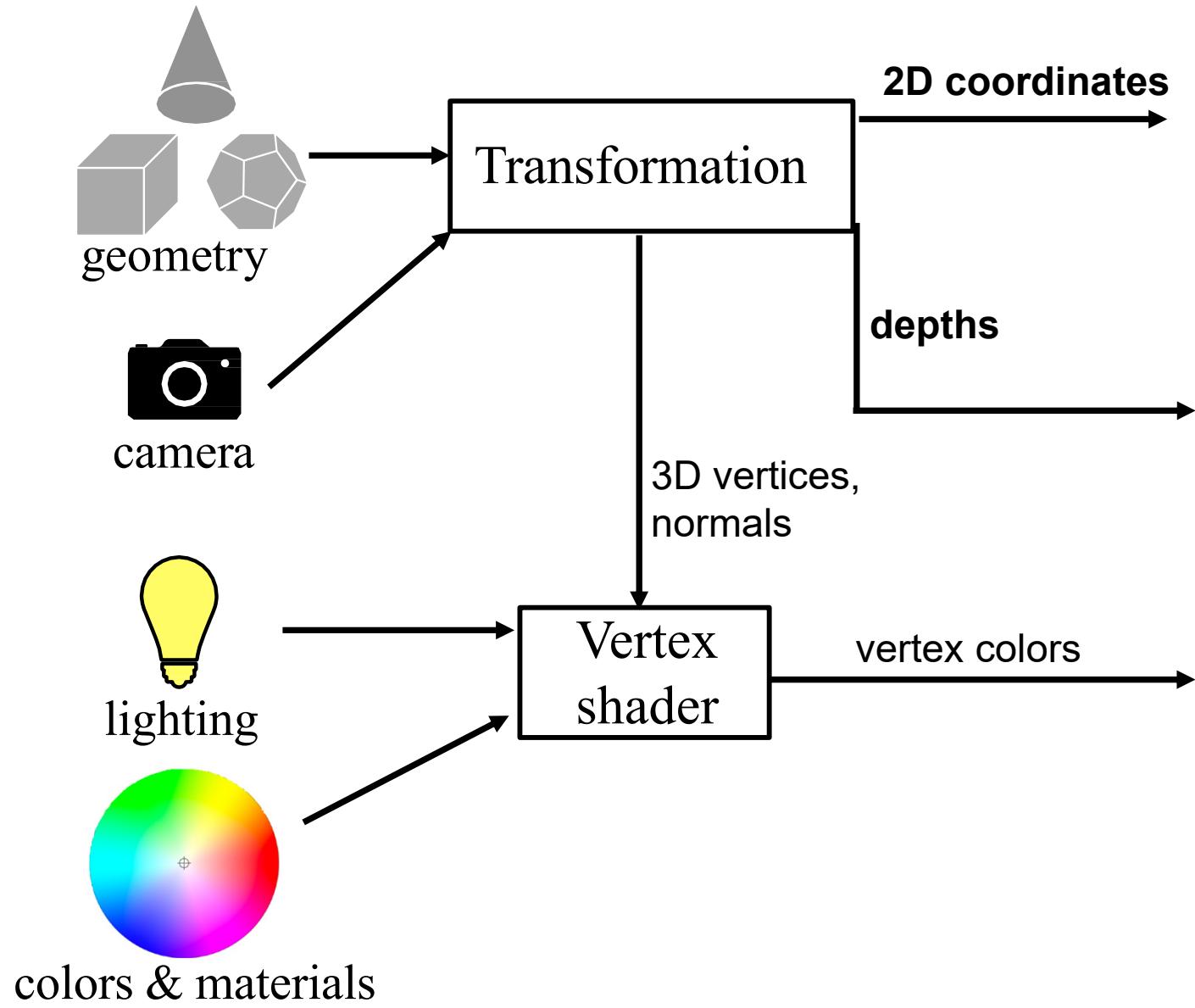
Use similar triangles

$$\frac{x}{z} = \frac{x'}{-n} \Rightarrow x' = -\frac{n \cdot x}{z}$$

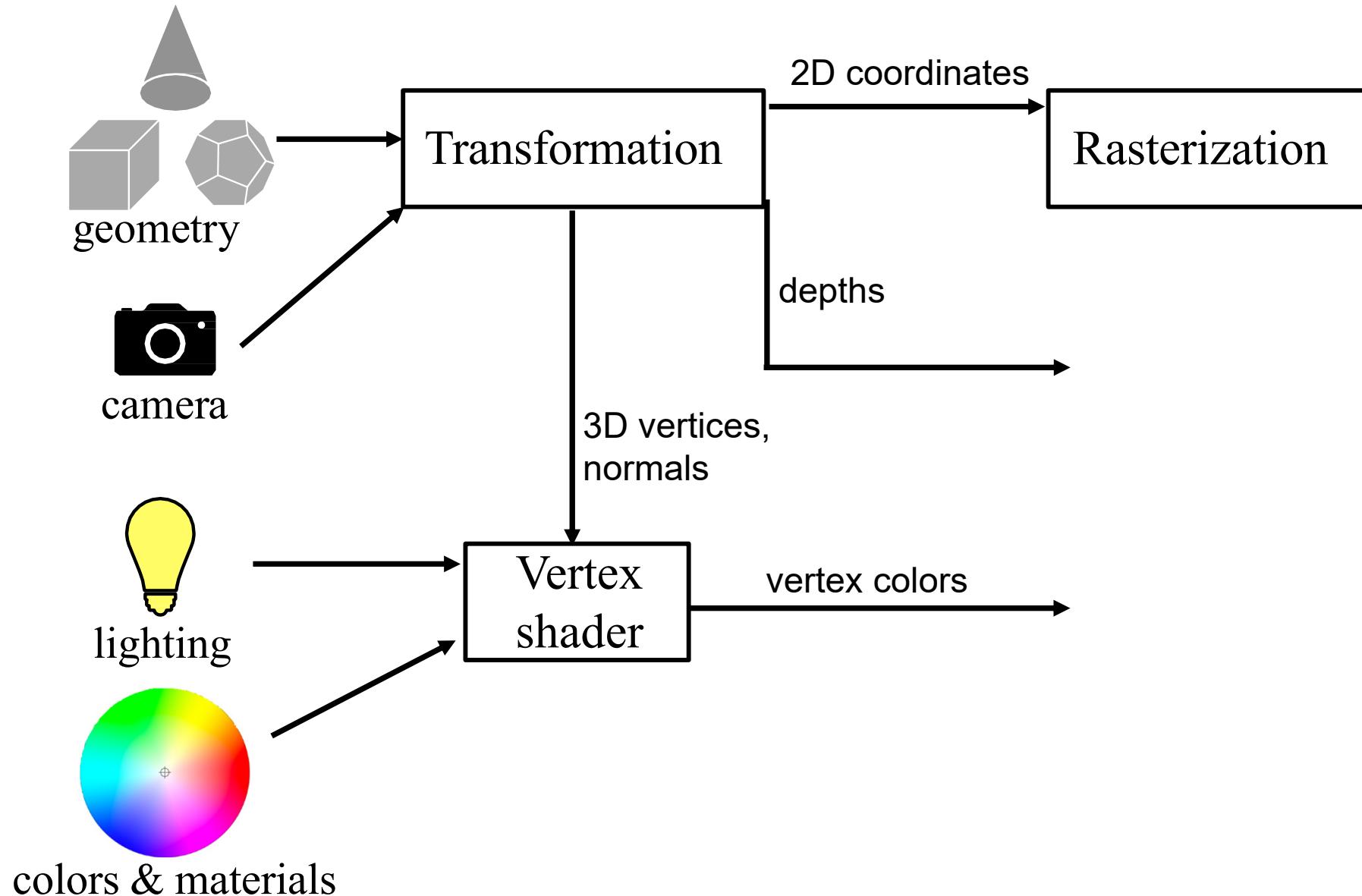
$$\frac{y}{z} = \frac{y'}{-n} \Rightarrow y' = -\frac{n \cdot y}{z}$$

DIFFERENTIABLE

# Rasterization pipeline

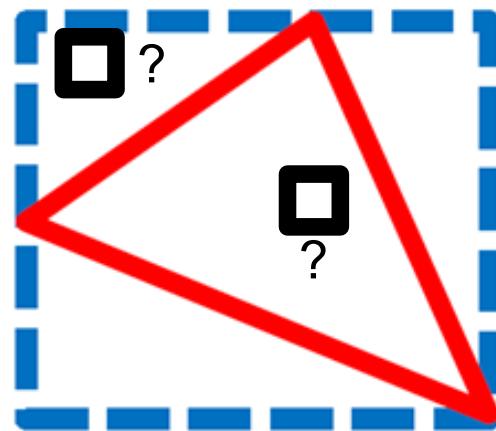


# Rasterization pipeline



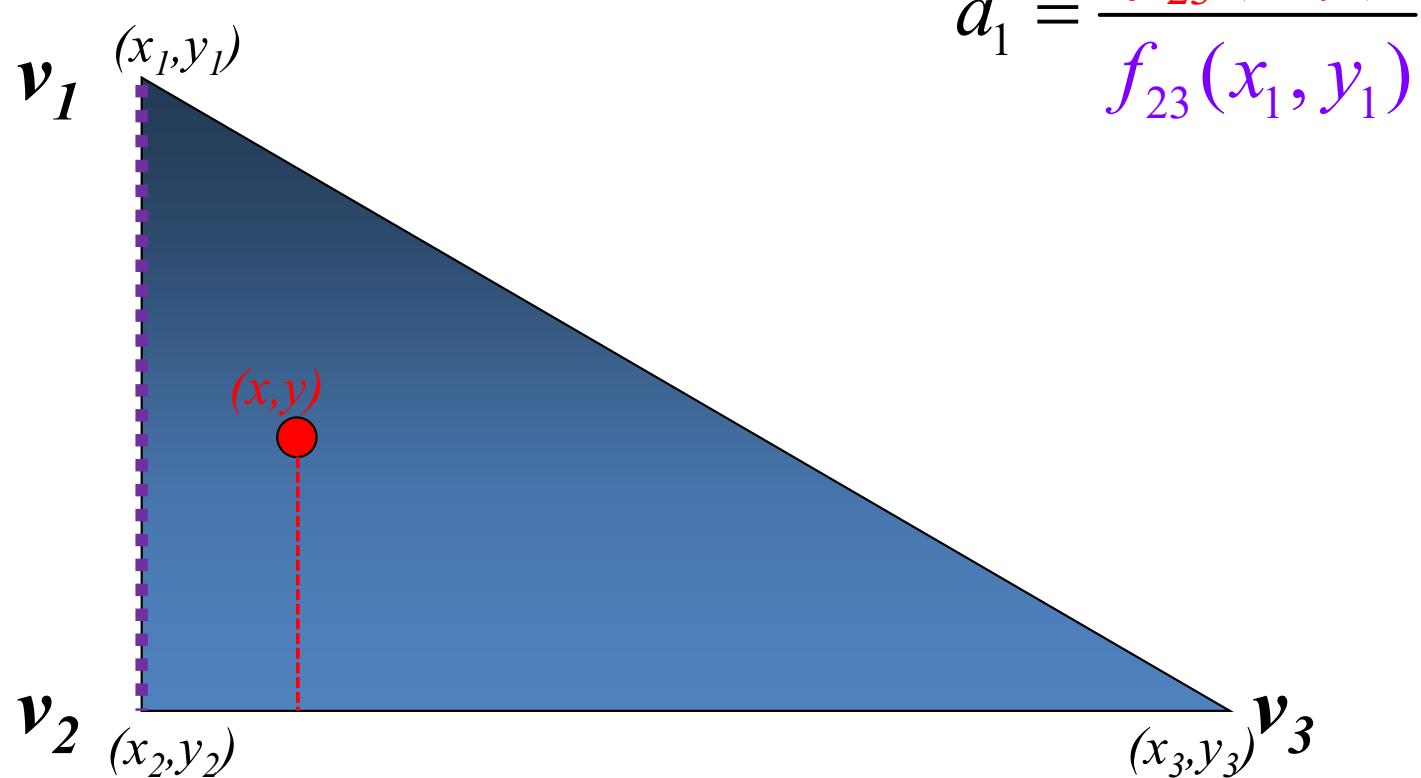
# Rasterization

Check if each pixel (its center point) is inside a triangle or not ... a **binary test**



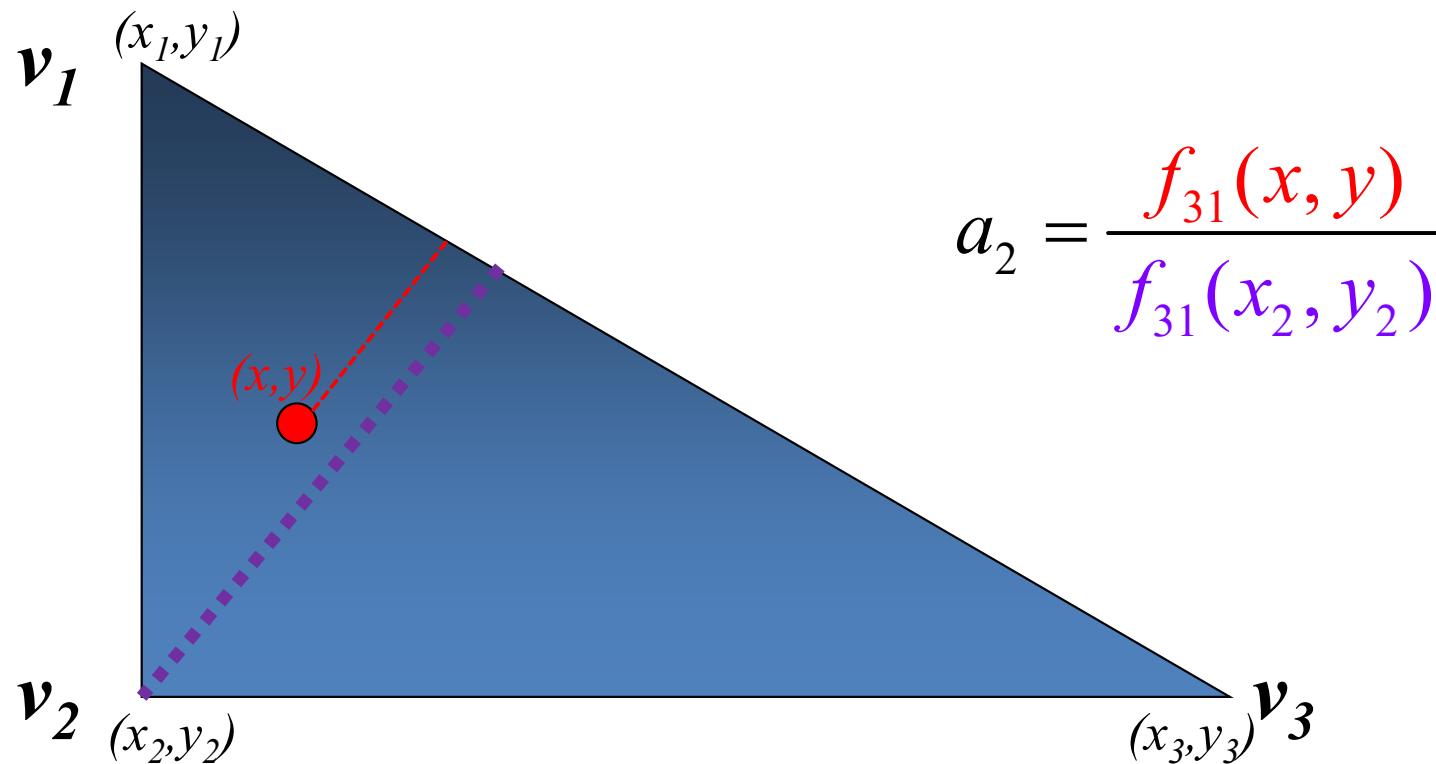
# Traditional rasterization

Check if a point is inside a triangle using **barycentric coordinates** and implicit line equations for edges:



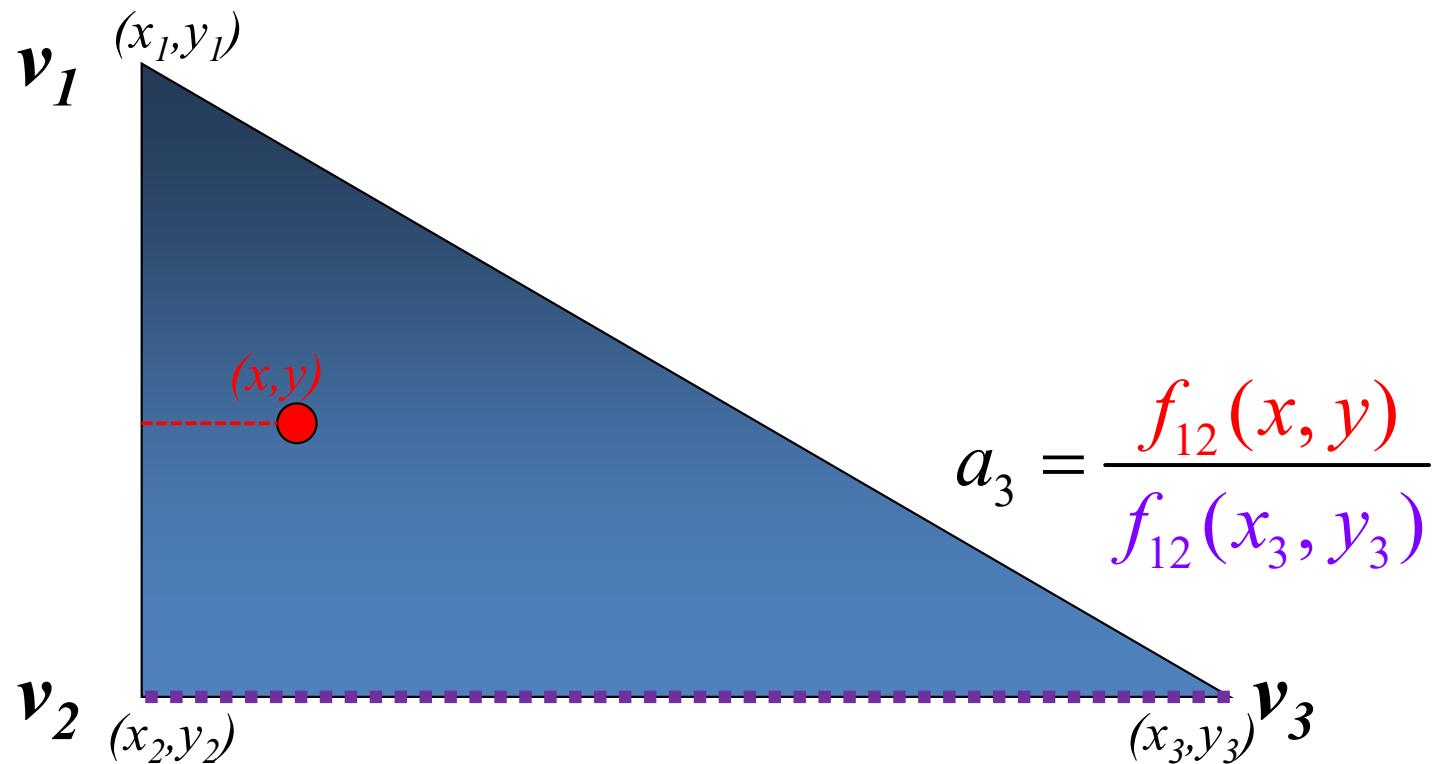
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Check if a point is inside a triangle using **barycentric coordinates** and implicit line equations for edges:



# Traditional rasterization

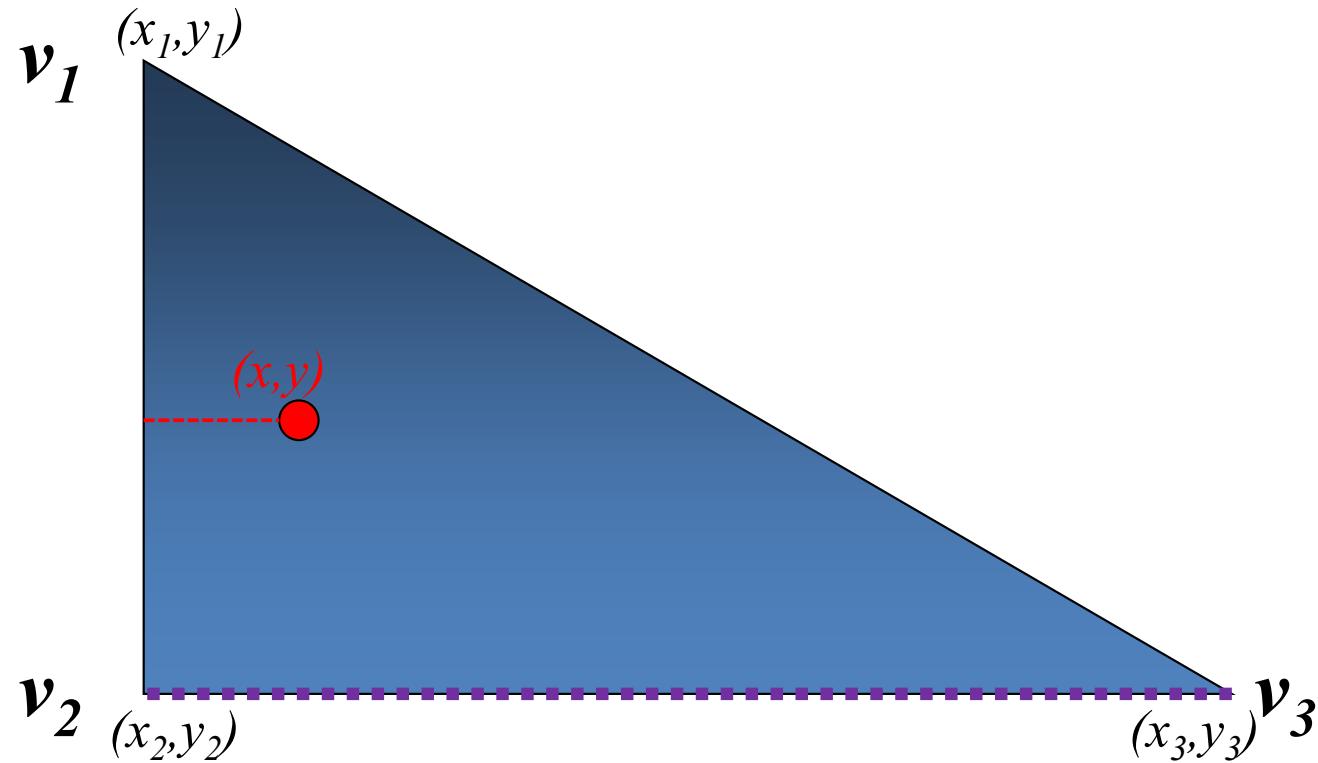
Check if a point is inside a triangle using **barycentric coordinates** and implicit line equations for edges:



# Traditional rasterization

If  $0 < a_1 < 1, 0 < a_2 < 1, 0 < a_3 < 1$

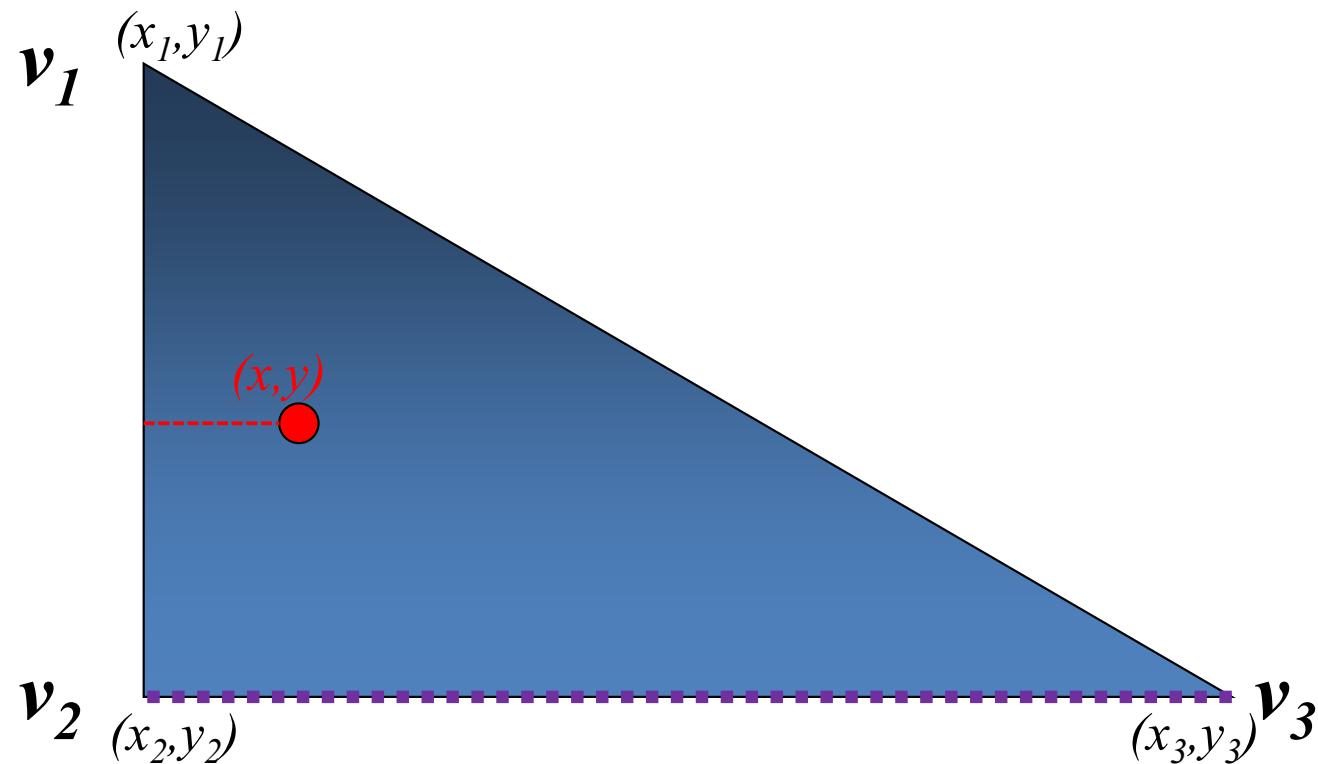
the pixel is inside (*send the fragment to the next stage!*)



# Traditional rasterization

If  $0 < a_1 < 1, 0 < a_2 < 1, 0 < a_3 < 1$

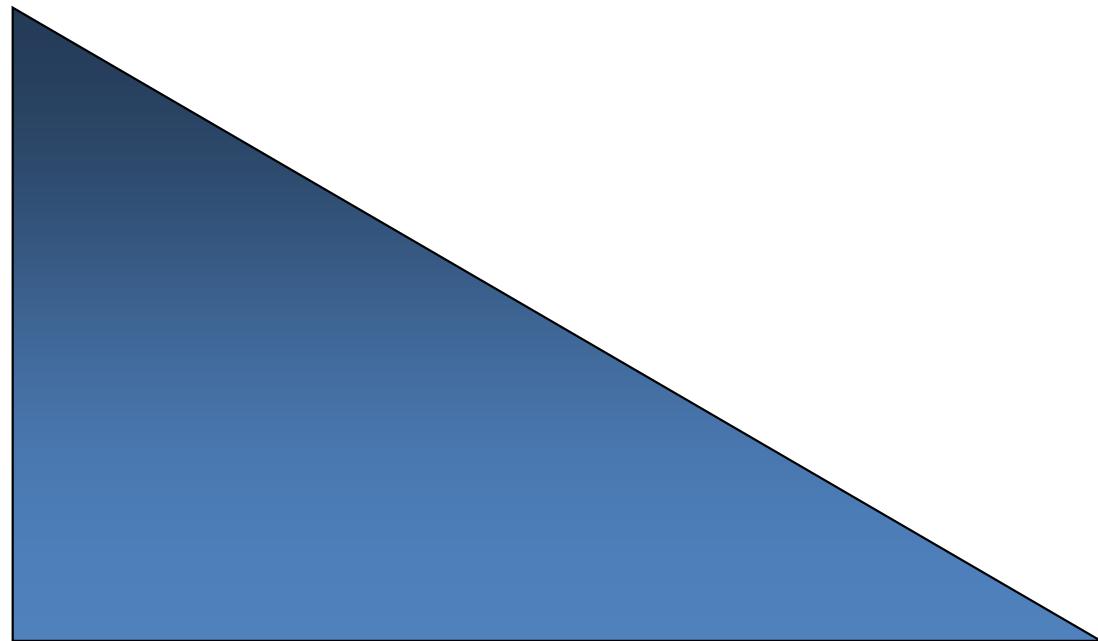
the pixel is inside (*send the fragment to the next stage!*)



**NON-DIFFERENTIABLE**

# Soft Rasterizer

Instead of making a hard decision (pixel belongs to the triangle or not), **soften it!**



# Soft Rasterizer

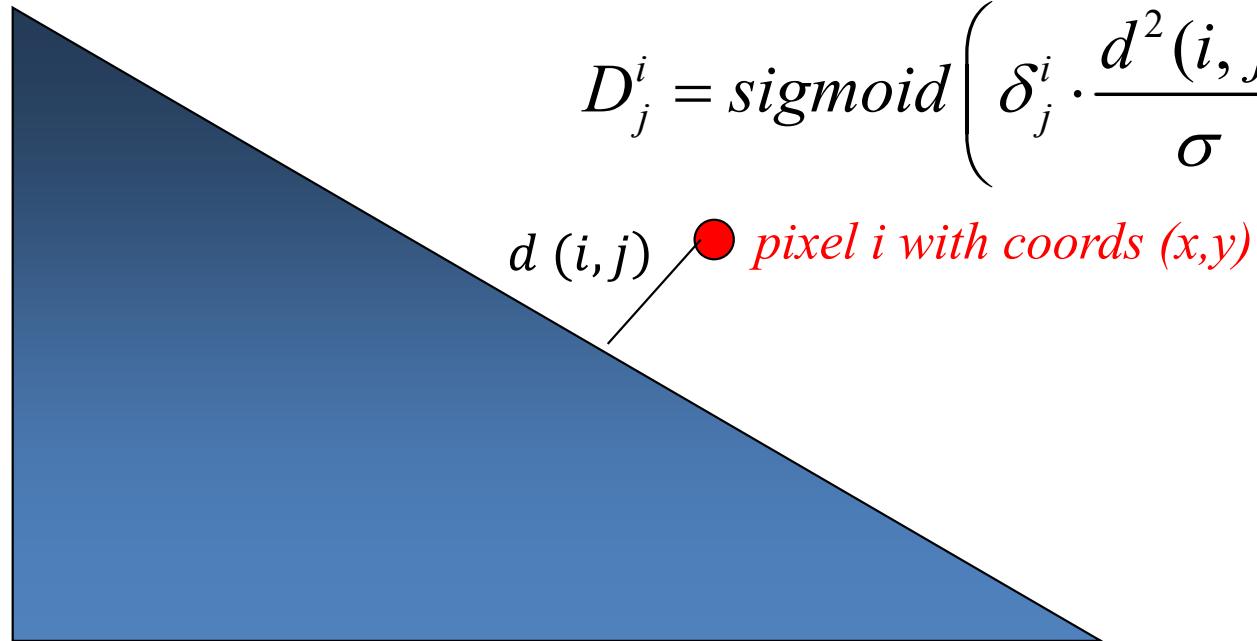
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# Soft Rasterizer

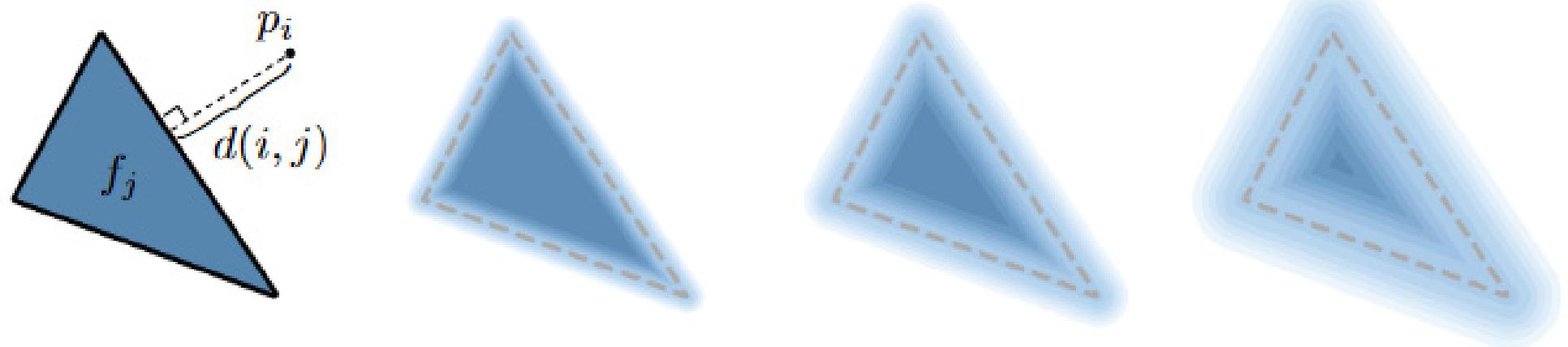
Given the Euclidean distance  $d(i,j)$  of pixel  $i$  to triangle  $j$ , a binary indicator  $\delta$  (values 1 / -1) returning whether the pixel belongs to the triangle or not, and a hyperparameter  $\sigma$ :

$$D_j^i = \text{sigmoid} \left( \delta_j^i \cdot \frac{d^2(i,j)}{\sigma} \right)$$

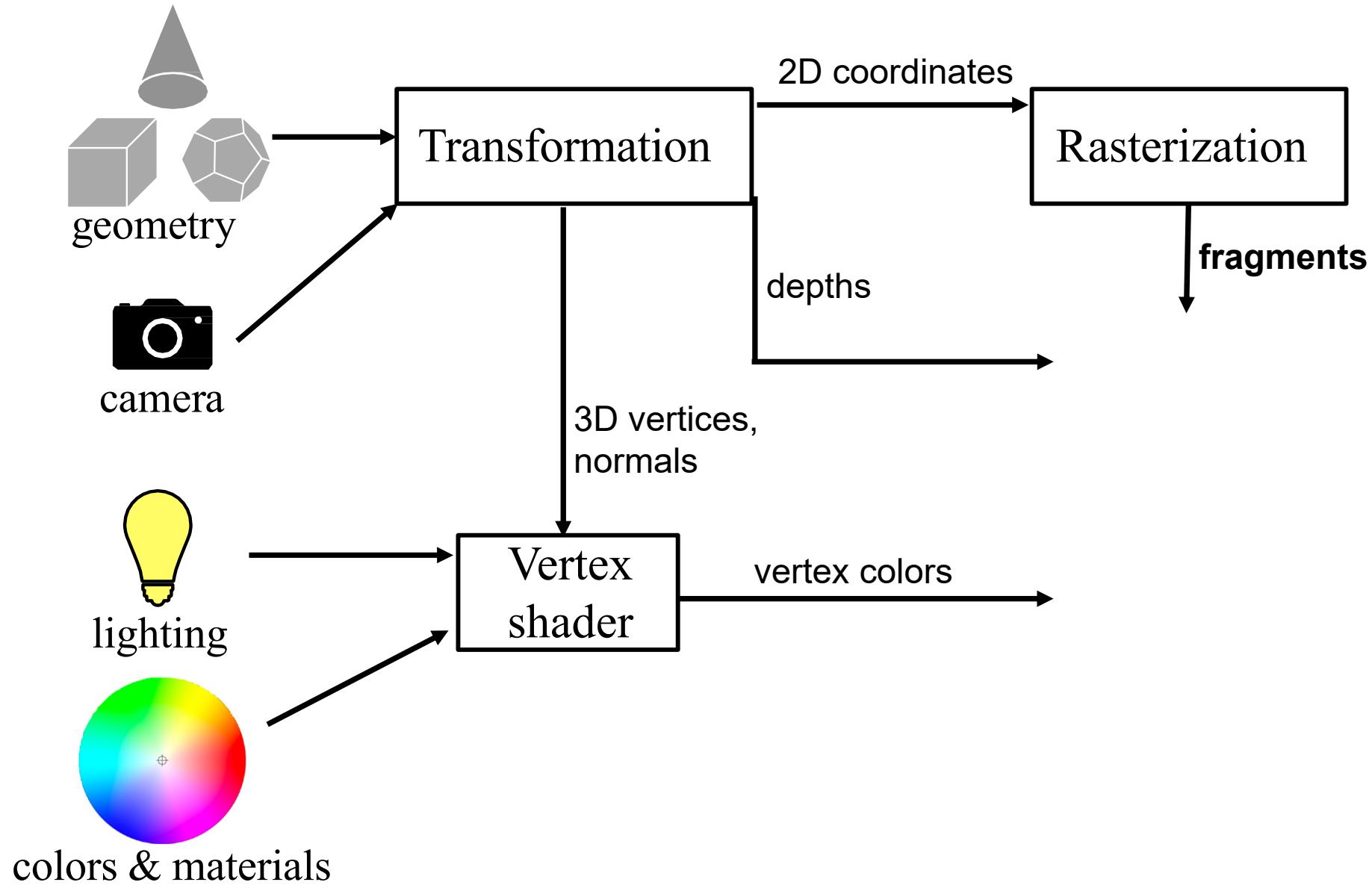


# Soft Rasterizer

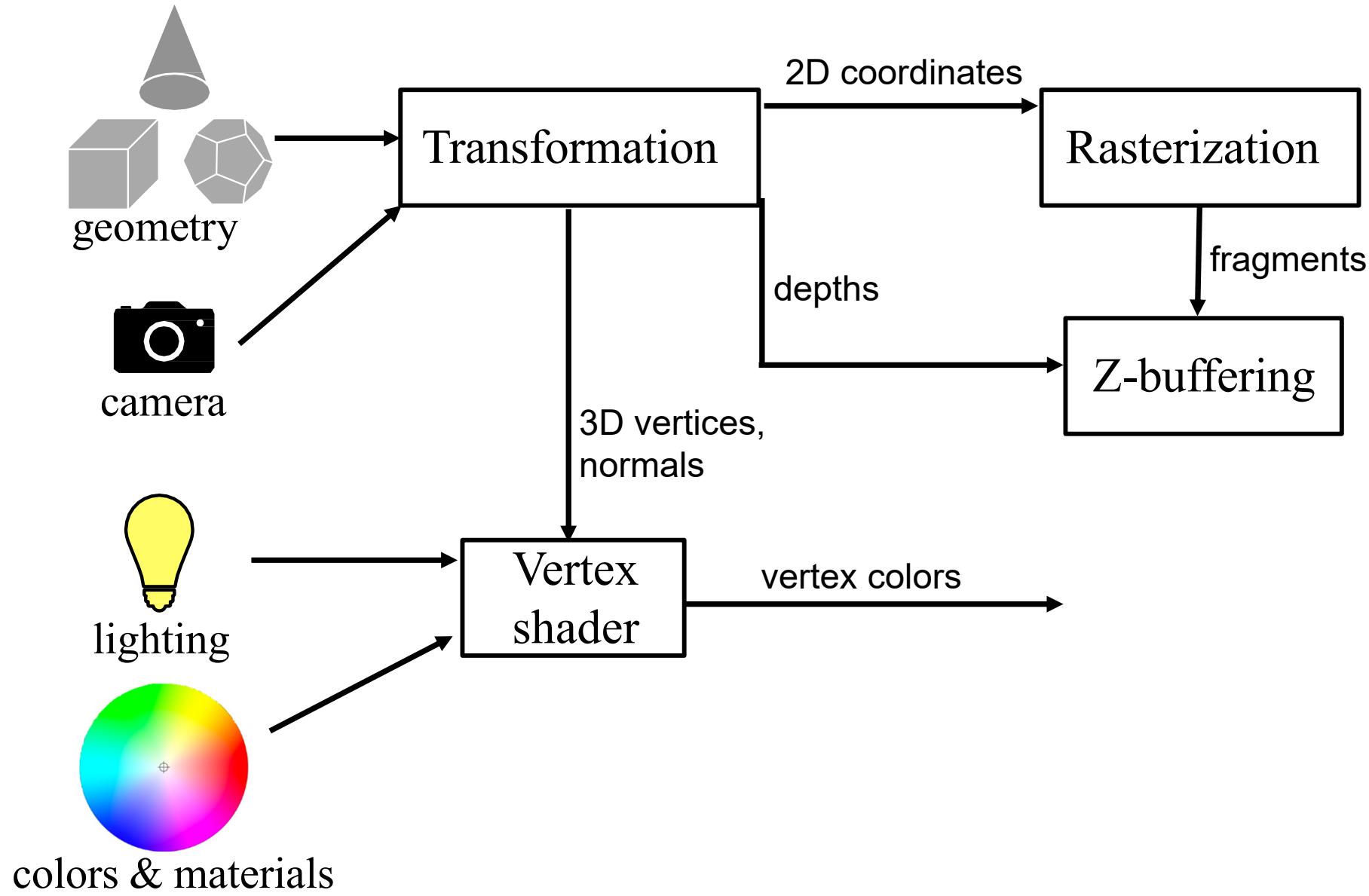
Effect of hyperparameter  $\sigma$ :



# Rasterization pipeline



# Rasterization pipeline



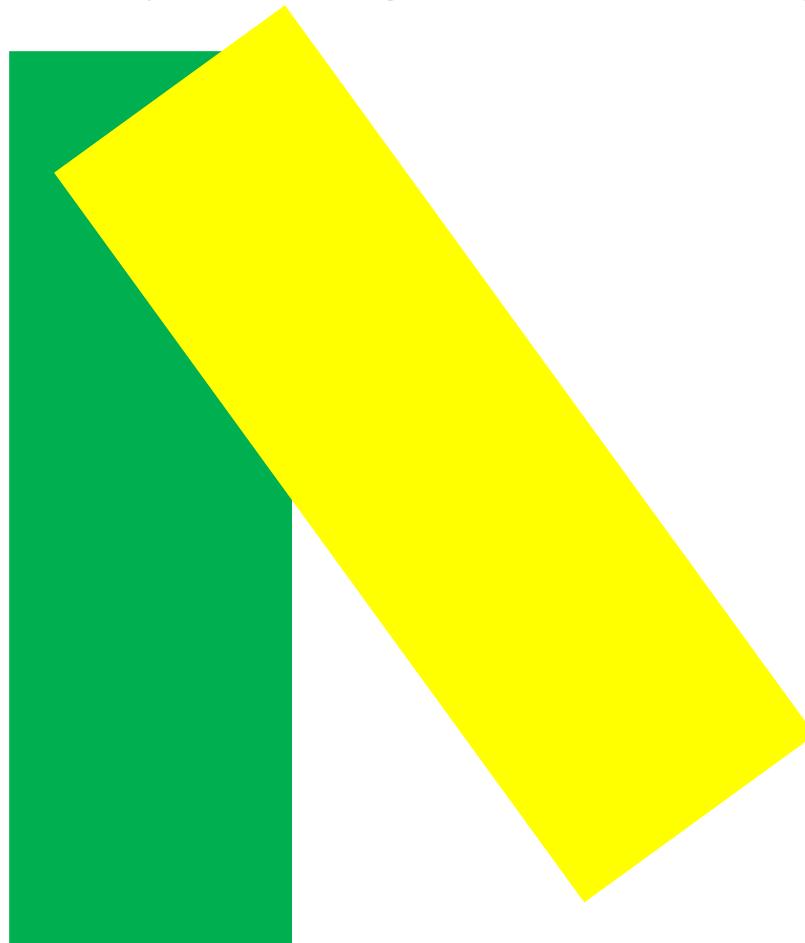
# Depth test

After rasterizing each polygon, we obtain **fragments** (pixels + depth + colors)... we need to decide which fragments are in front of others depending on their depth



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After rasterizing each polygon, we obtain **fragments** (pixels + depth + colors)... we need to decide which fragments are in front of others depending on their depth



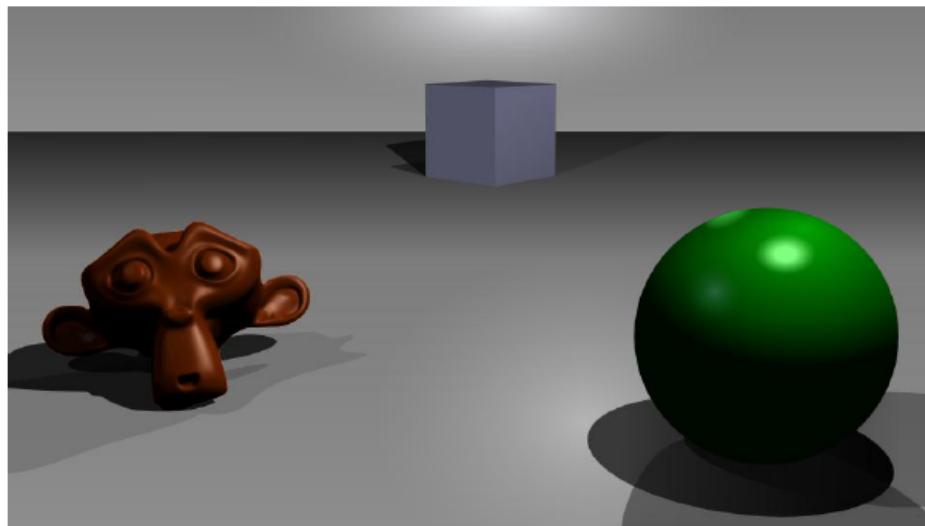
# Depth test

After rasterizing each polygon, we obtain **fragments** (pixels + depth + colors)... we need to decide which fragments are in front of others depending on their depth



# The Z-buffer (or depth buffer)

**Stores depth for each fragment [we *do not throw out the z-coordinate* during our pipeline]**



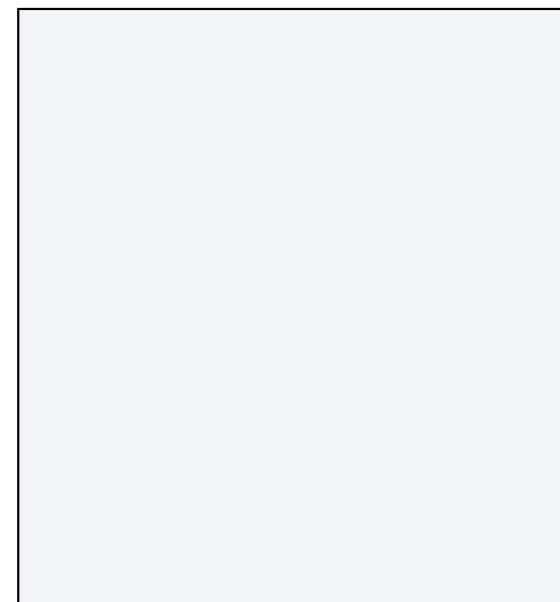
**Color buffer**



**Depth buffer**

# Z-Buffer Algorithm

$\infty$							
$\infty$							
$\infty$							
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$\infty$							
$\infty$							

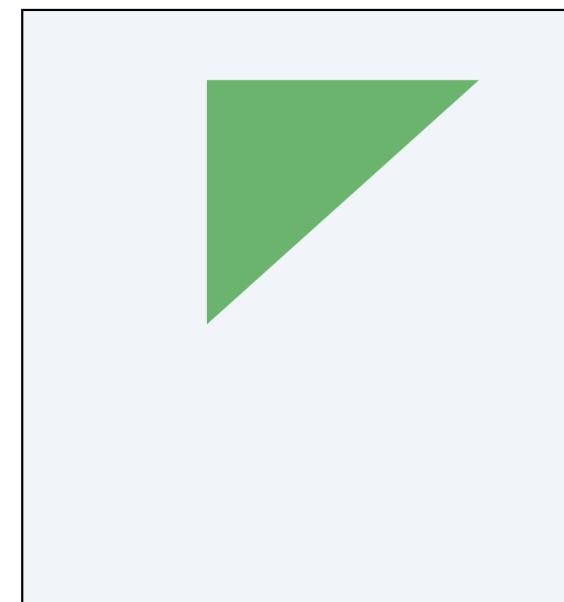


# Z-Buffer Algorithm

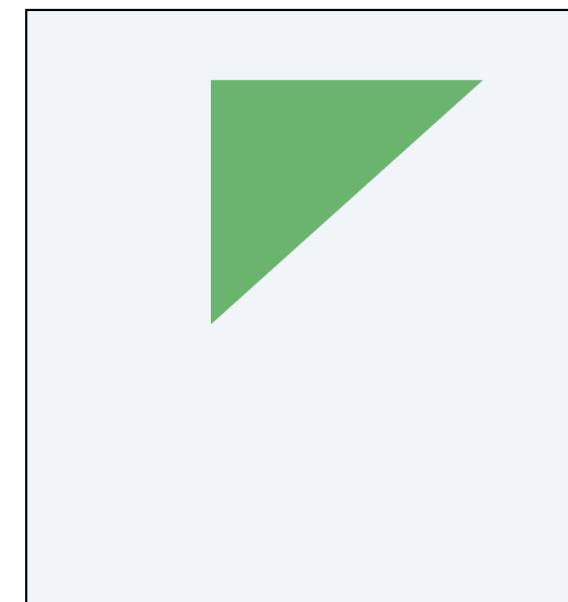
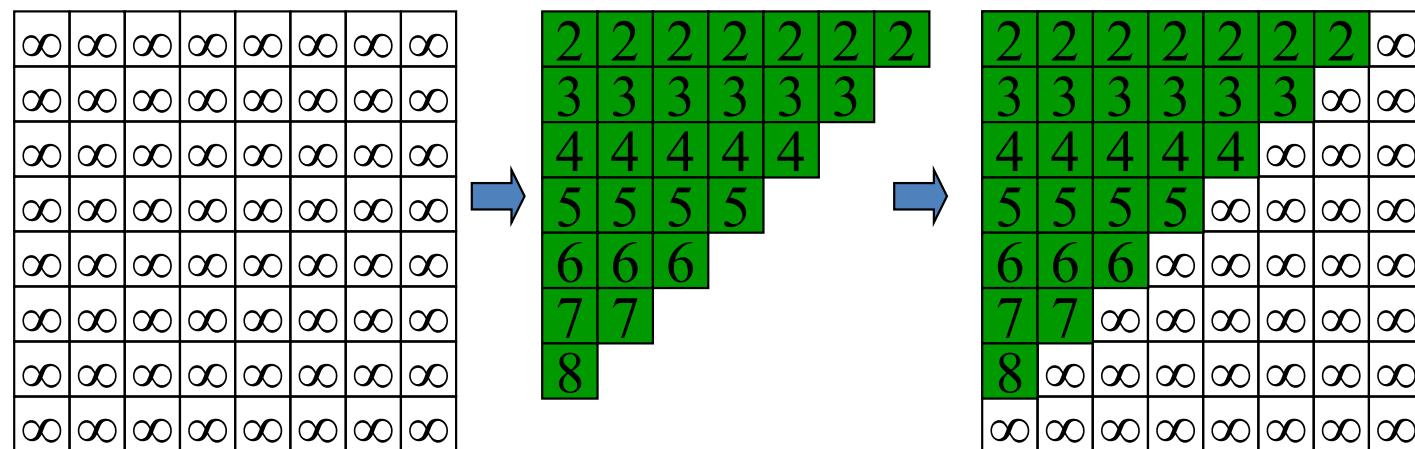
$\infty$								
$\infty$								
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$\infty$								
$\infty$								
$\infty$								
$\infty$								
$\infty$								
$\infty$								



2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	
5	5	5	5	5		
6	6	6				
7	7					
8						



# Z-Buffer Algorithm



# Z-Buffer Algorithm

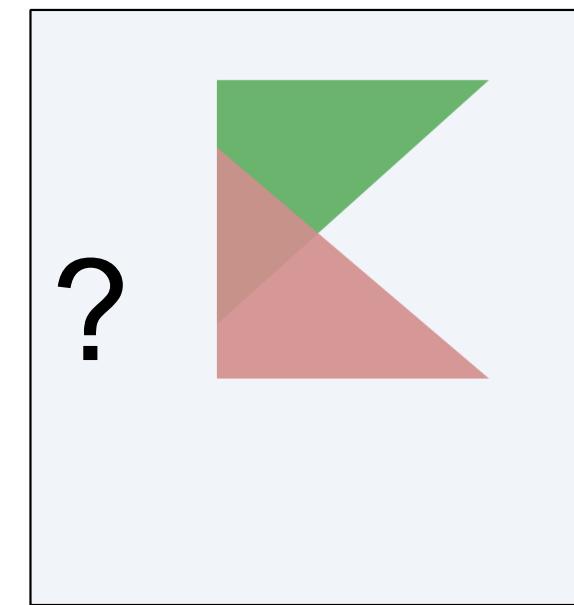
$\infty$								
$\infty$								
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$\infty$								
$\infty$								
$\infty$								
$\infty$								
$\infty$								

2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	$\infty$
4	4	4	4	4	4	$\infty$	$\infty$
5	5	5	5	5	$\infty$	$\infty$	$\infty$
6	6	6	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
7	$\infty$						
8	$\infty$						

2	2	2	2	2	2	2	2
3	3	3	3	3	3	$\infty$	$\infty$
4	4	4	4	4	$\infty$	$\infty$	$\infty$
5	5	5	5	$\infty$	$\infty$	$\infty$	$\infty$
6	6	6	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
7	$\infty$						
8	$\infty$						
$\infty$							

2	2	2	2	2	2	2	2
3	3	3	3	3	3	$\infty$	$\infty$
4	4	4	4	4	$\infty$	$\infty$	$\infty$
5	5	5	5	$\infty$	$\infty$	$\infty$	$\infty$
6	6	6	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
7	7	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
8	$\infty$						
$\infty$							

8
7
6
5
4
3
2

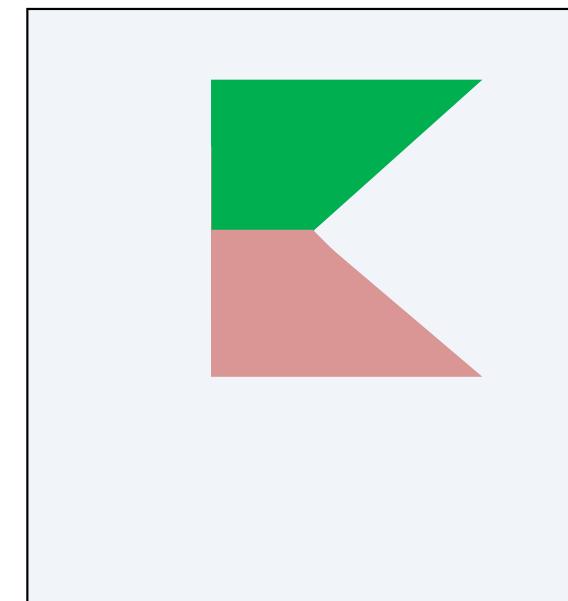


# Z-Buffer Algorithm

$\infty$								
$\infty$								
$\infty$								
$\infty$								
$\infty$								
$\infty$								
$\infty$								
$\infty$								

2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	$\infty$
4	4	4	4	4	4	4	$\infty$
5	5	5	5	5	5	5	$\infty$
6	6	6	6	6	6	6	$\infty$
7	7	7	7	7	7	7	$\infty$
8							$\infty$

2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	$\infty$
4	4	4	4	4	4	4	$\infty$
5	5	5	5	5	5	5	$\infty$
6	6	6	6	6	6	6	$\infty$
7	7	7	7	7	7	7	$\infty$
8	$\infty$						
$\infty$							



2	2	2	2	2	2	2	2	$\infty$
3	3	3	3	3	3	3	$\infty$	$\infty$
4	4	4	4	4	4	$\infty$	$\infty$	$\infty$
5	5	5	5	5	$\infty$	$\infty$	$\infty$	$\infty$
6	6	6	6	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
7	7	$\infty$						
8	$\infty$							
$\infty$								

8								
7	7							
6	6	6						
5	5	5	5					
4	4	4	4	4	4			
3	3	3	3	3	3	3		
2	2	2	2	2	2	2	2	

2	2	2	2	2	2	2	2	$\infty$
3	3	3	3	3	3	3	$\infty$	$\infty$
4	4	4	4	4	4	4	$\infty$	$\infty$
5	5	5	5	5	5	5	$\infty$	$\infty$
6	6	6	6	6	6	6	$\infty$	$\infty$
7	7	7	7	7	7	7	$\infty$	$\infty$
8	8	8	8	8	8	8	$\infty$	$\infty$
3	3	3	3	3	3	3	3	$\infty$
2	2	2	2	2	2	2	2	$\infty$

# The Z-Buffer Algorithm [Z-test]

**For each rasterized primitive j (triangle, lines)**

Is the **depth**  $z_i^j$  of fragment i about to be drawn **greater** than the **recorded depth** in the z-buffer?

**If yes, throw it away!**

**Otherwise, draw pixel and update depth!**

```
if (  $z_i^j < z\_buffer[x, y]$  )
{
    z_buffer[x,y]=  $z_i^j$ ;
    I[x,y]=c[x,y];
}
```

# The Z-Buffer Algorithm [Z-test]

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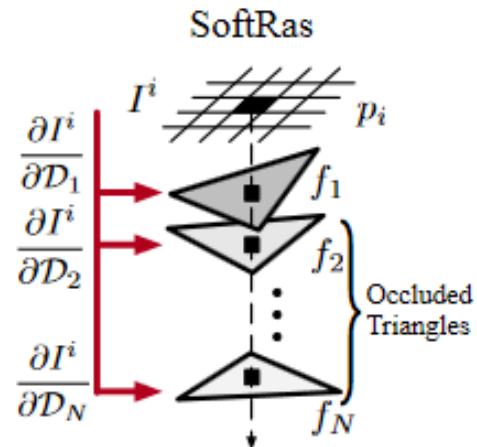
**NON-DIFFERENTIABLE**

# Soft Rasterizer

Given the Euclidean distance  $d(i,j)$  of pixel  $i$  to triangle  $j$ , a binary indicator  $\delta$  (values -1 / 1) returning whether the pixel belongs to the triangle or not, and a hyperparameter  $\sigma$ :

$$D_j^i = \text{sigmoid}\left(\delta_j^i \cdot \frac{d^2(i, j)}{\sigma}\right)$$

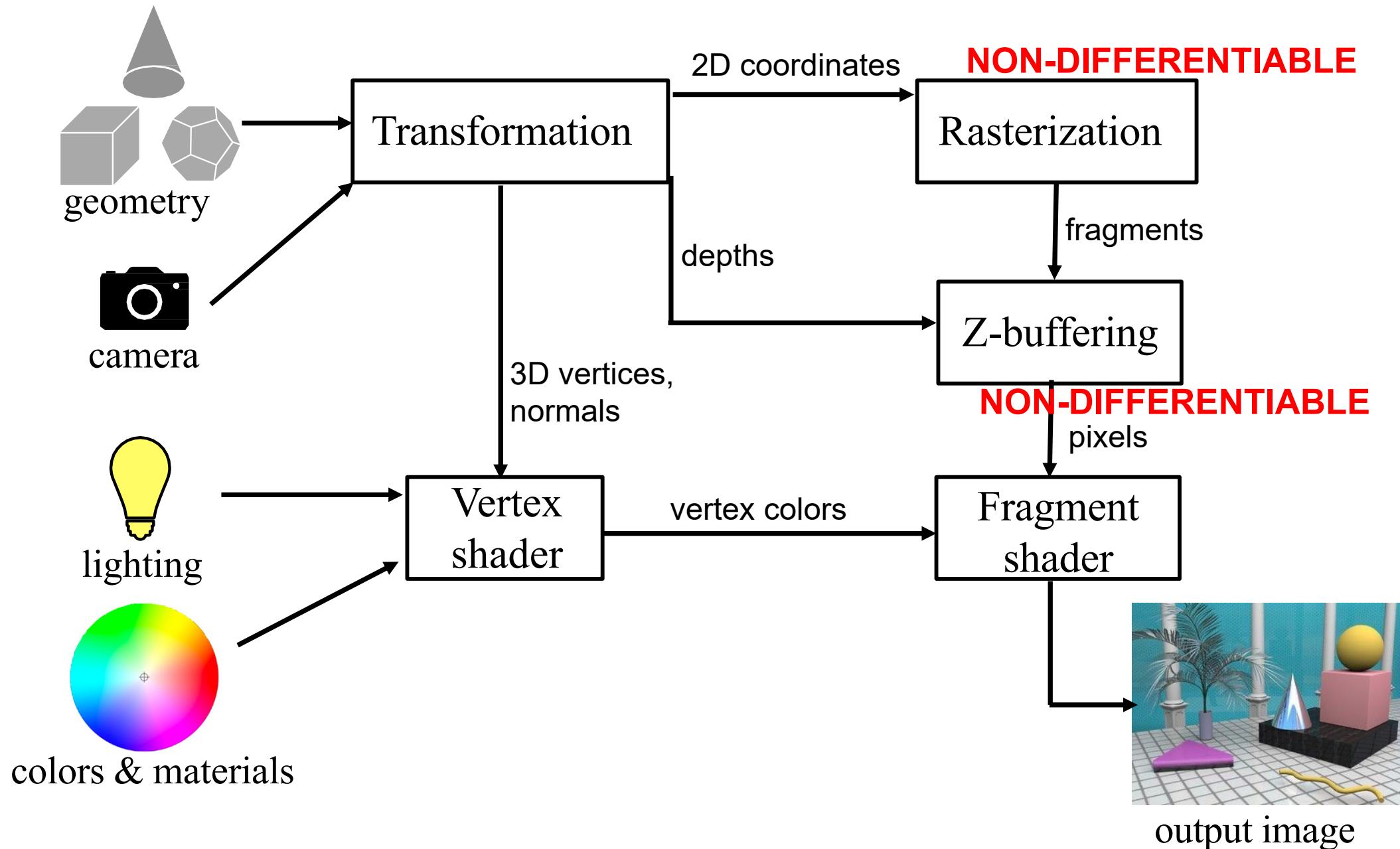
Probability  
of pixel belong to triangle



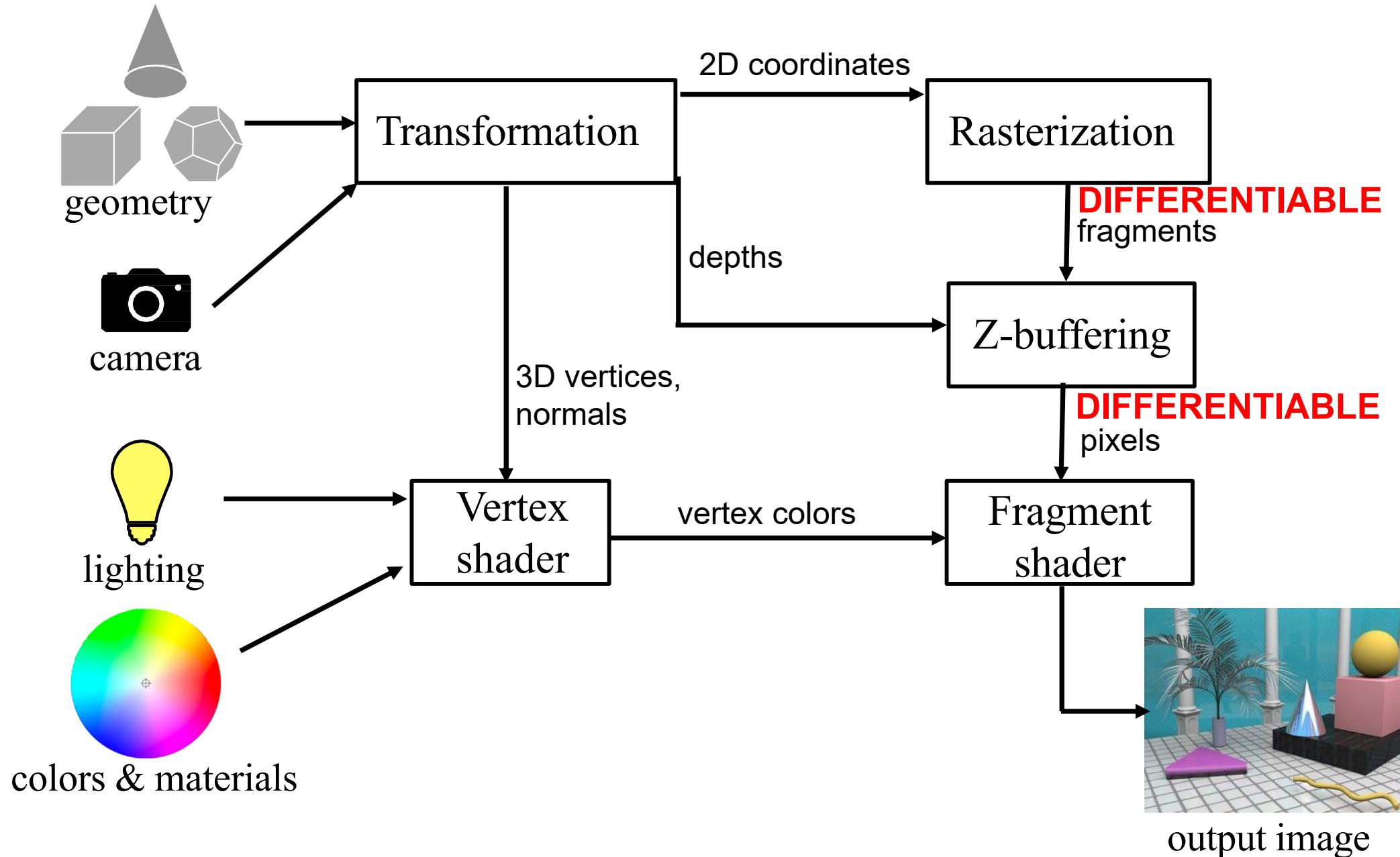
$$w_j^i = \frac{D_j^i \exp(-z_j^i / \gamma)}{\sum_k D_k^i \exp(-z_k^i / \gamma) + \text{const}}$$

Probability  
of pixel being drawing  
based on its  
inverse depth

# Traditional rasterization pipeline



# Soft Rasterizer



# Results: Single Image Reconstruction

Input



3D Model



w/ color



# Results: Mesh deformation

Deforming Car to Airplane



Target image



Multi-view silhouette difference



Deformed mesh

*(Animated GIF  
seen better in video)*

# Differentiable rasterizers

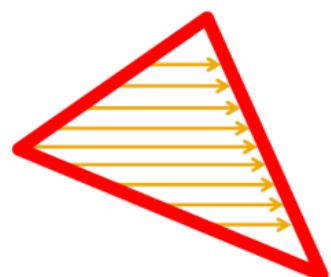
Well suited for differentiable rendering of meshes

Yet, for 3D shape generation we need to generative mesh decoders...  
need initialization with predefined templates (e.g. see Pixel2Mesh)

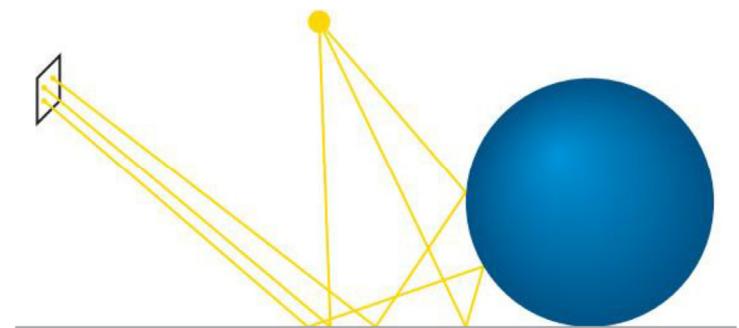
# Reminder: Rendering pipelines

The process of generating a 2D image from a 3D representation

**Rasterization**  
for each polygon  
project it onto 2D plane  
for each pixel in polygon  
shade the pixel

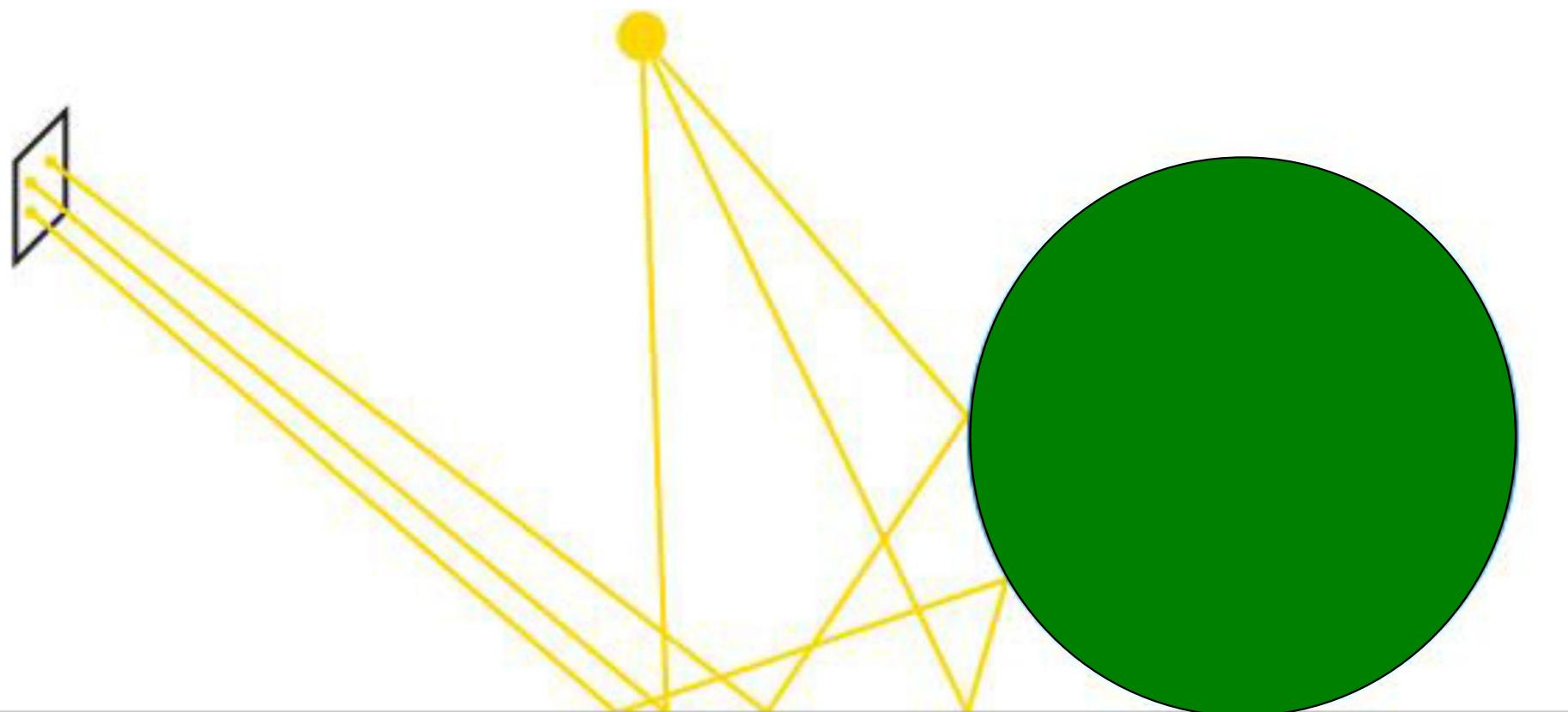


**Ray tracing**  
for each pixel throw ray  
for each object  
calculate intersection  
shade the pixel

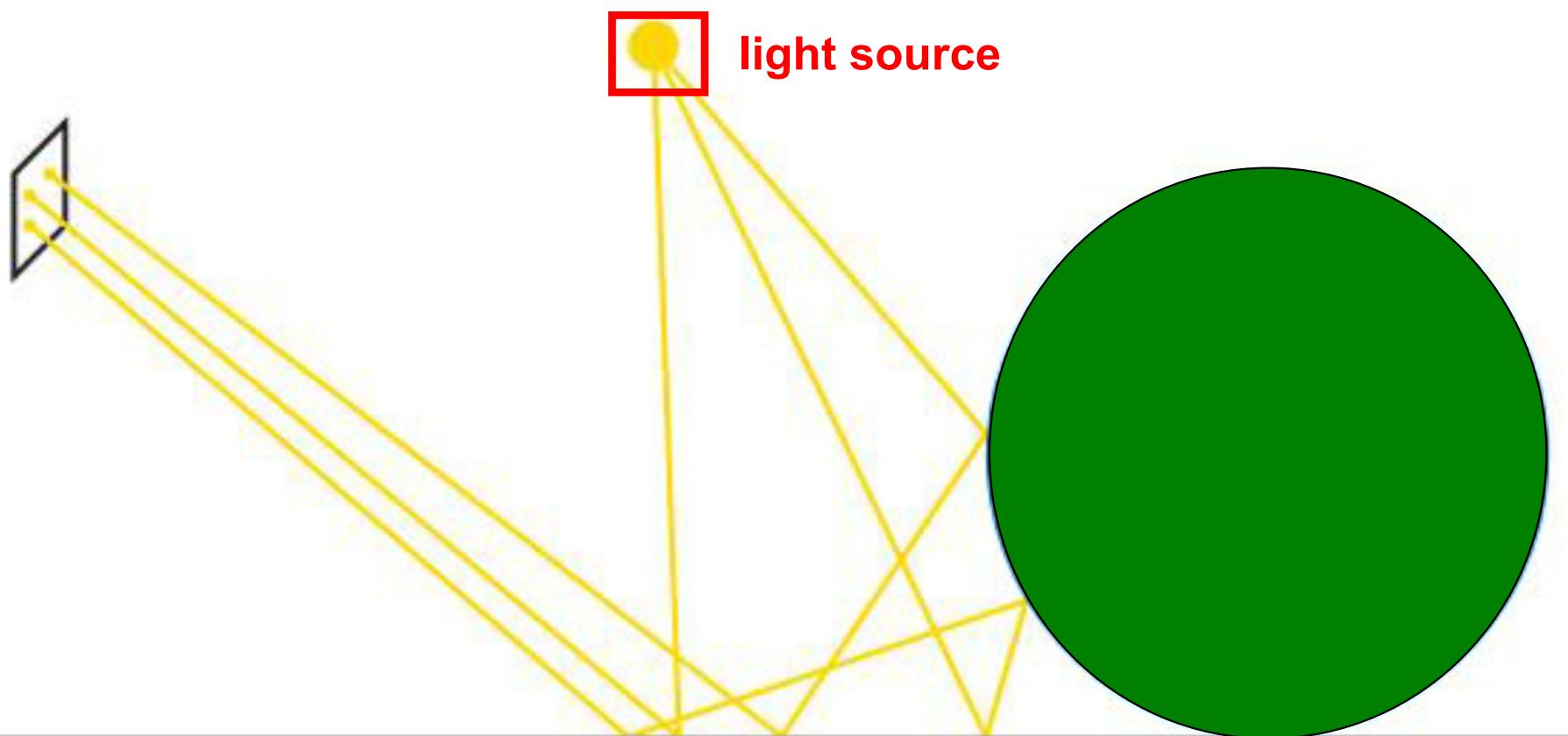


# Paths of light

In nature, light sources emit rays of light that bounce on objects. Think of a "ray" as a stream of photons traveling along a path.

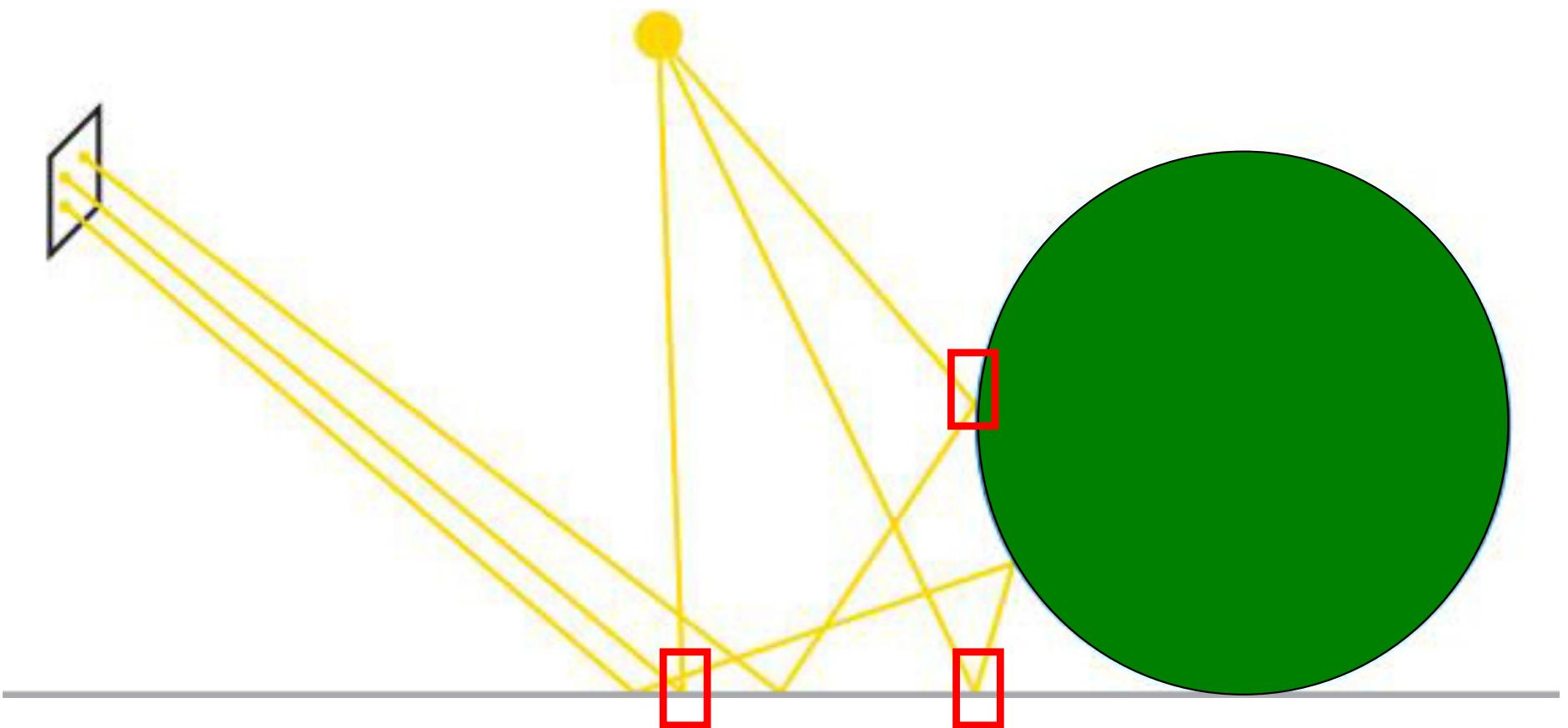


# Paths of light



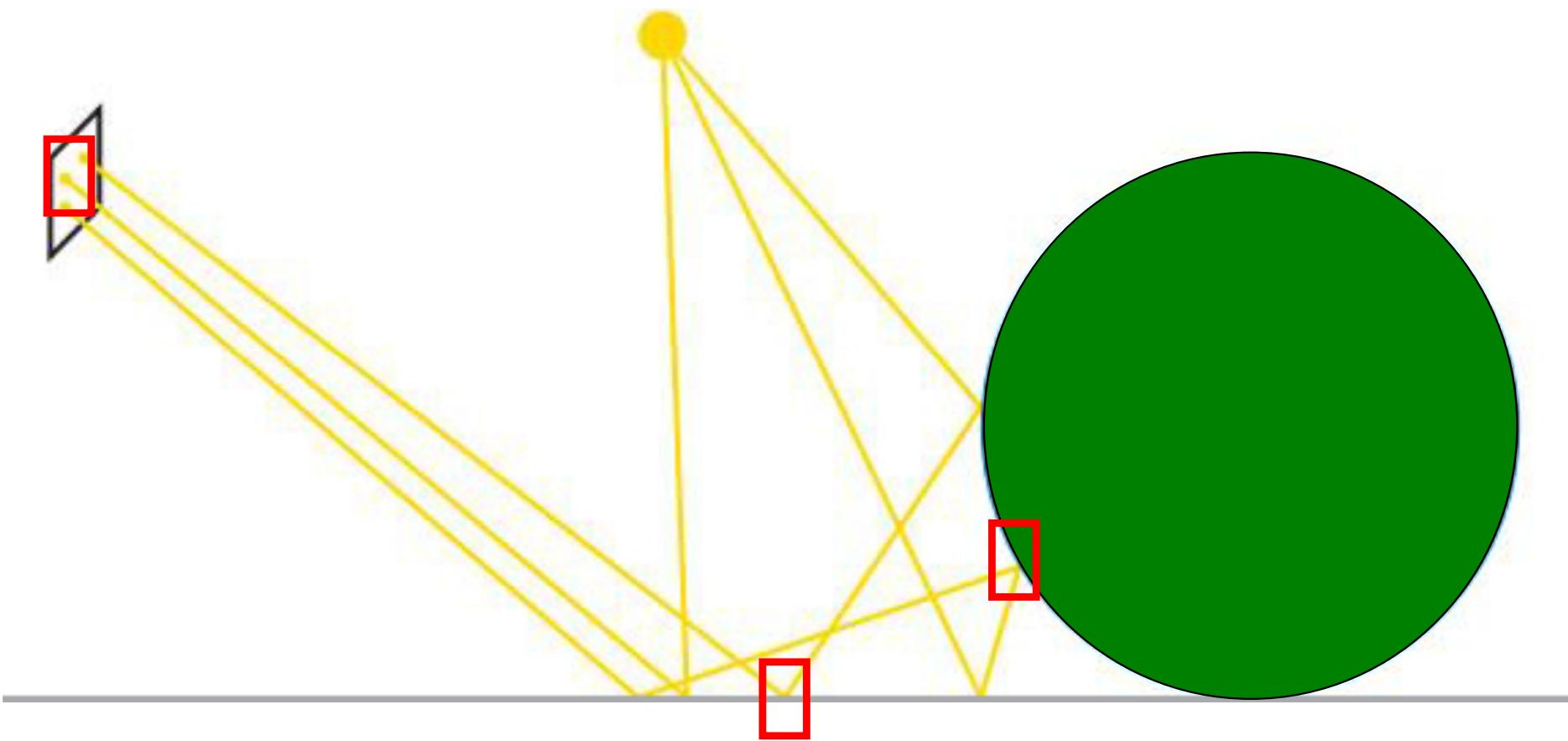
# Paths of light

**Rays interact with objects in a scene**



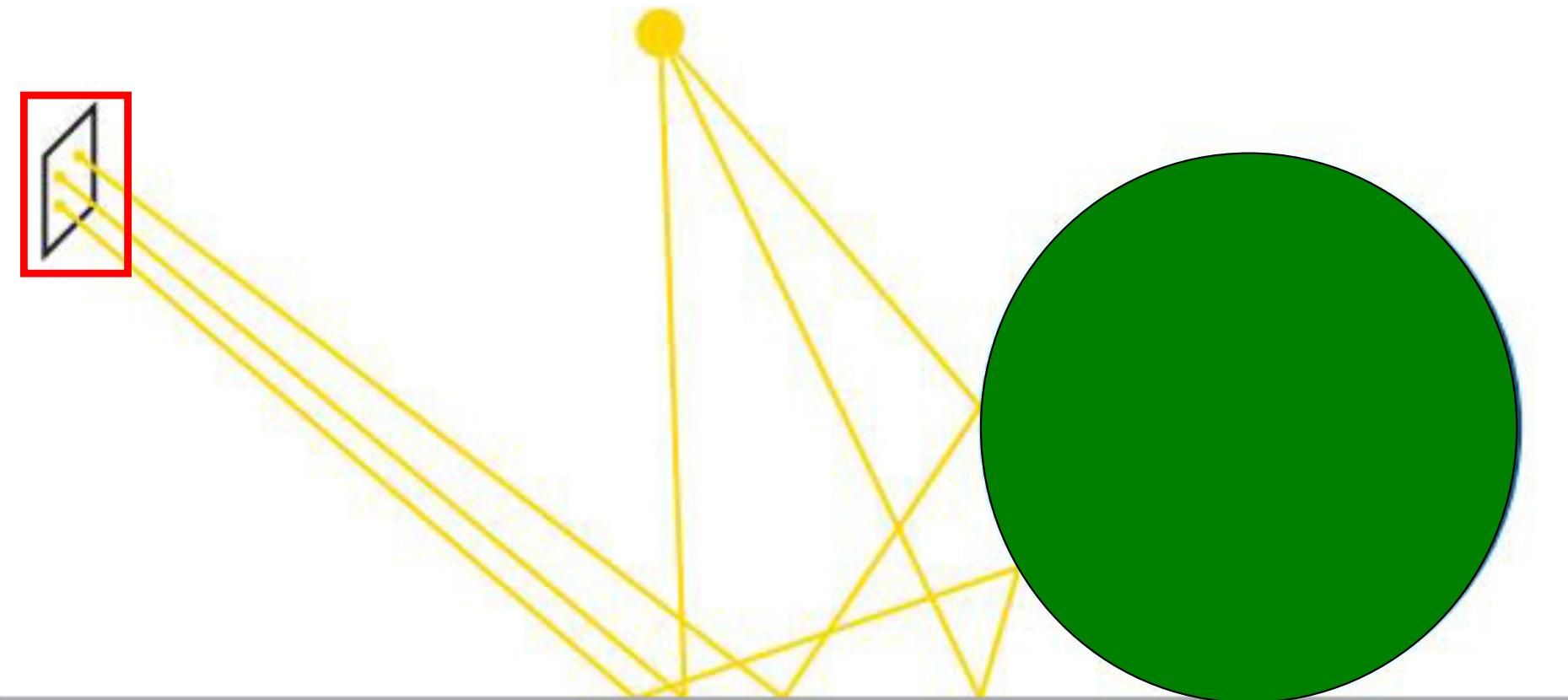
# Paths of light

**Rays interact with objects in a scene**



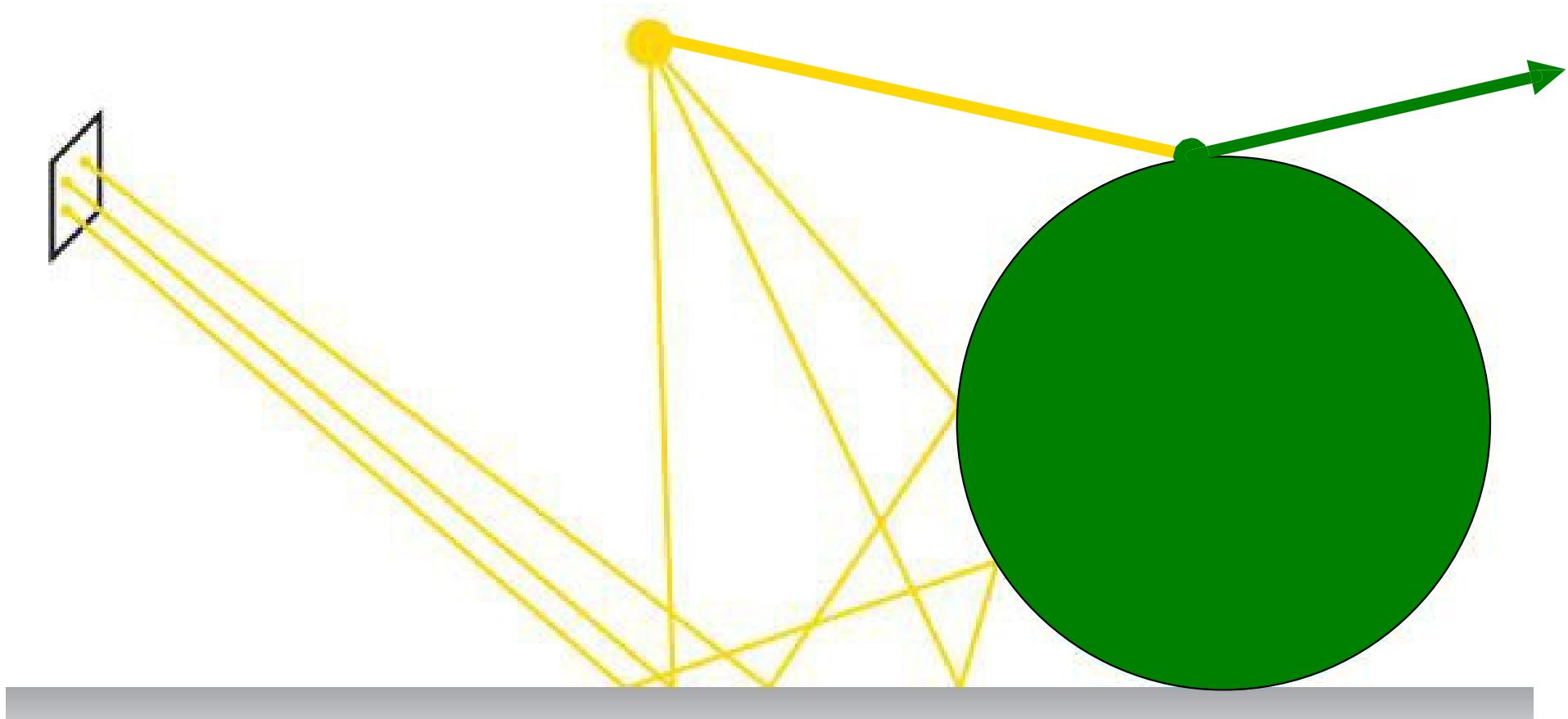
# Paths of light

**Some rays end up in our camera plane**



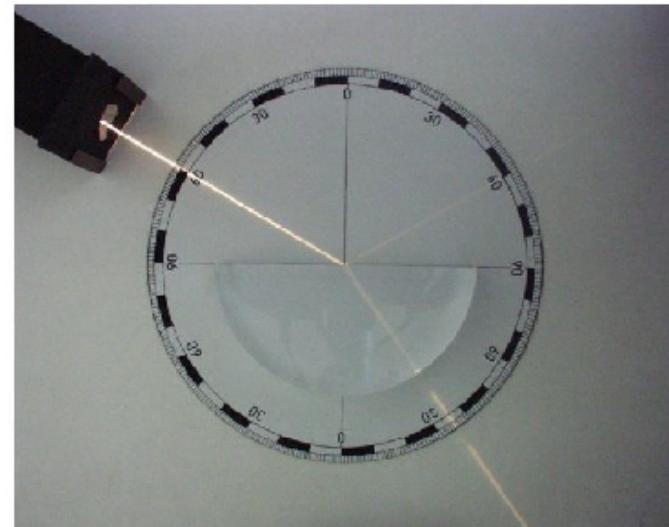
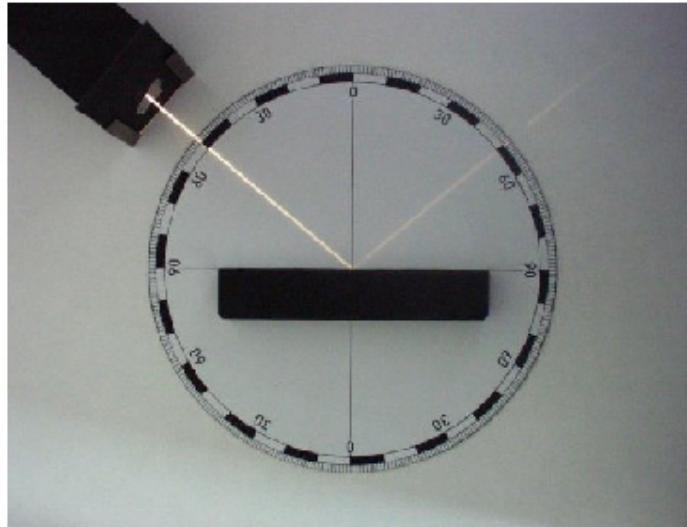
# Is "forward" ray tracing good idea?

However, most rays never reach the camera! Huge unnecessary computation...



# "Backward" ray tracing

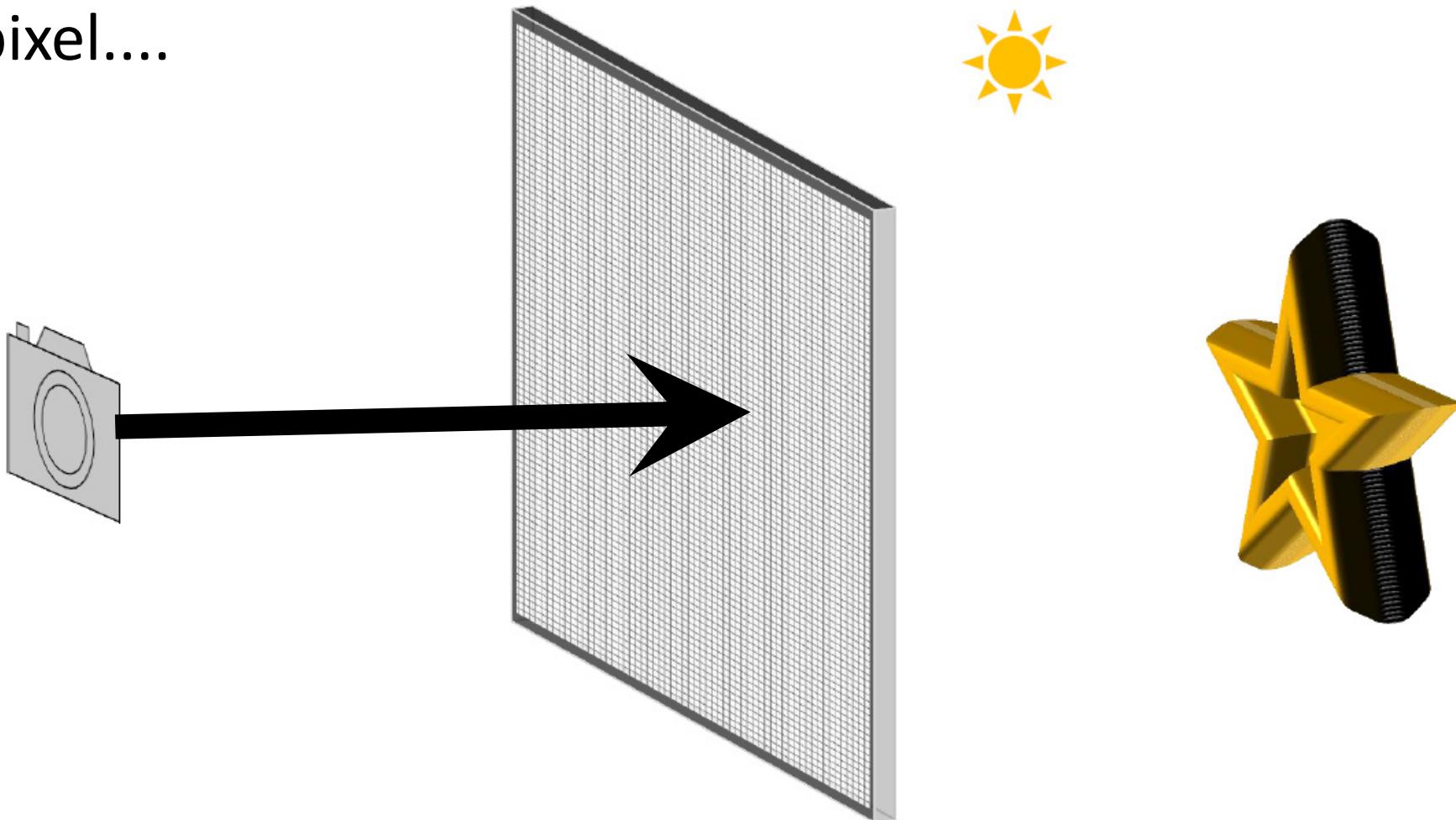
Many light effects are reversible... Let's trace rays from the **camera to light** instead!



note: it is possible to combine forward and backward ray tracing (bidirectional path tracing)... particularly useful for indoor scenes

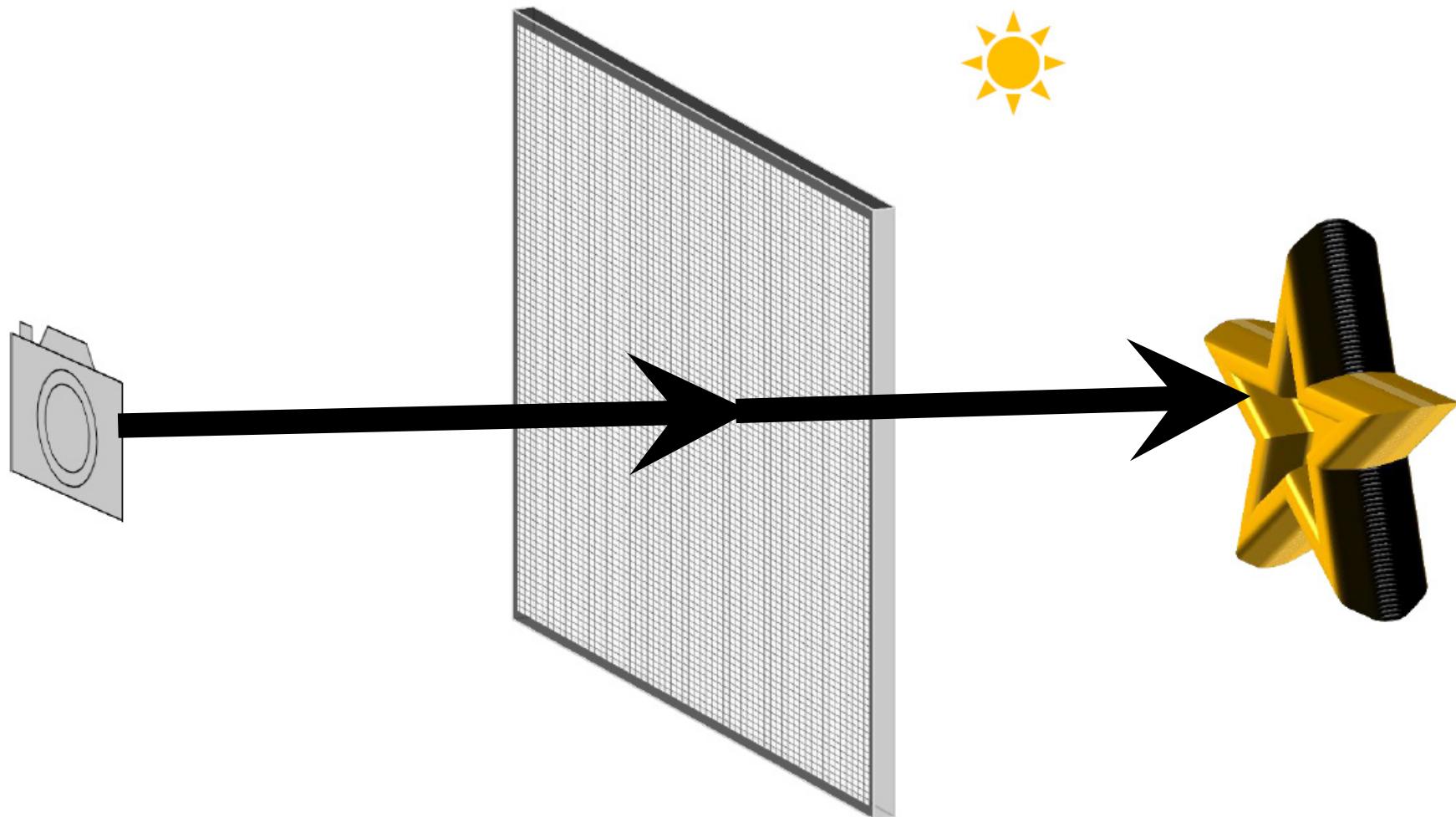
# Ray tracing algorithm

Place your camera. Place an image array in front of it.  
Send a ray from camera towards the center of each  
pixel....



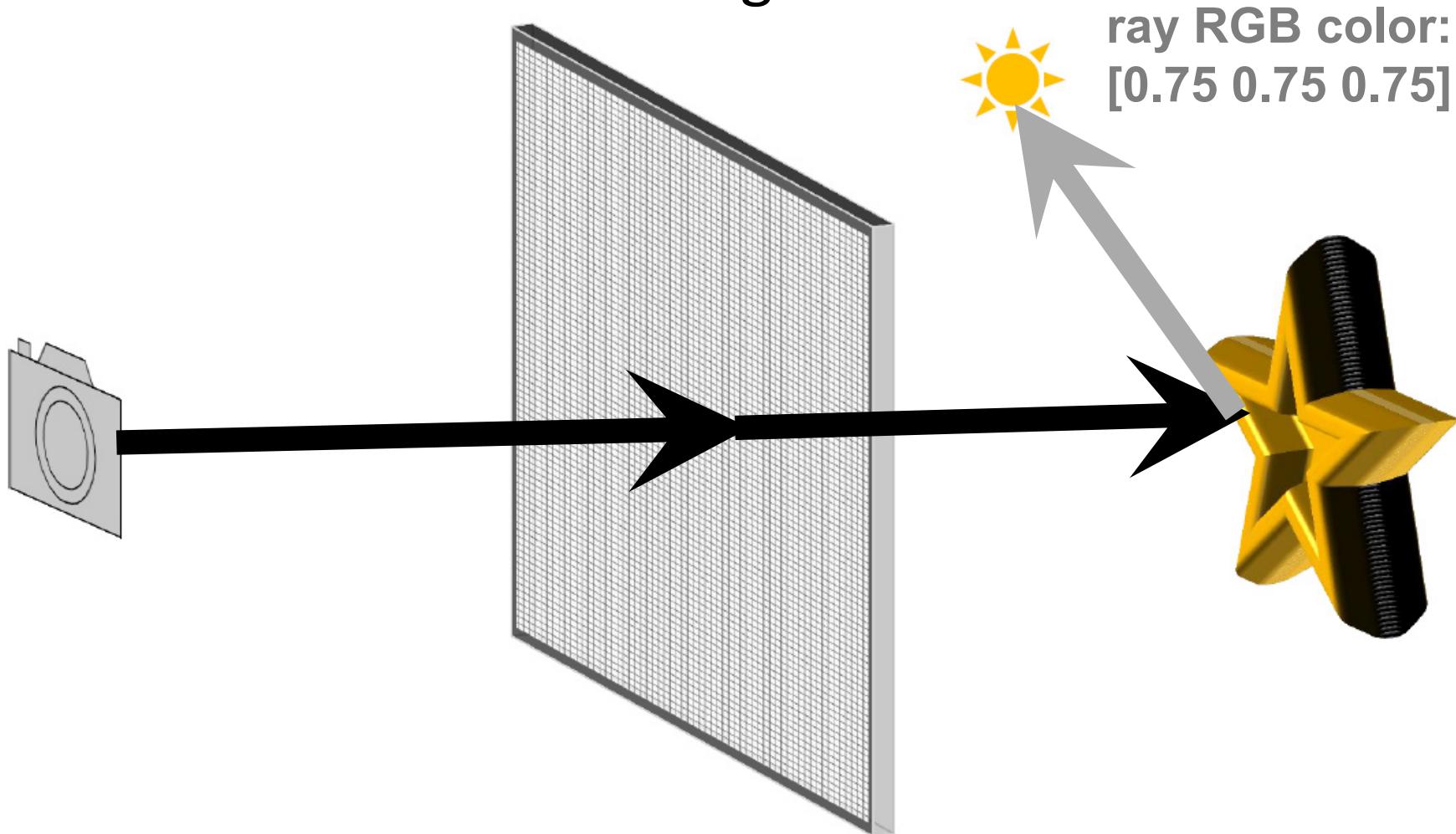
# Ray tracing algorithm

Ray keeps going ... until it intersects with an object.



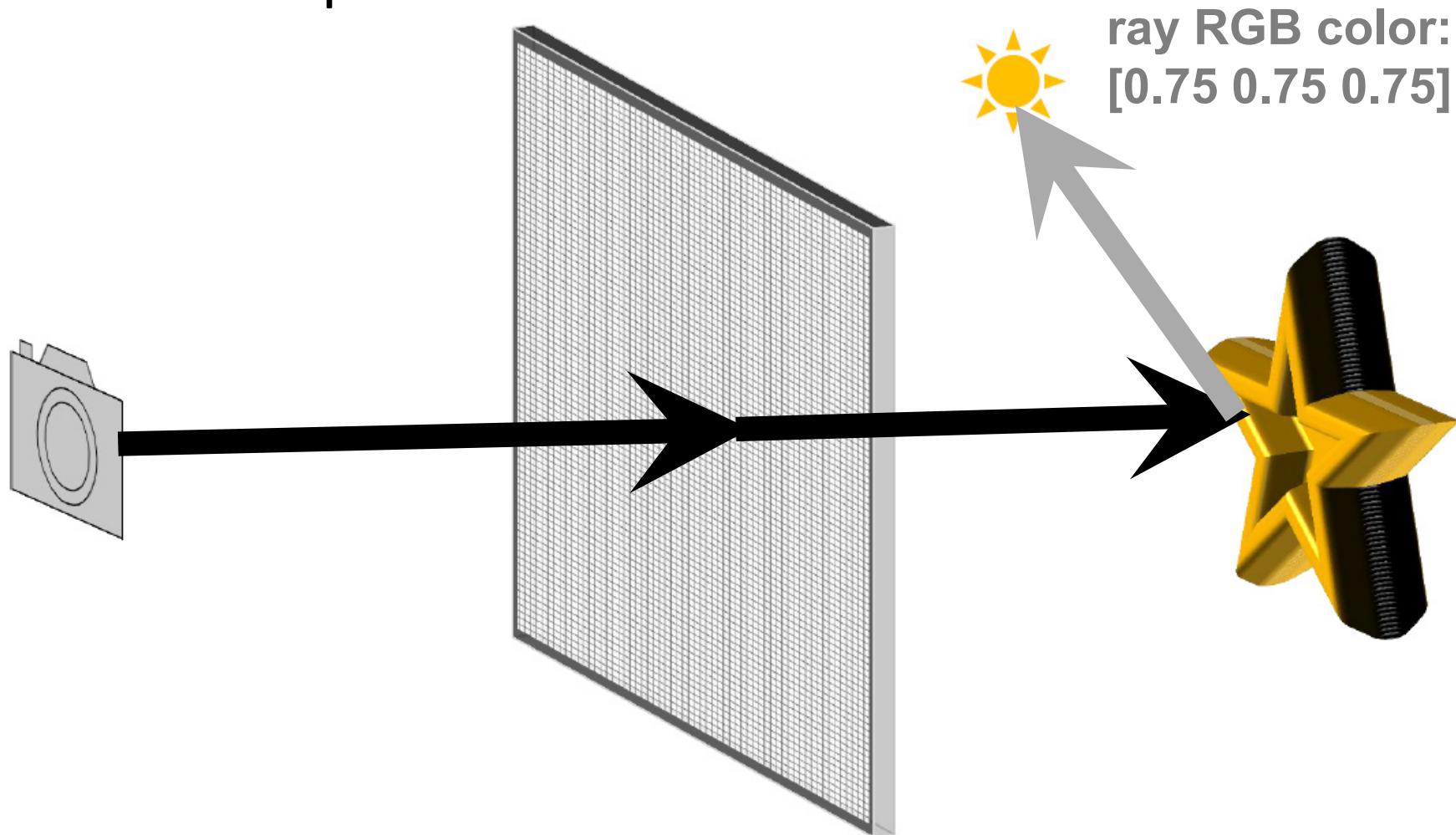
# Ray tracing algorithm

Then compute a new **light ray** from the point of intersection towards each light.



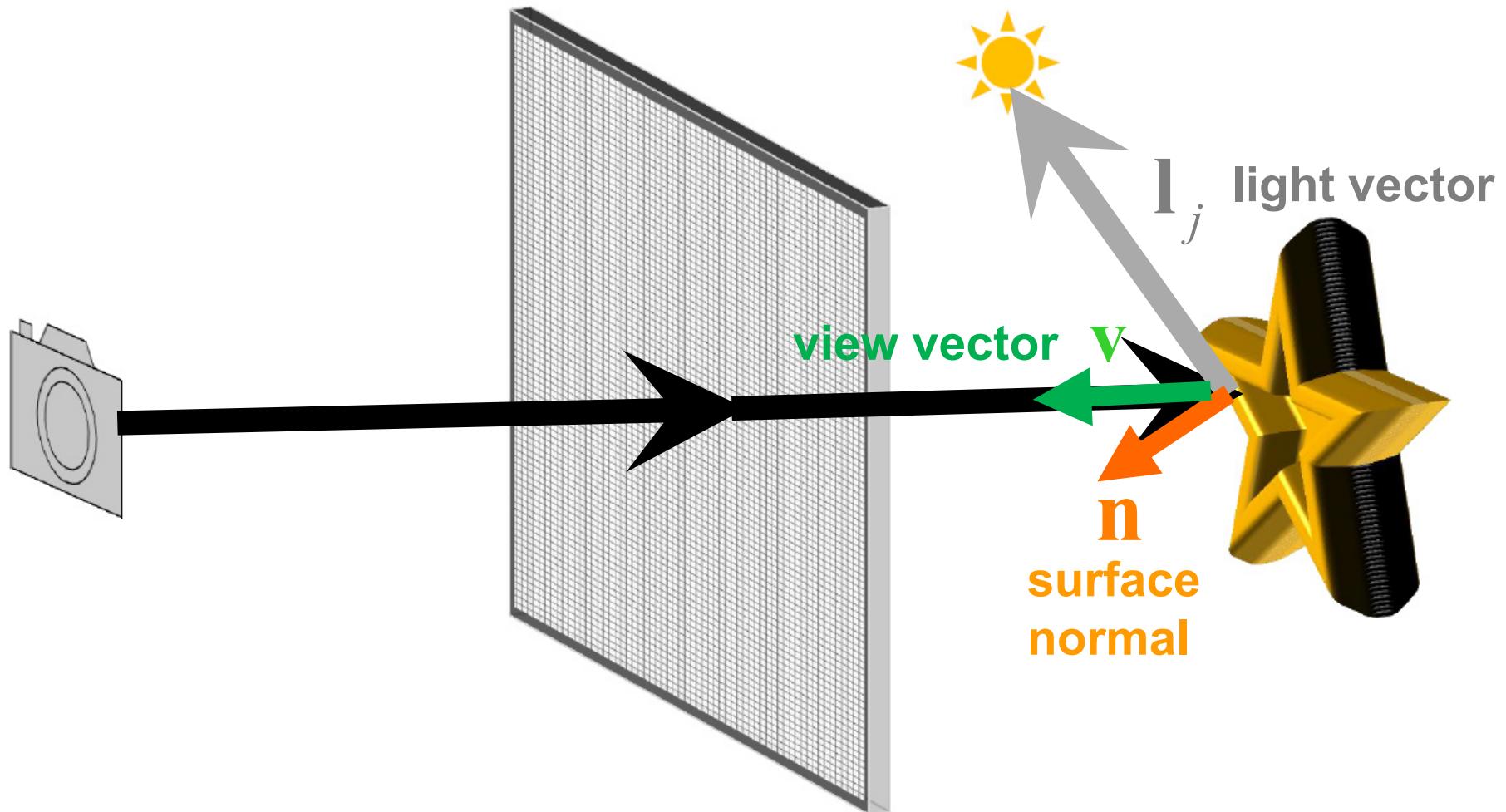
# Ray tracing algorithm

Evaluate **shading model** based on the material of the intersection point...



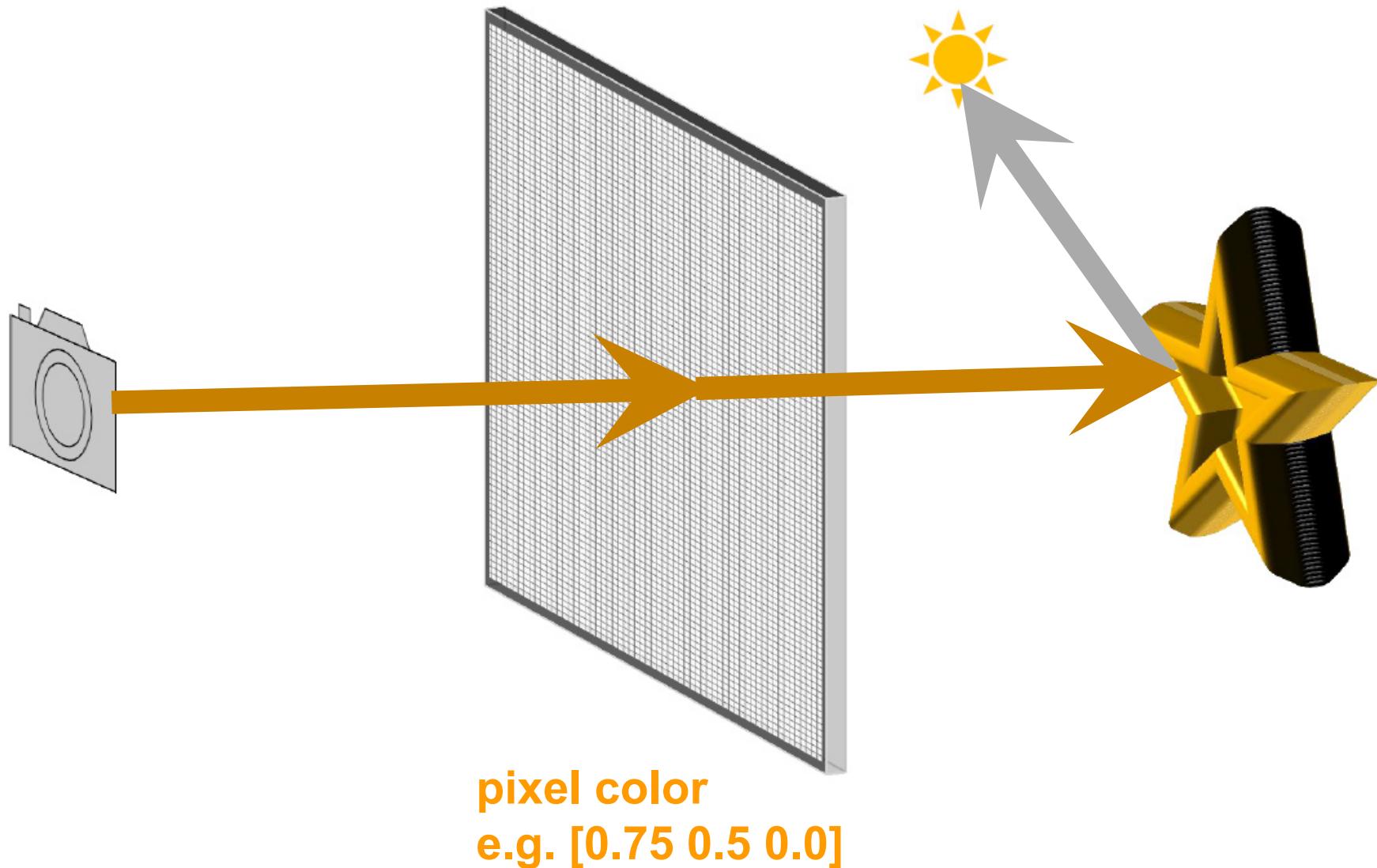
# Ray tracing algorithm

Evaluate **shading model** based on the material of the intersection point...



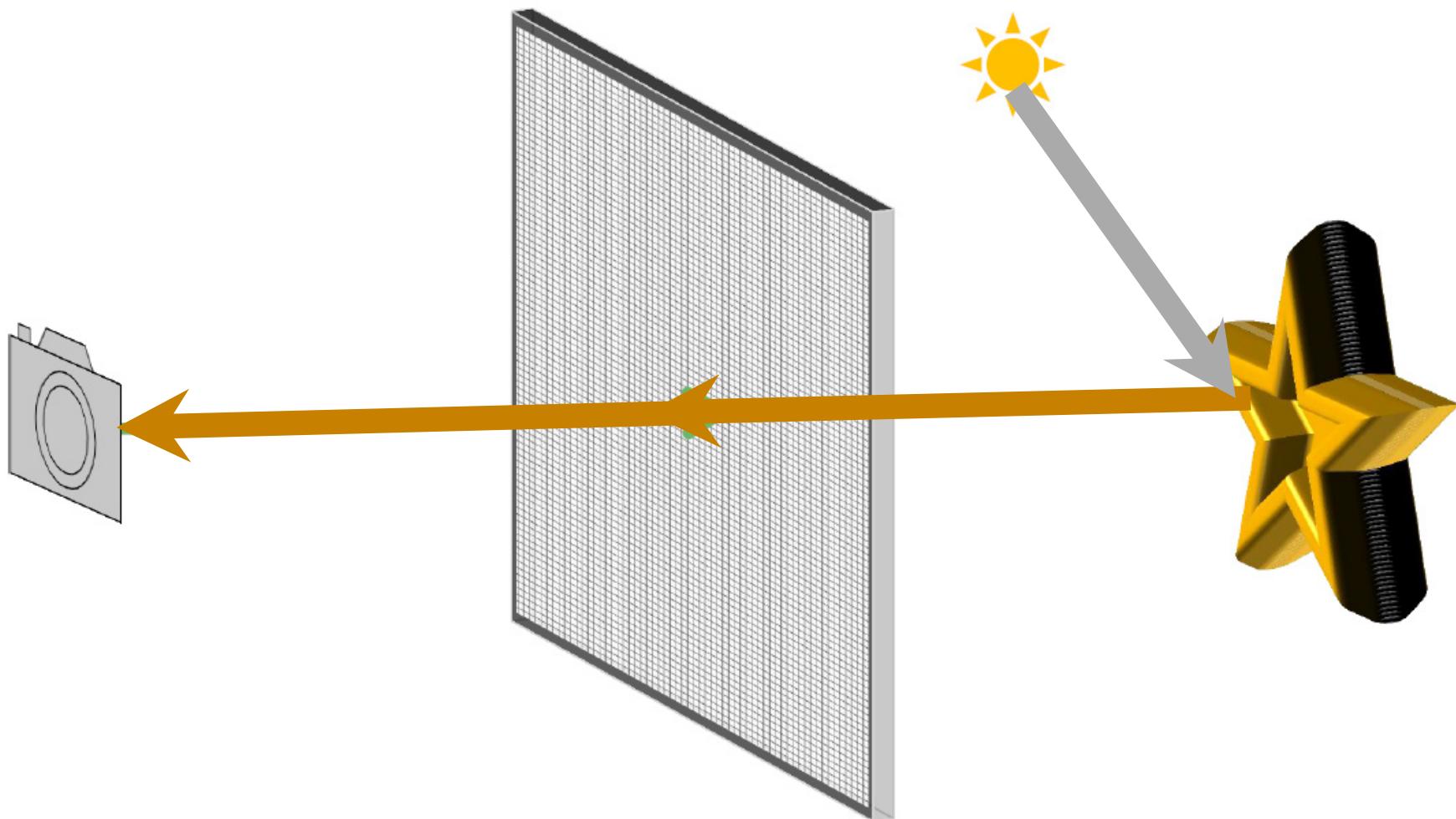
# Ray tracing algorithm

Update pixel color!



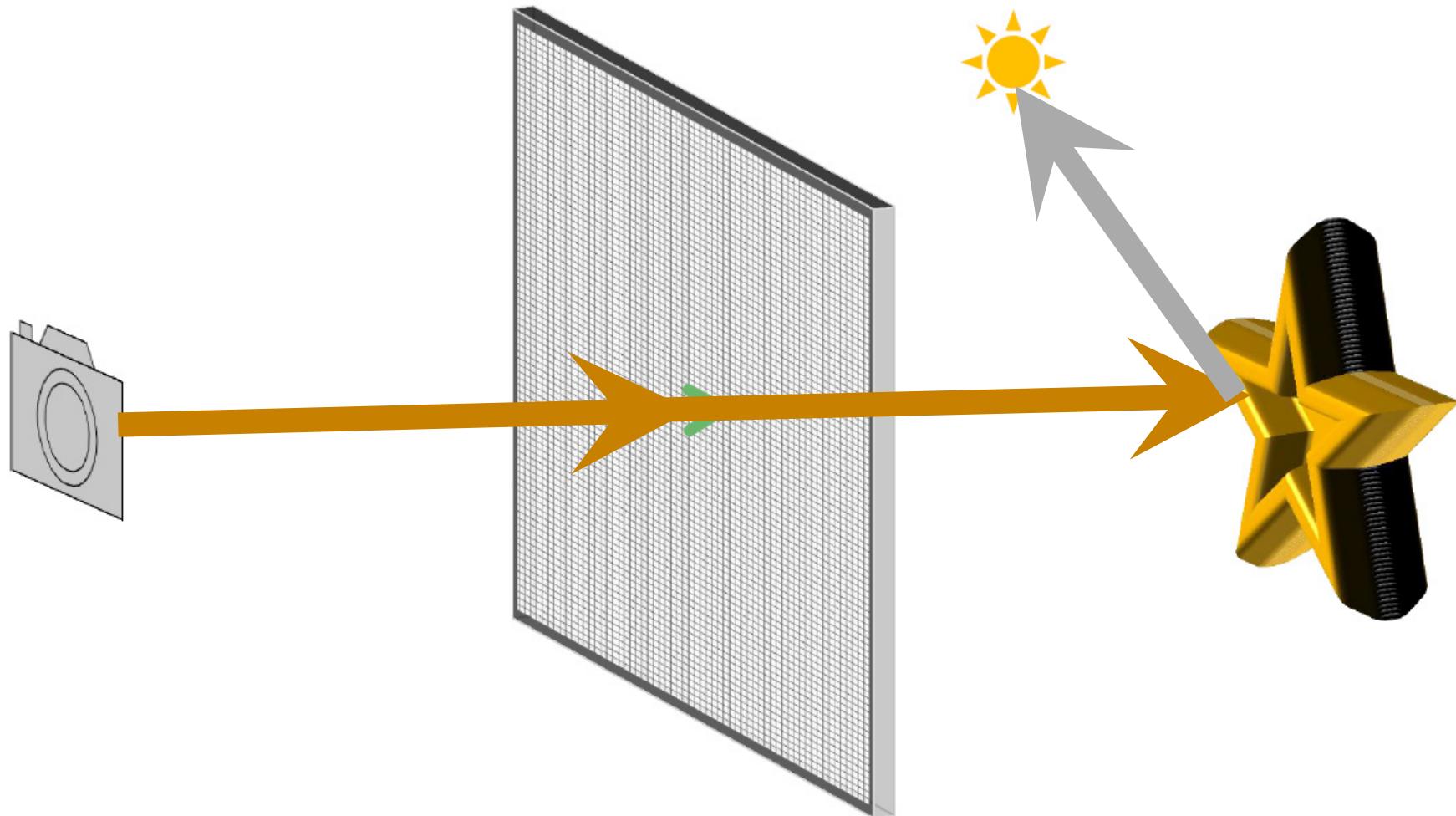
# Ray tracing algorithm

This is the “**actual**” ray we traced in a “**backwards**” manner.



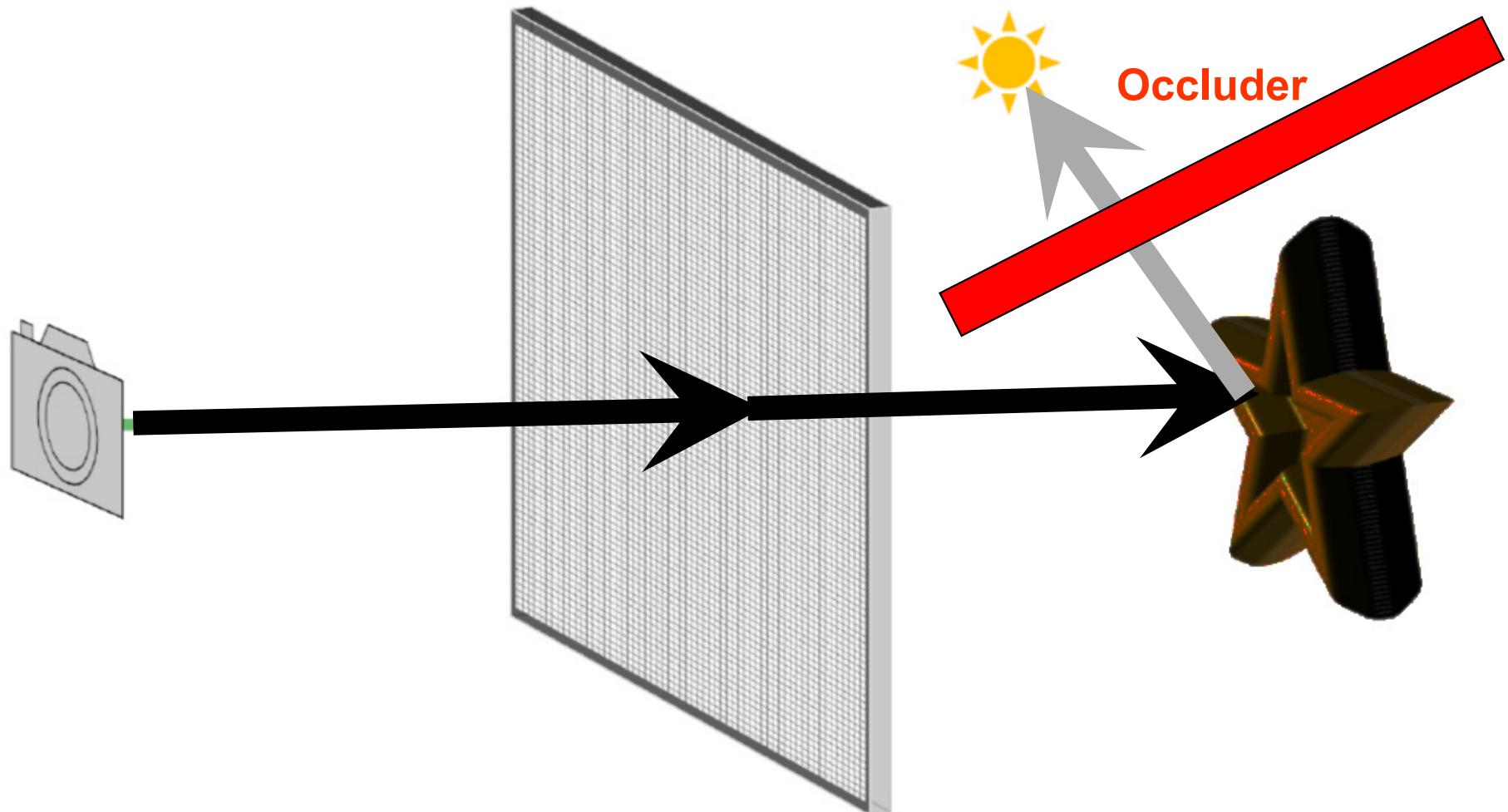
# Ray tracing algorithm

As we discussed, ray tracing is **reversible**.



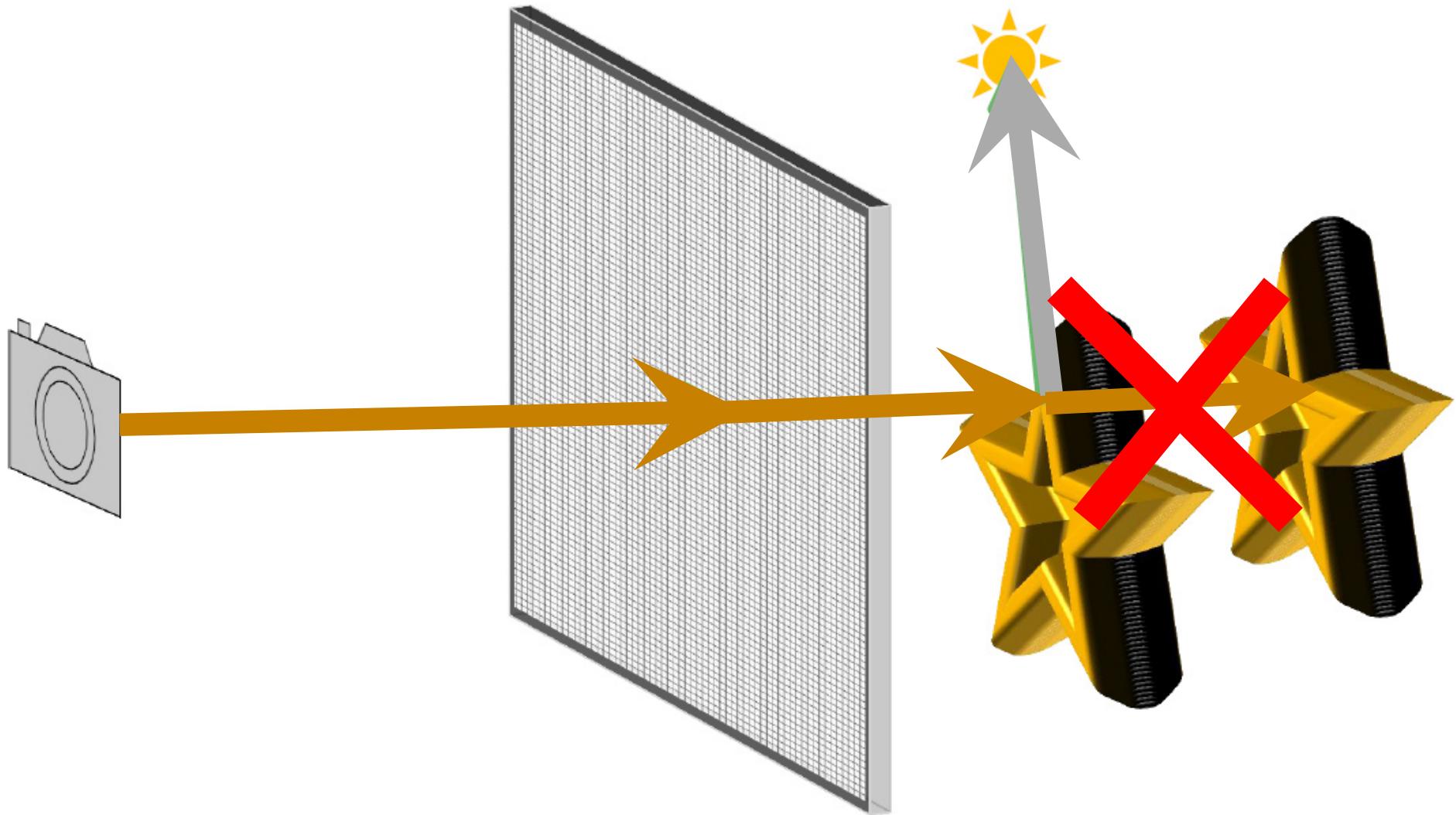
# Shadow rays

If the light ray is blocked by another object, then the color of ray is set to black (or an ambient color)



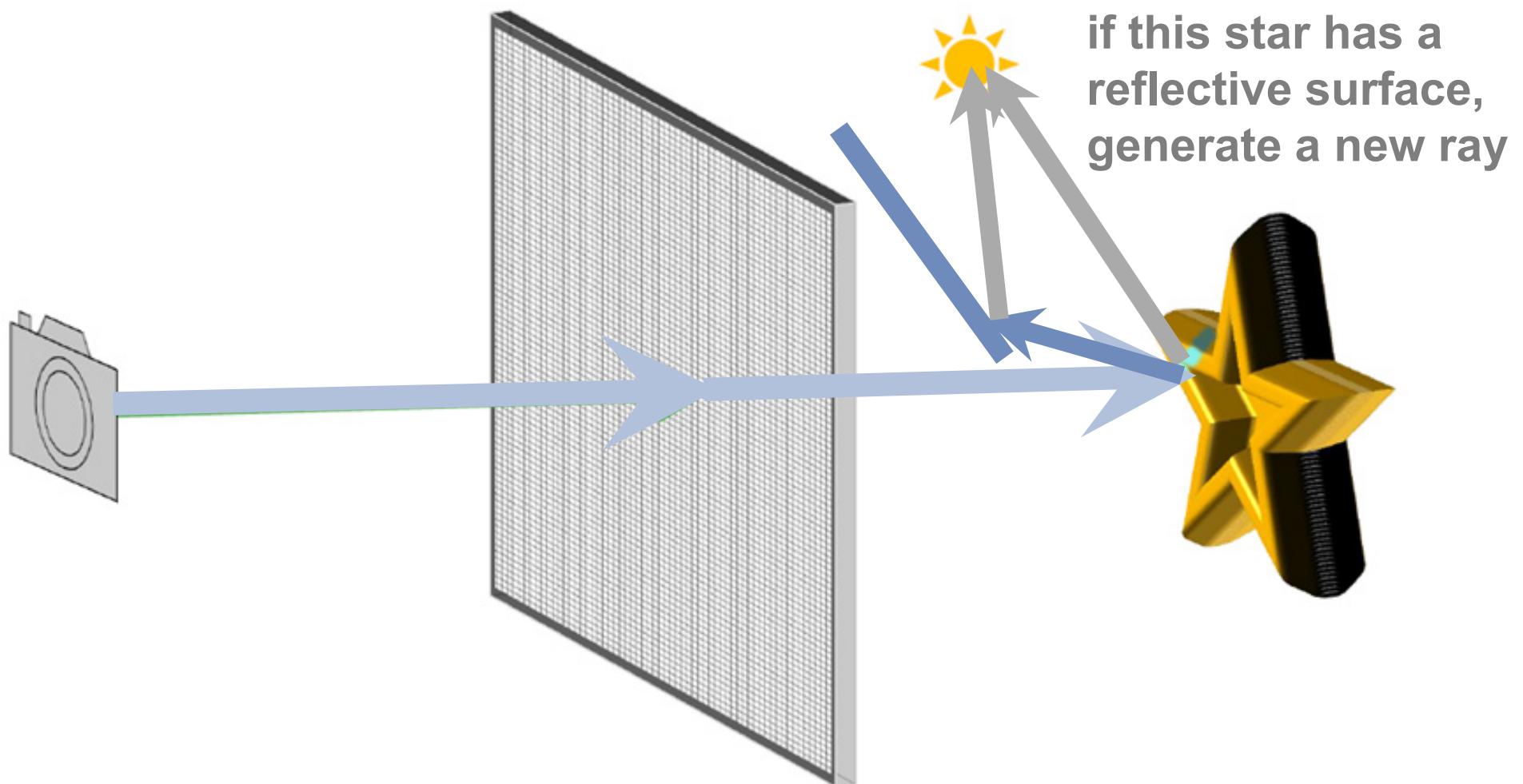
# Multiple objects?

Consider only the closest intersection point during ray tracing!

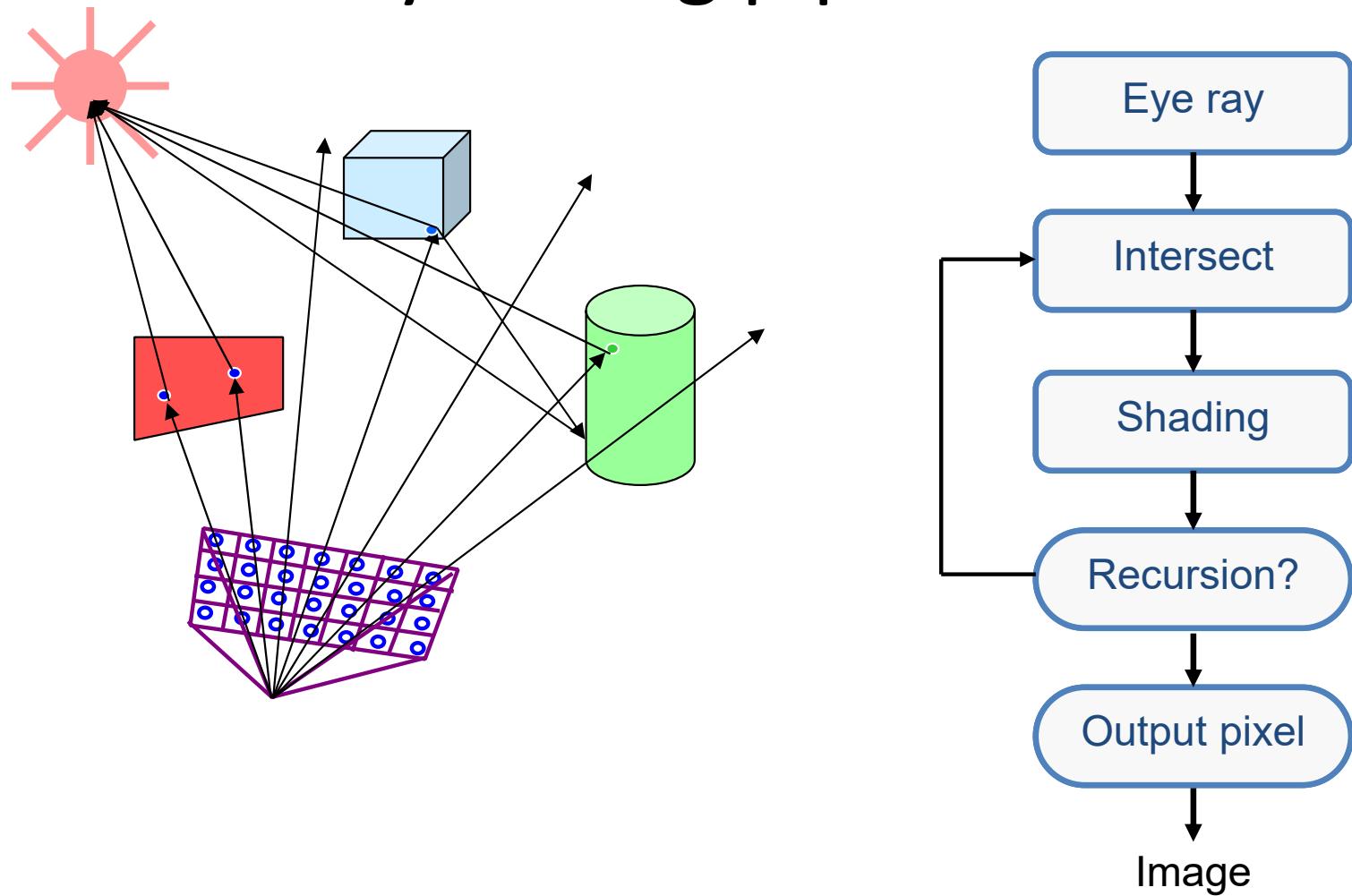


# Reflections?

Basic idea: at each intersection point, **generate a new ray recursively!**



# Ray tracing pipeline



# Differentiable Ray Marching for Implicit Surfaces

## Signed Distance Function

2.3 • 1.7 • 0.9 • 0.2 •

1.2 • 0.4 • 0.1 • -0.8 •

0.3 • -0.5 • -0.7 • -1.4 •

0.2 • -0.9 • -1.7 • -2.5 •

# Differentiable Ray Marching for Implicit Surfaces

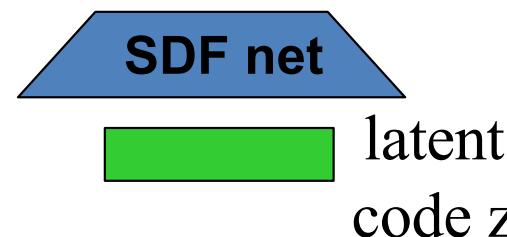
## Signed Distance Function

2.3 • 1.7 • 0.9 • 0.2 •

1.2 • 0.4 • 0.1 • -0.8 •

0.3 • -0.5 • -0.7 • -1.4 •

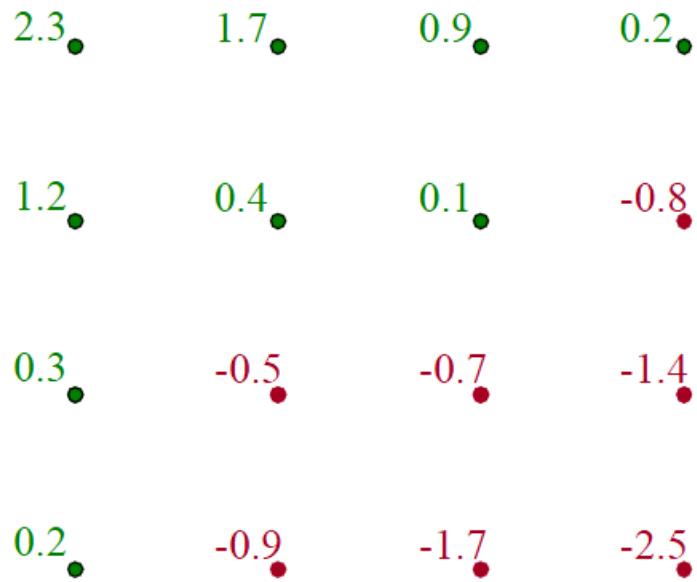
0.2 • -0.9 • -1.7 • -2.5 •



DIST: Rendering Deep Implicit Signed Distance Function with Differentiable Sphere Tracing, Liu et al., CVPR 2020

# Differentiable Ray Marching for Implicit Surfaces

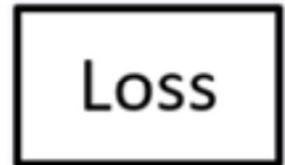
Signed Distance Function



latent  
code z



Shaded  
image

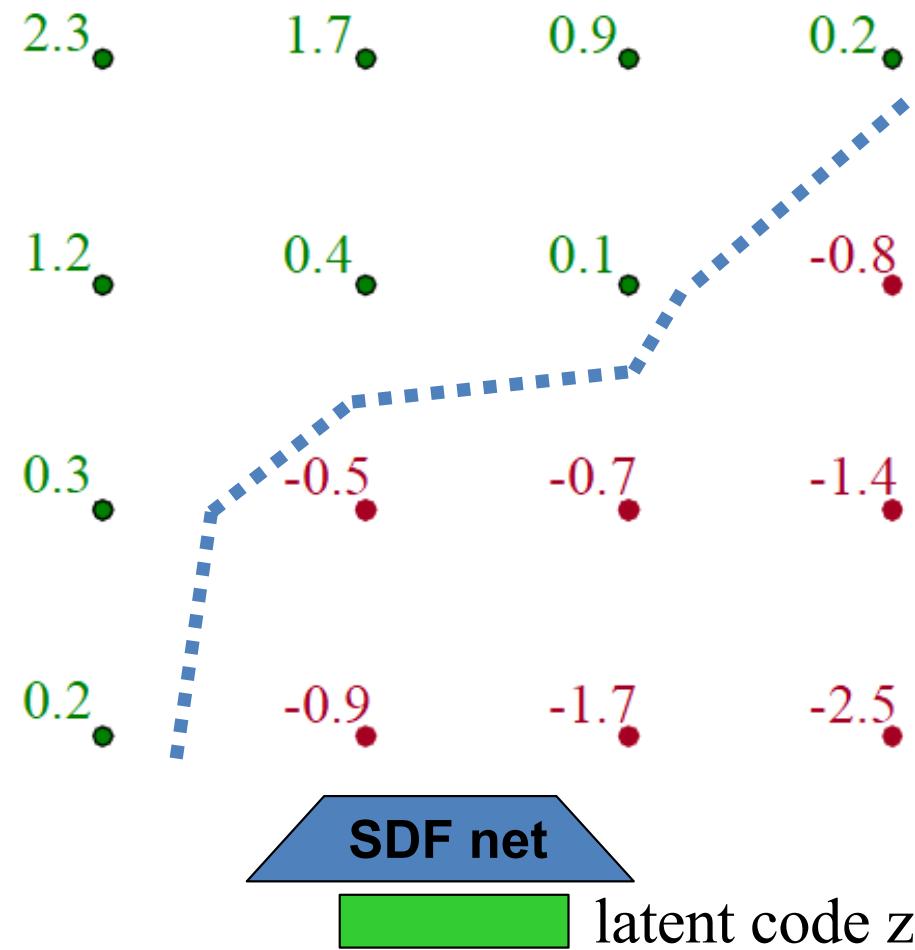


Depth  
image

DIST: Rendering Deep Implicit Signed Distance Function with Differentiable Sphere Tracing, Liu et al., CVPR 2020

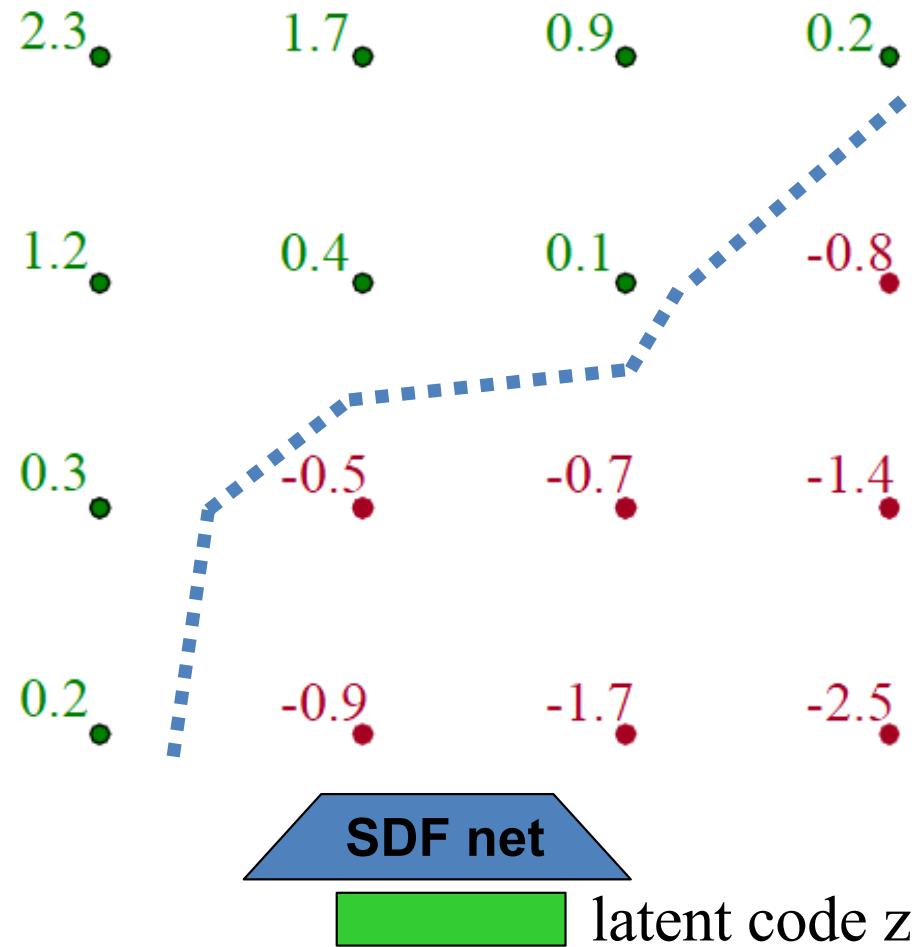
# How to perform ray tracing for implicits?

Given a continuous SDF function (here visualized as grid)...



# How to perform ray tracing for implicits?

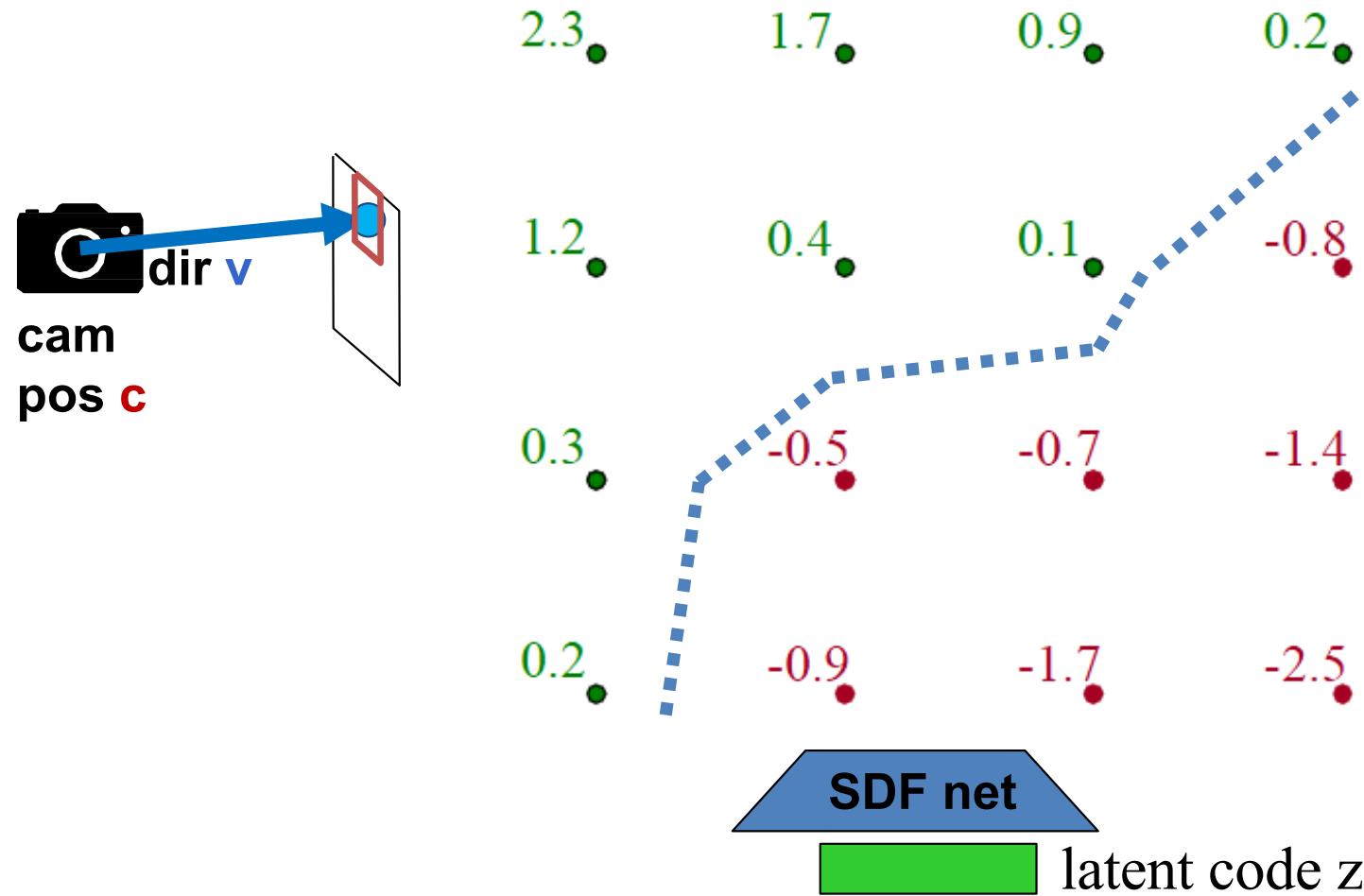
and a camera position and camera plane



DIST: Rendering Deep Implicit Signed Distance Function with Differentiable Sphere Tracing, Liu et al., CVPR 2020

# How to perform ray tracing for implicits?

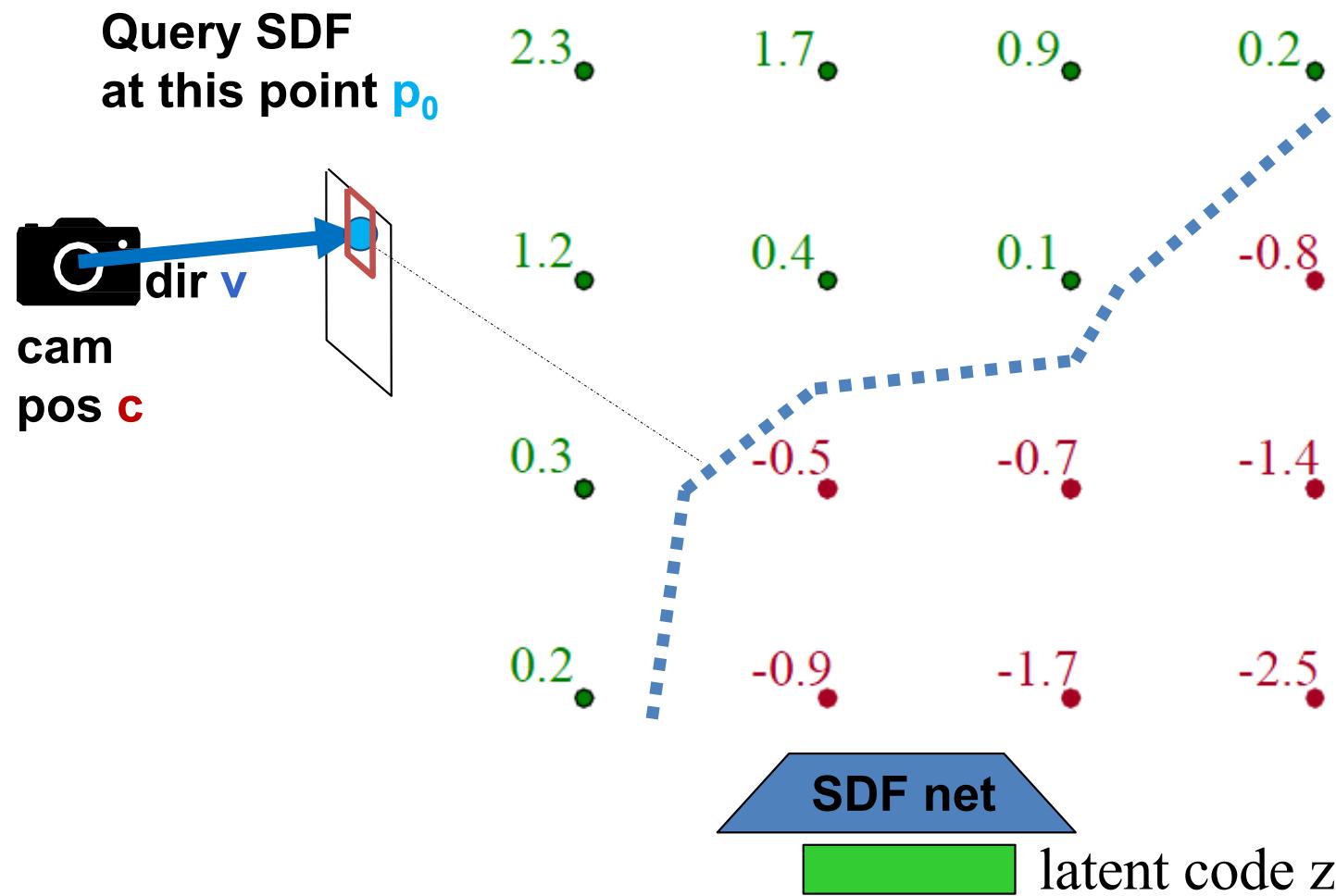
Shoot a ray for each pixel...



DIST: Rendering Deep Implicit Signed Distance Function with Differentiable Sphere Tracing, Liu et al., CVPR 2020

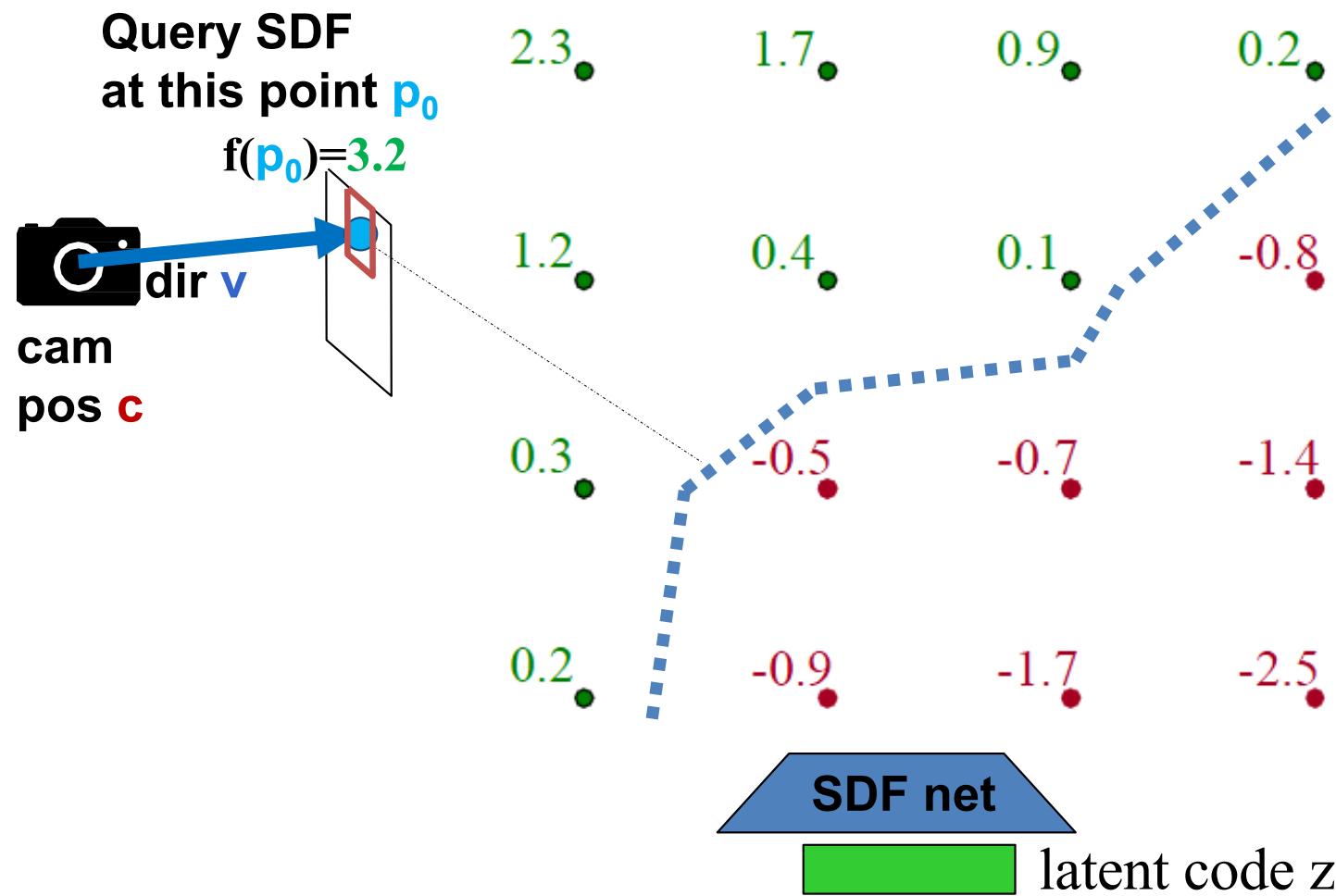
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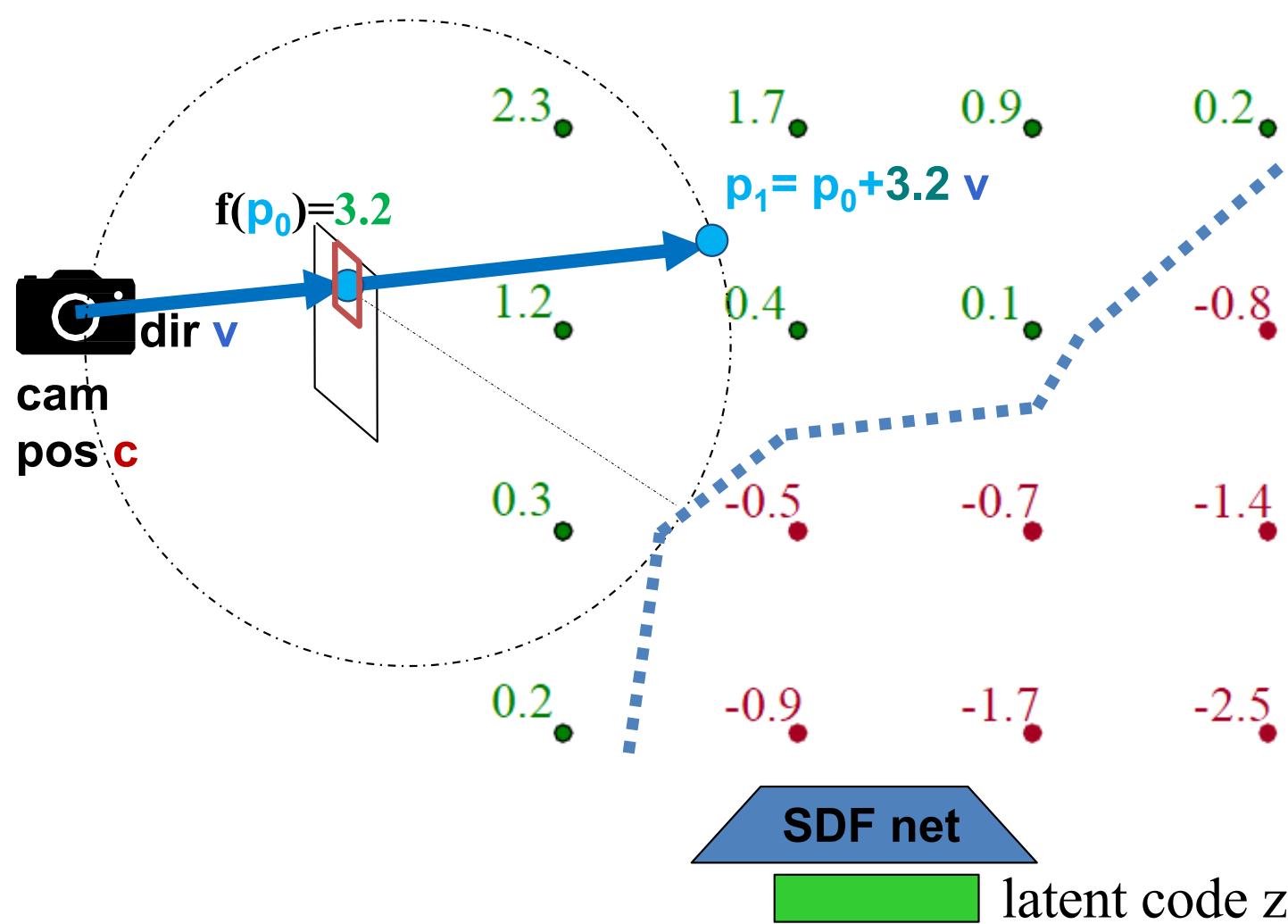
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DIST: Rendering Deep Implicit Signed Distance Function with Differentiable Sphere Tracing, Liu et al., CVPR 2020

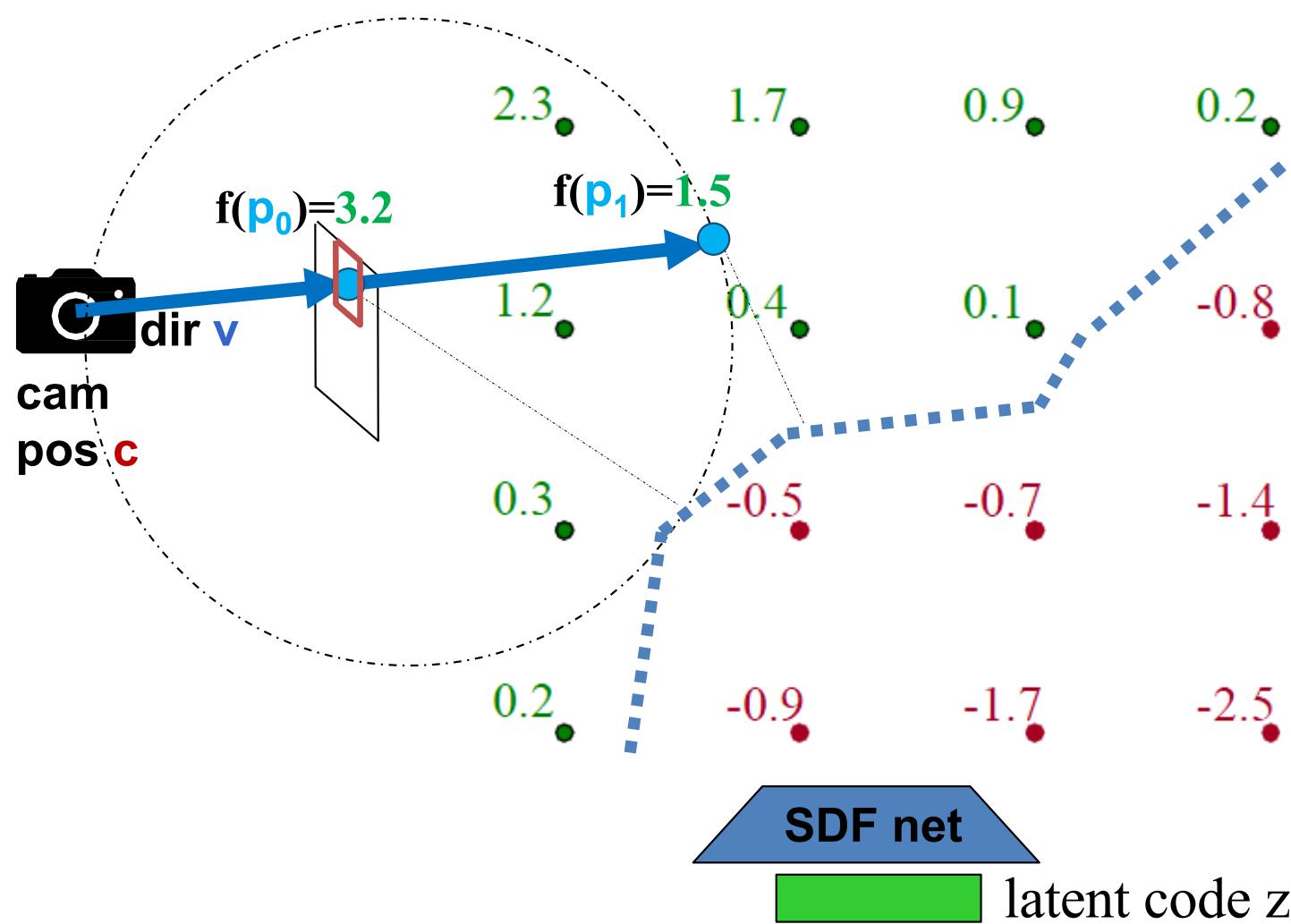
# How to perform ray tracing for implicits?

Follow the path of the ray taking steps according to the SDF (“sphere” ray tracing)



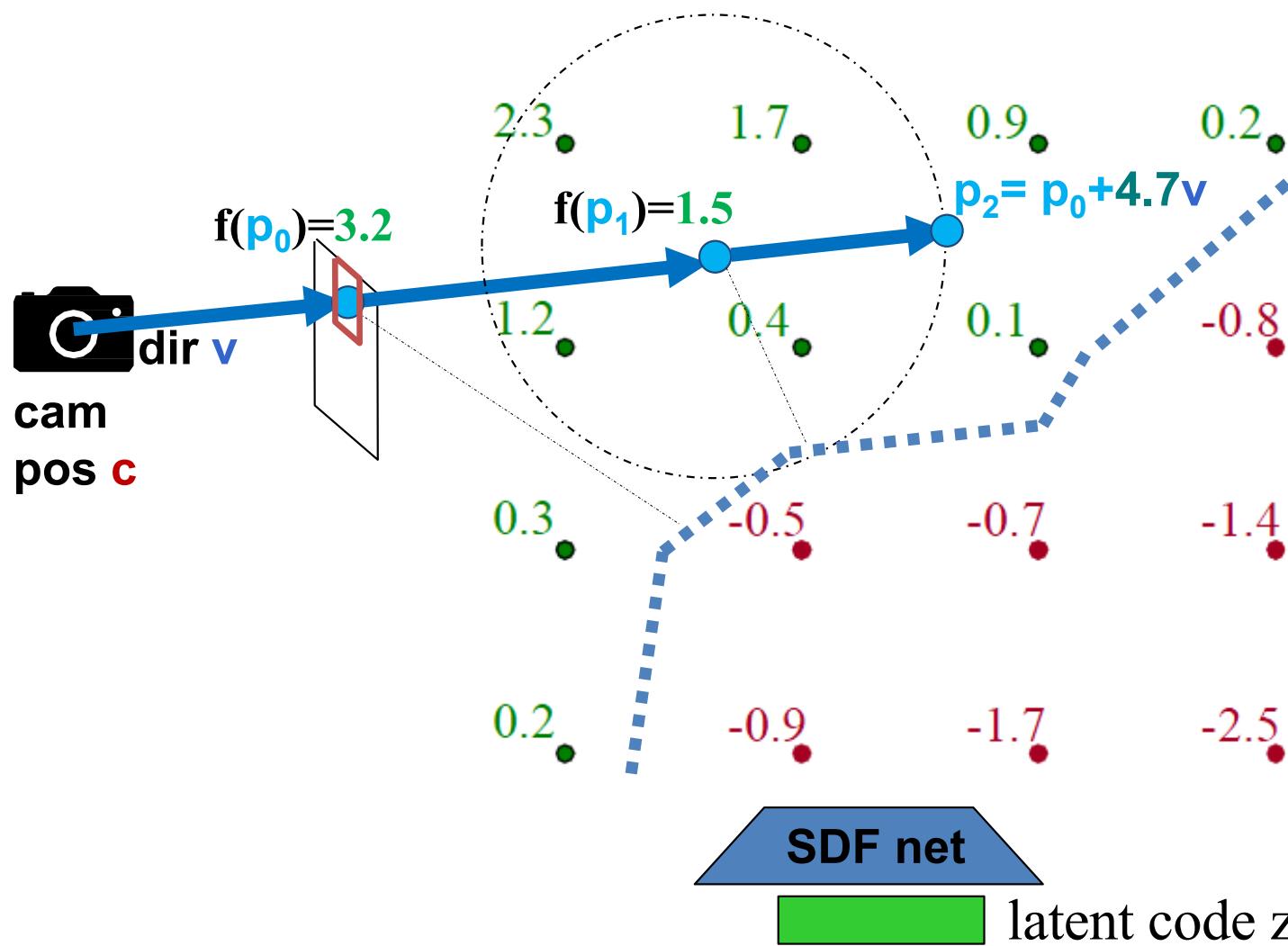
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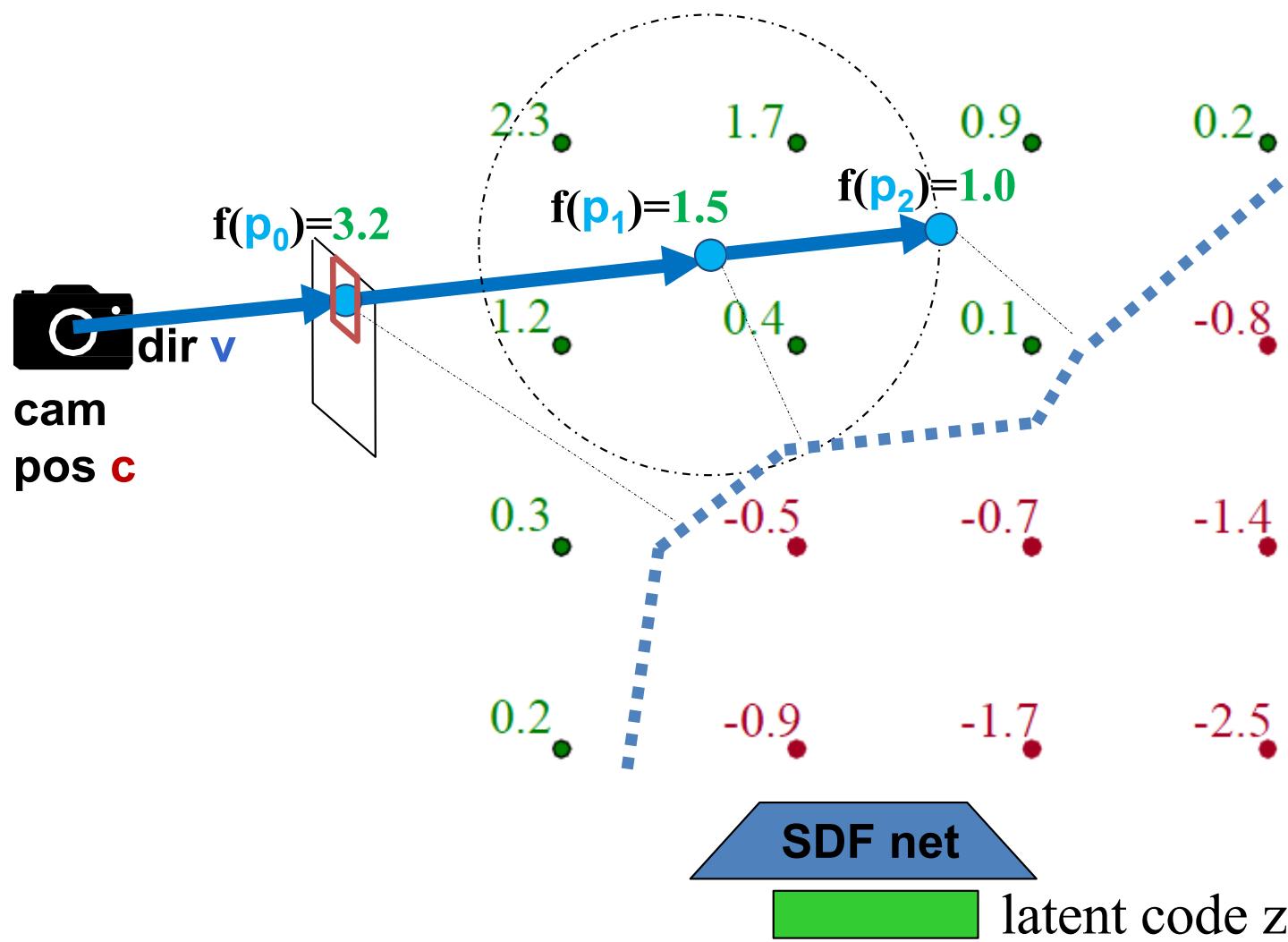
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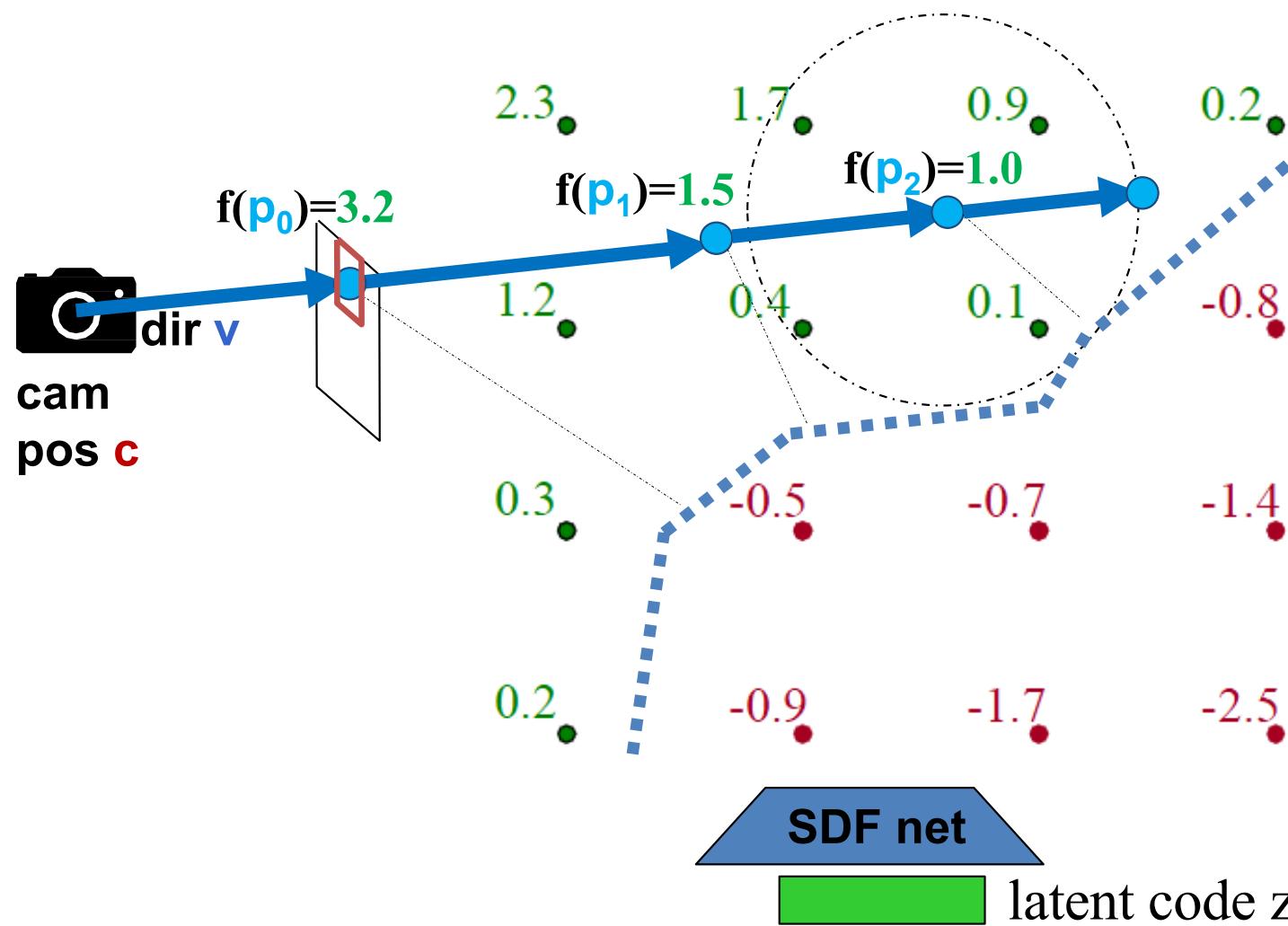
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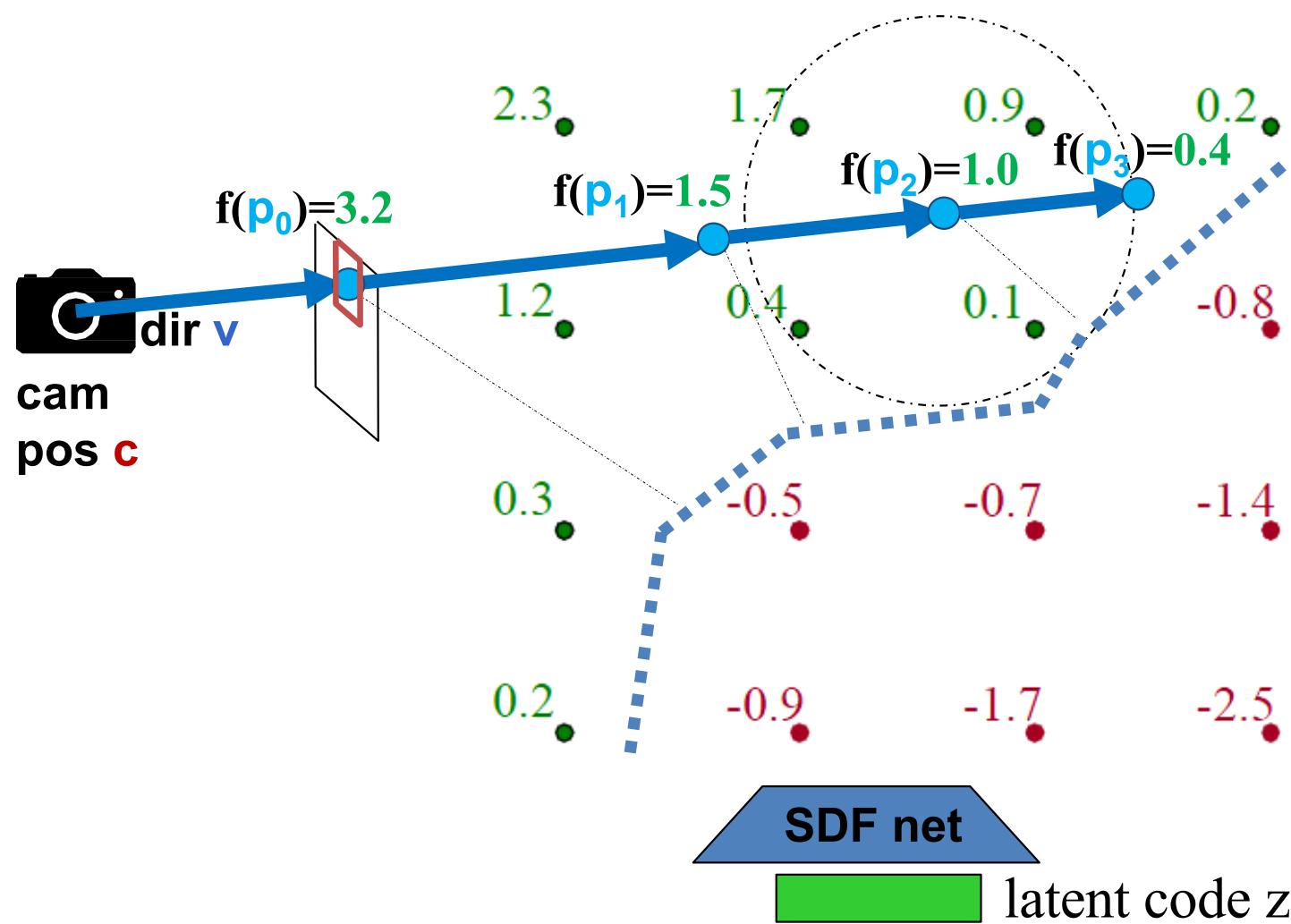
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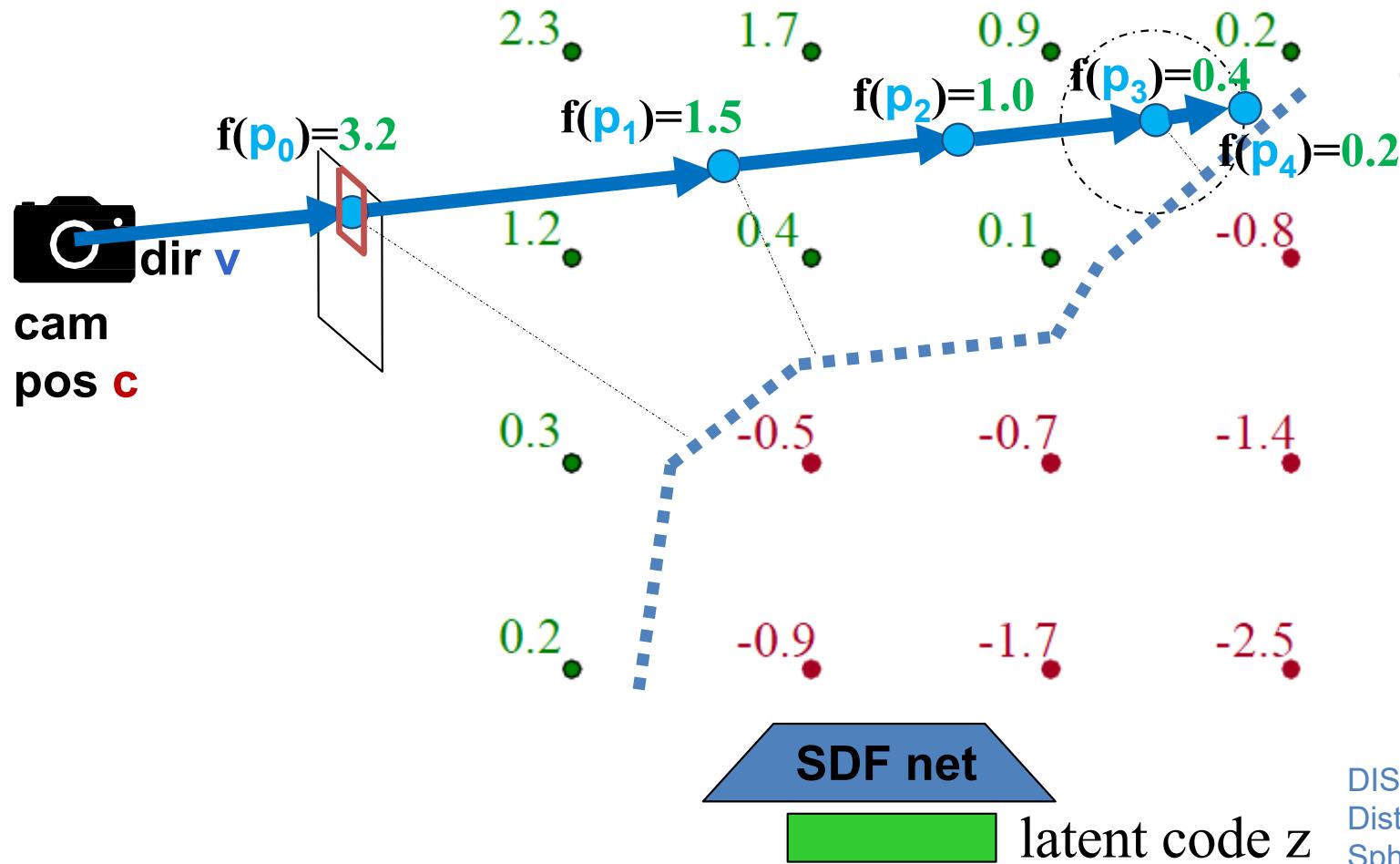
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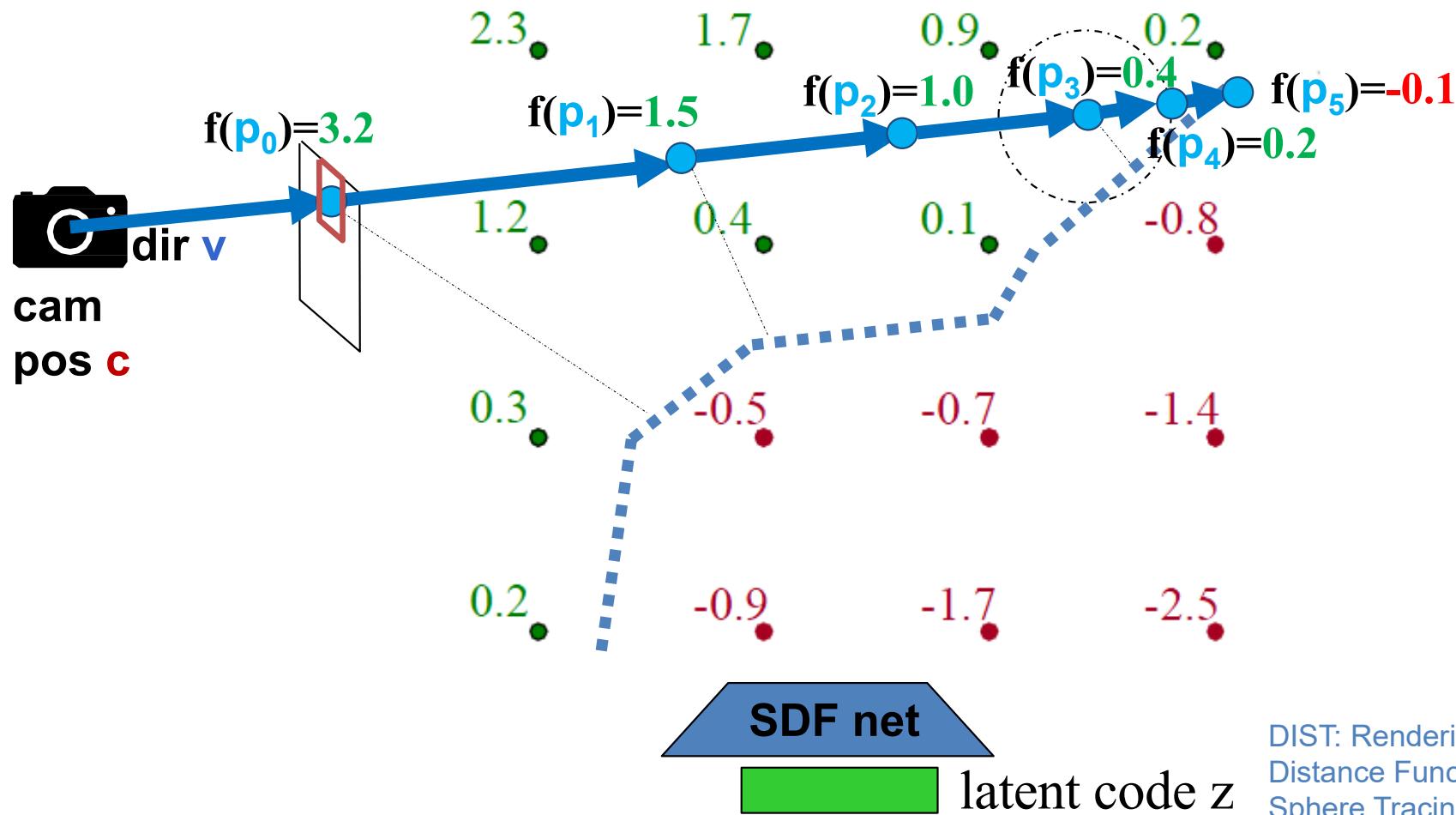
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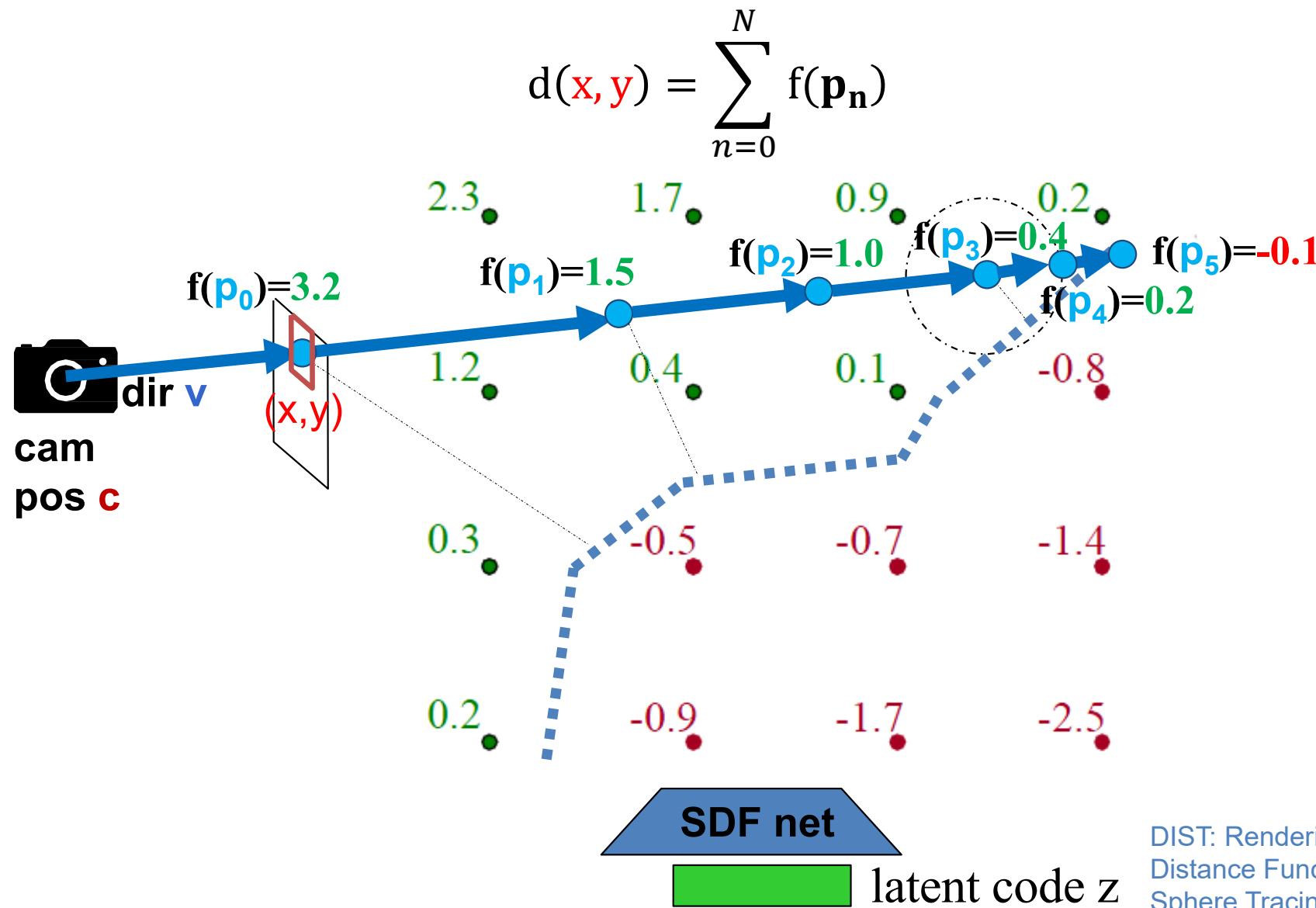
# How to perform ray tracing for implicits?

... stop when you are very close to the surface based on a threshold OR when the ray comes out of the grid limits (hits background)



# How to perform ray tracing for implicits?

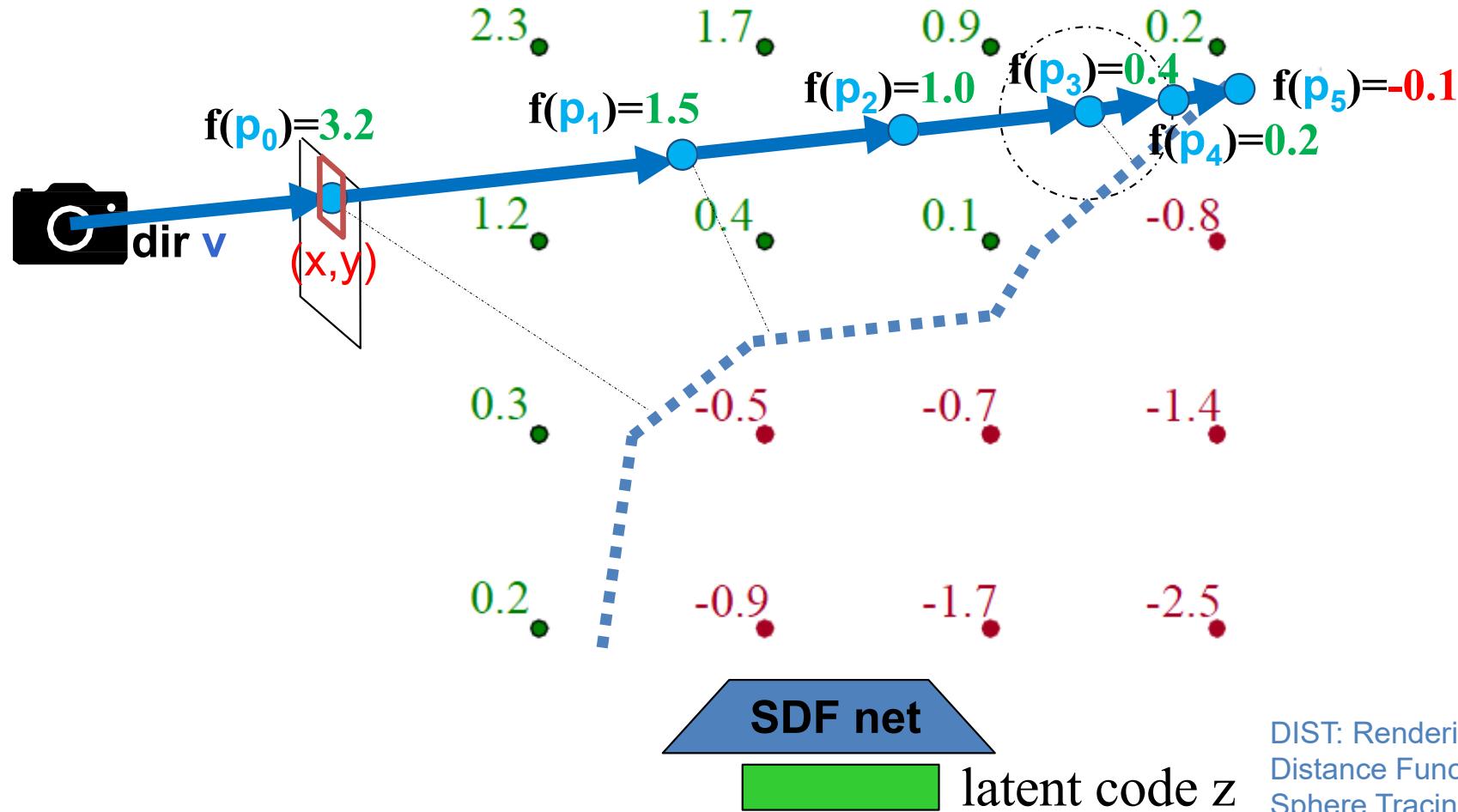
Distance traveled for that ray:



# Differentiable sphere ray tracing

Distance traveled is **differentiable!** ( $\theta$  are the network parameters)

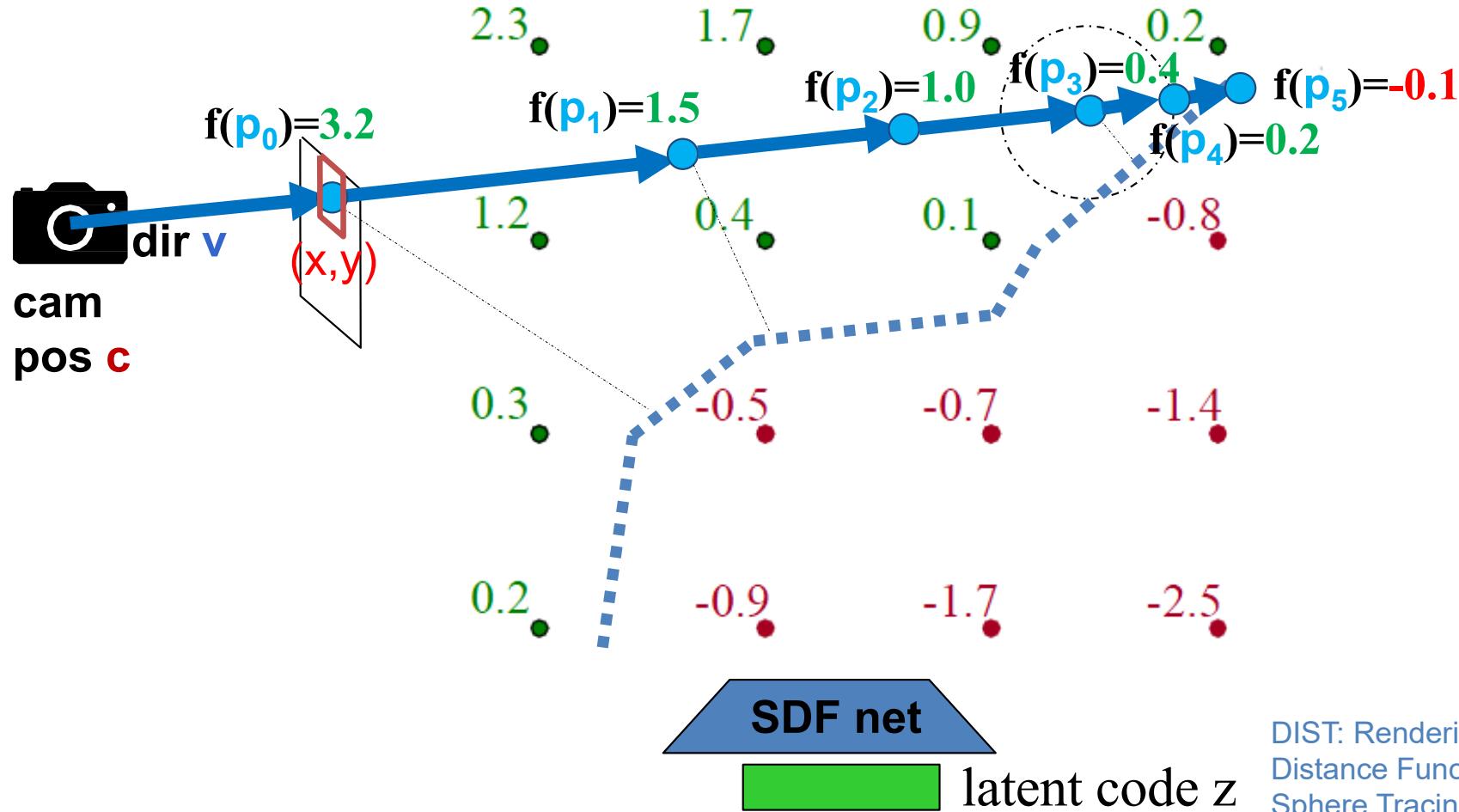
$$\frac{\partial d(\mathbf{x}, \mathbf{y})}{\partial \theta} = \sum_{n=0}^N \frac{\partial f(\mathbf{p}_n)}{\partial \theta}$$



# How to perform ray tracing for implicits?

Intersection point can be computed from camera position  $\mathbf{c}$ , ray direction  $\mathbf{v}$ , and distance traveled:

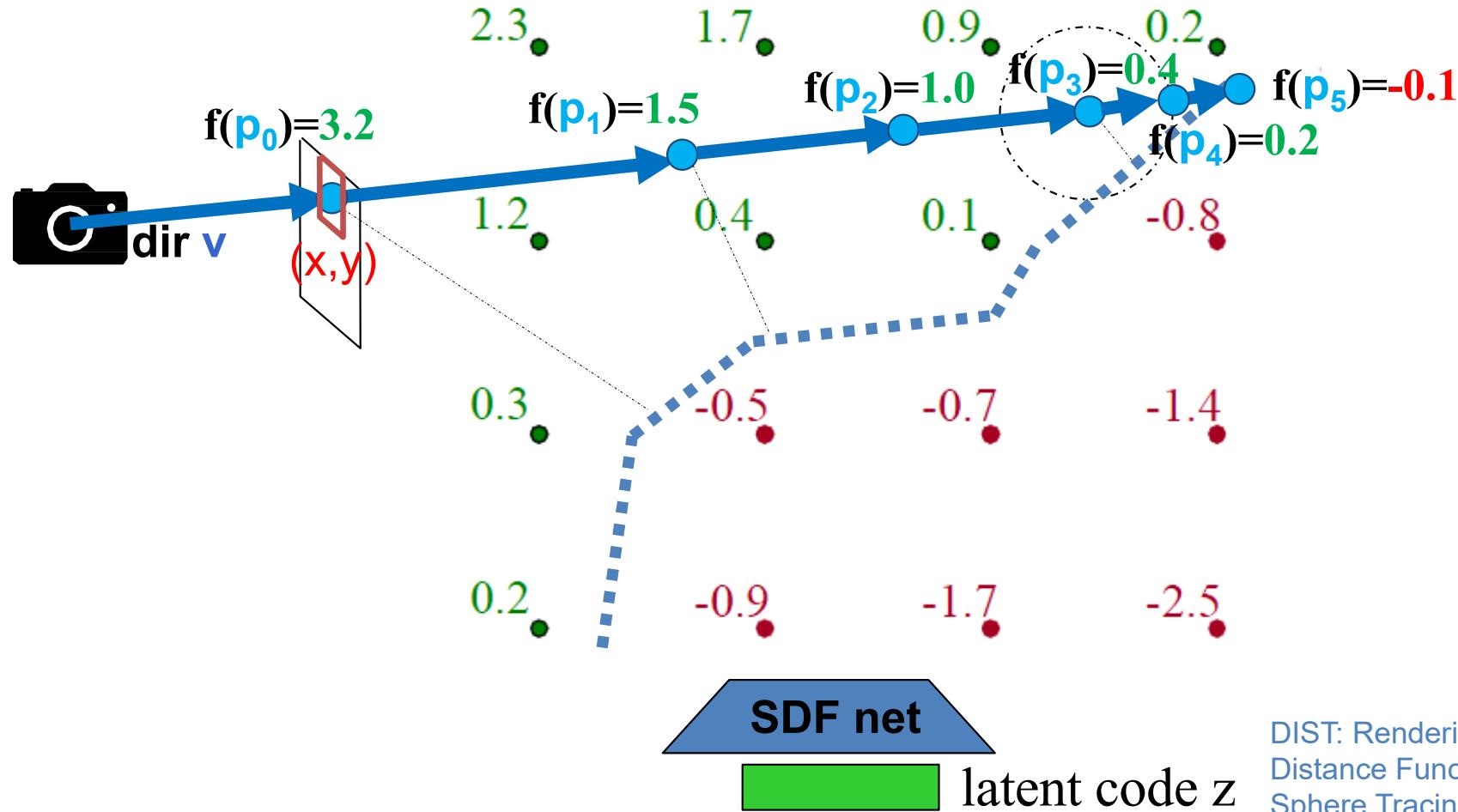
$$\mathbf{p}(x, y) = \mathbf{c} + d(x, y) \mathbf{v} \quad (\text{differentiable})$$



# Differentiable sphere ray tracing

Surface normal observed at pixel  $(x, y)$  ... also **differentiable**:

$$\mathbf{n}(\mathbf{x}, \mathbf{y}) = \frac{\nabla f(\mathbf{p}_N)}{\|\nabla f(\mathbf{p}_N)\|} \quad (\dots \text{thus basic shading can also be differentiable})$$



# A few results

Synthetic #1



Synthetic #2



Real-world #1



Real-world #2



DIST: Rendering Deep Implicit Signed Distance Function with Differentiable Sphere Tracing, Liu et al., CVPR 2020

# A few results

Synthetic #1

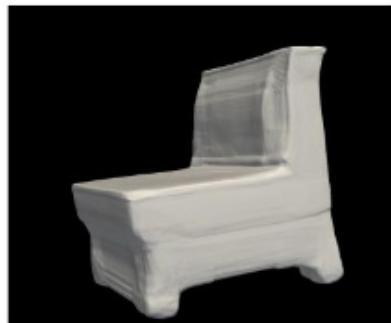


Synthetic #2



Appearance/texture is missed  
Intersection points might be unreliable  
(esp at early stages of training)

Real-world #1



Real-world #2



# Neural Volumetric Rendering

# Neural Volumetric **Rendering**

querying the radiance value  
along rays through 3D space



What color?

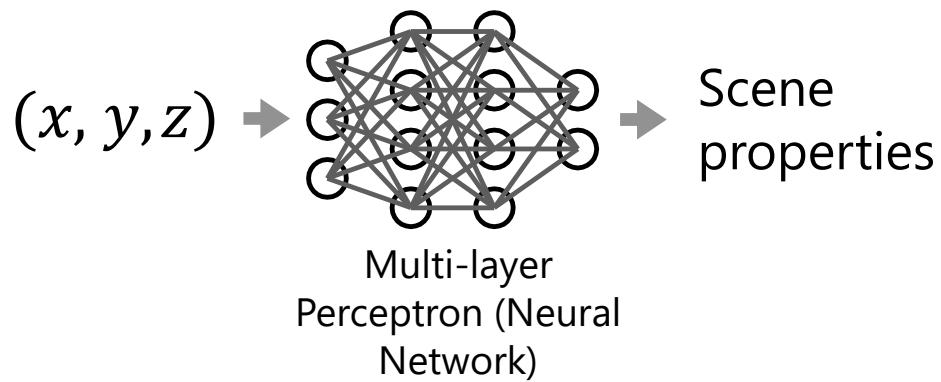
# Neural Volumetric Rendering

continuous, differentiable  
rendering model without  
concrete ray/surface intersections



# Neural Volumetric Rendering

using a neural network as a scene representation, rather than a voxel grid of data





Given a set of sparse views of an object with known camera poses

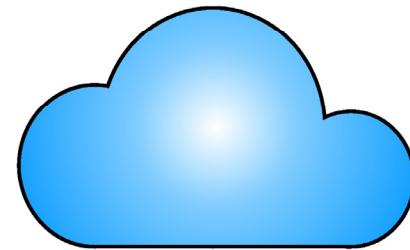
Optimize a NeRF model



3D reconstruction viewable from any angle

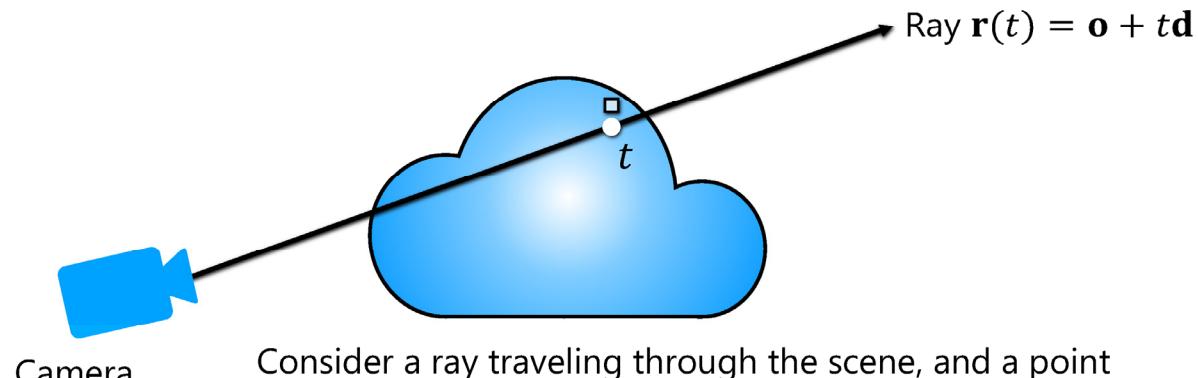
NeRF: Representing Scenes as Neural Radiance Fields for View Synthesis, Mildenhall et al. ECCV20

# Volumetric formulation for NeRF



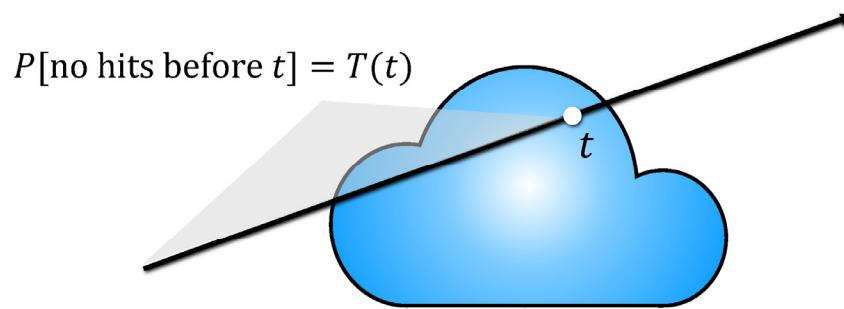
Scene is a cloud of colored fog

# Volumetric formulation for NeRF



NeRF: Representing Scenes as  
Neural Radiance Fields for View  
Synthesis, Mildenhall et al. ECCV20

# Volumetric formulation for NeRF



But  $t$  may also be blocked by earlier points along the ray.  $T(t)$ : probability that the ray didn't hit any particles earlier.

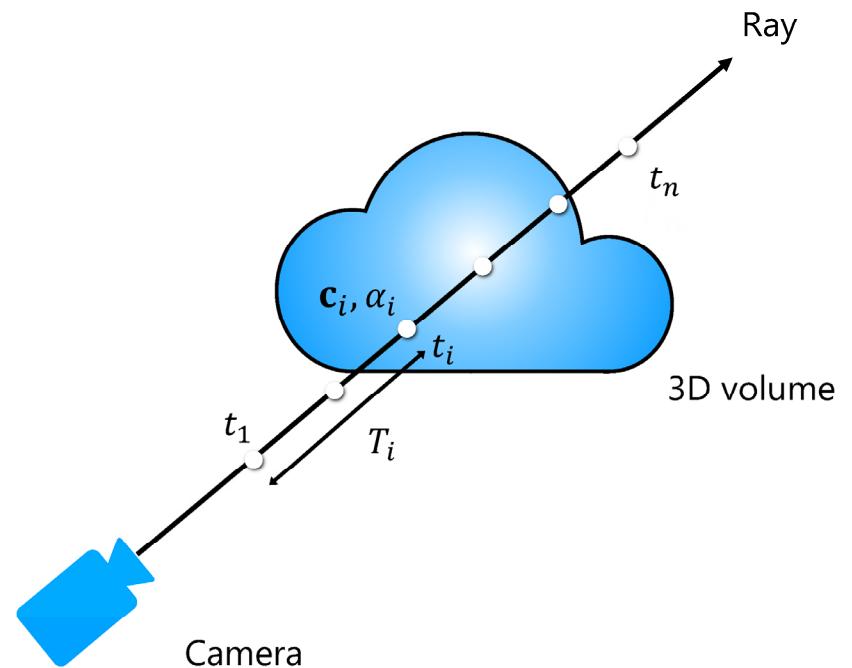
$T(t)$  is called "transmittance"

# Volume rendering estimation: integrating color along a ray

Rendering model for ray  $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$ :

$$\mathbf{c} \approx \sum_{i=1}^n T_i \alpha_i \mathbf{c}_i$$

final rendered color along ray      weights      colors

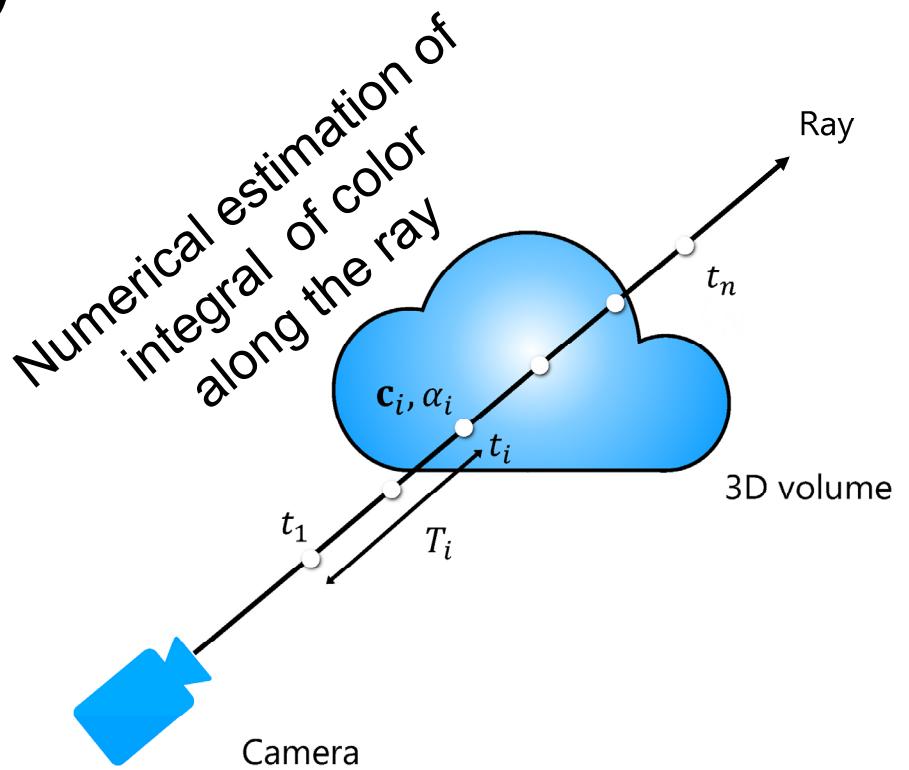


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NeRF: Representing Scenes as Neural Radiance Fields for View Synthesis, Mildenhall et al. ECCV20

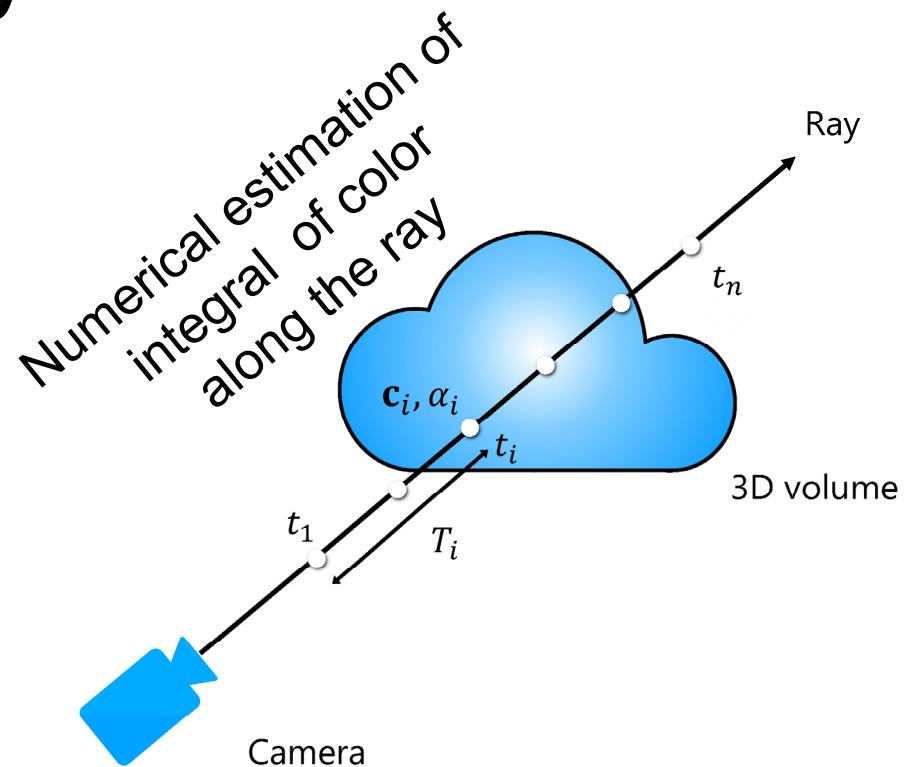
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final rendered color along ray      weights      colors

Computing the color for a set of rays through the pixels of an image yields a rendered image



NeRF: Representing Scenes as Neural Radiance Fields for View Synthesis, Mildenhall et al. ECCV20

# Volume rendering estimation: integrating color along a ray

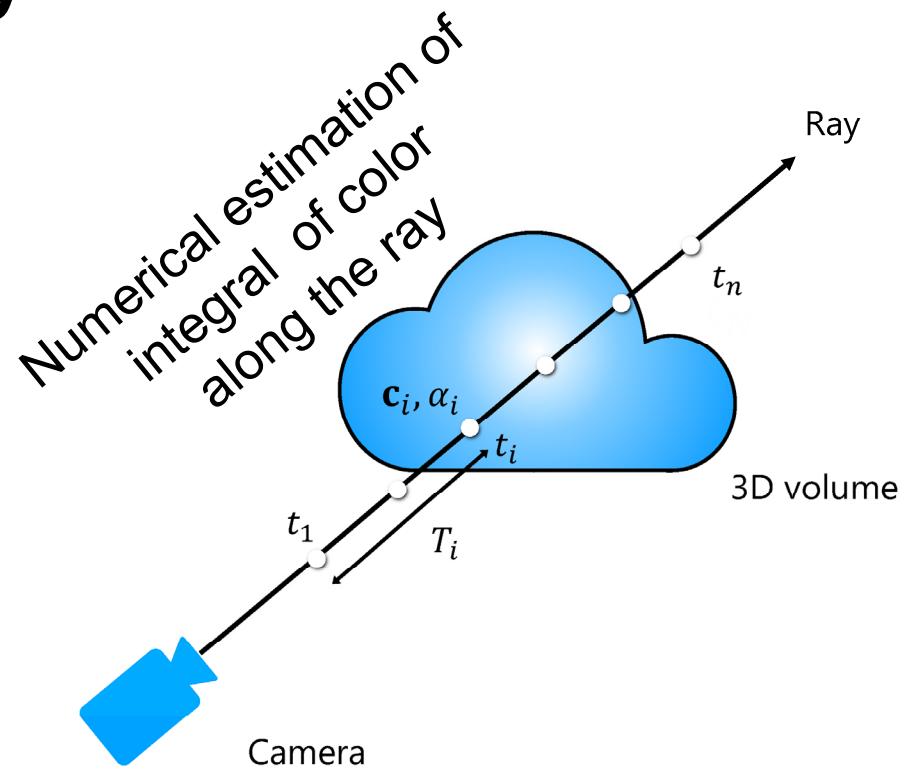
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$$\mathbf{c} \approx \sum_{i=1}^n T_i \alpha_i \mathbf{c}_i$$

final rendered color along ray      weights      colors

How much light is blocked earlier along ray:

$$T_i = \prod_{j=1}^{i-1} (1 - \alpha_j)$$



# Volume rendering estimation: integrating color along a ray

Rendering model for ray  $\mathbf{r}(t) = \mathbf{o} + t\mathbf{d}$ :

$$\mathbf{c} \approx \sum_{i=1}^n T_i \alpha_i \mathbf{c}_i$$

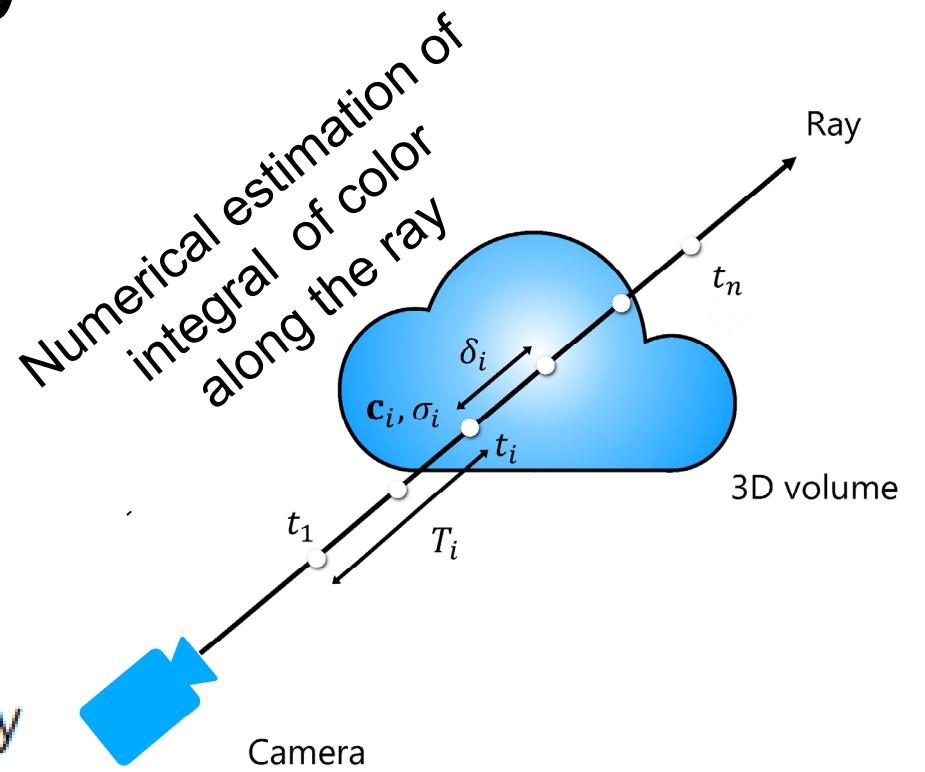
final rendered color along ray      weights      colors

How much light is blocked earlier along ray:

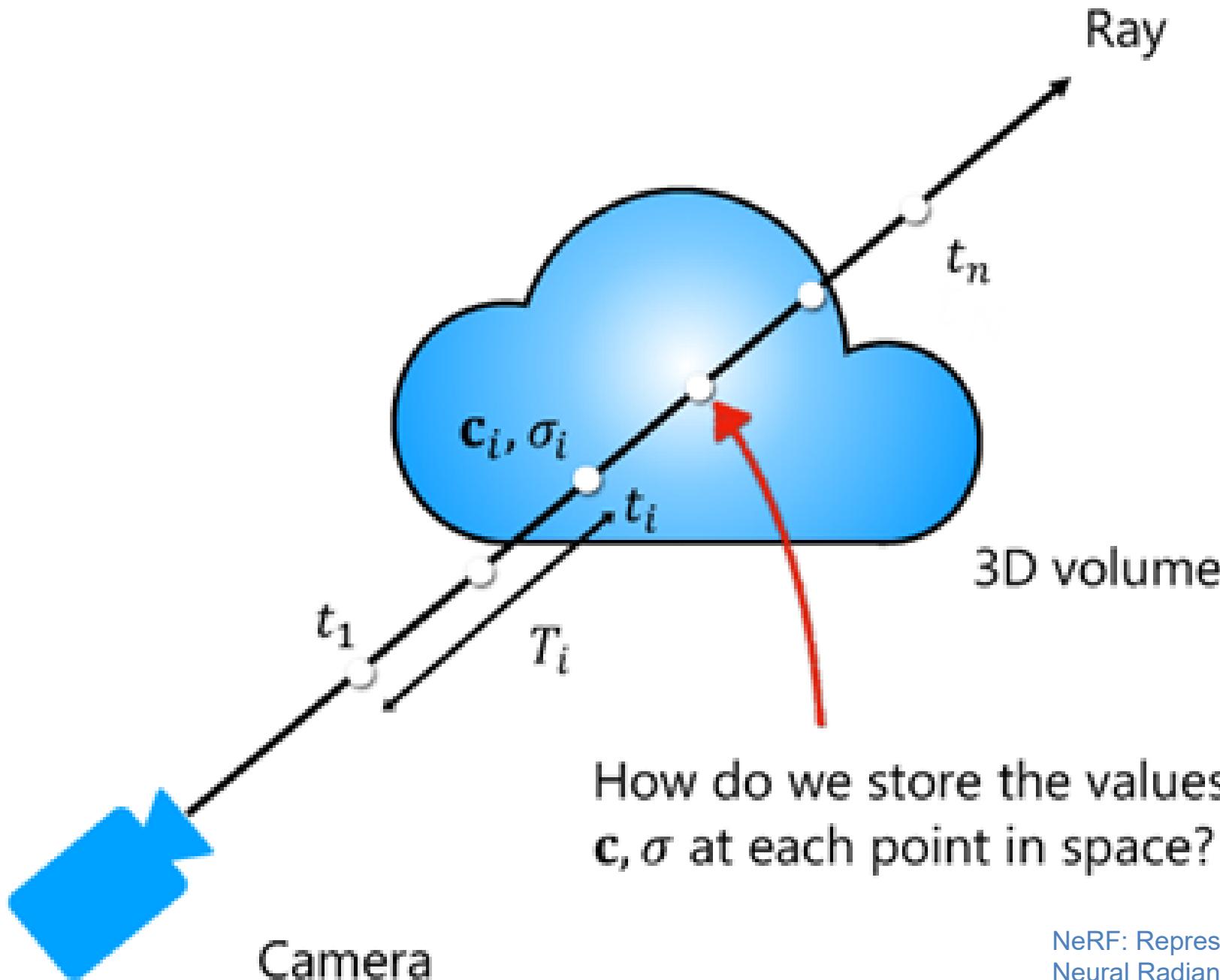
$$T_i = \prod_{j=1}^{i-1} (1 - \alpha_j)$$

$\alpha$  is derived from a stored volume density sigma ( $\sigma$ ) multiplied by the distance between samples delta  $\delta$ :

$$\alpha_i = 1 - \exp(-\sigma_i \delta_i)$$



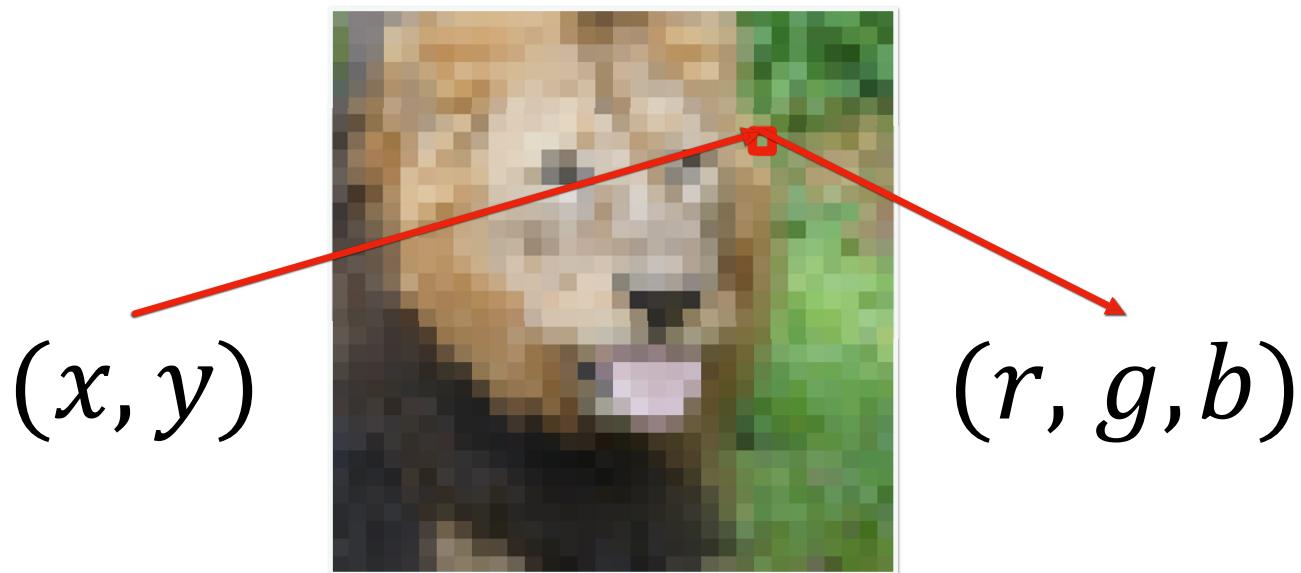
NeRF: Representing Scenes as Neural Radiance Fields for View Synthesis, Mildenhall et al. ECCV20



How do we store the values of  
 $c, \sigma$  at each point in space?

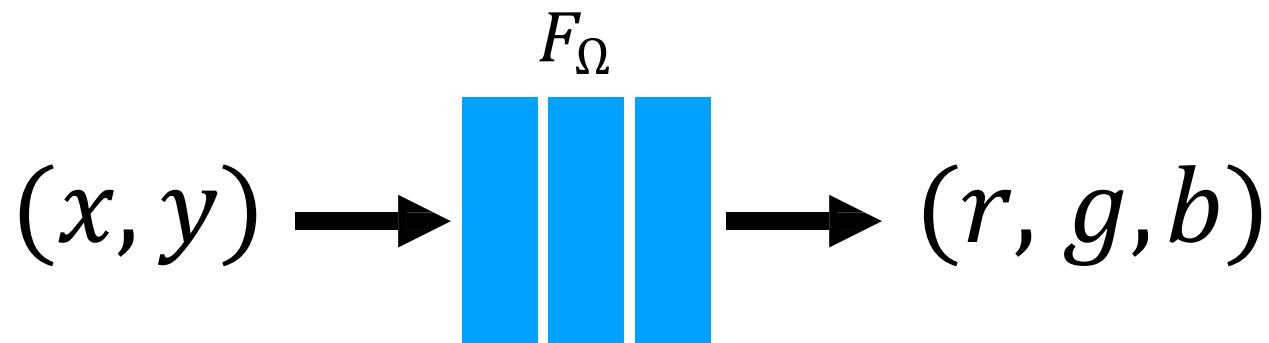
NeRF: Representing Scenes as  
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Synthesis, Mildenhall et al. ECCV20

## Toy problem: storing 2D image data



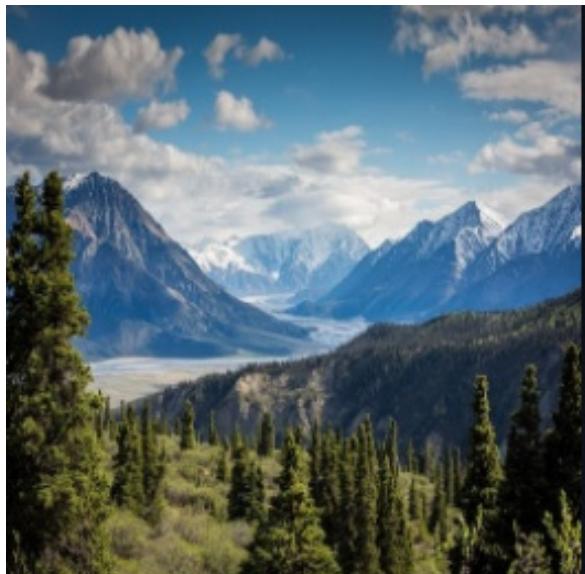
Usually we store an image as a  
2D grid of RGB color values

Toy problem: storing 2D image data

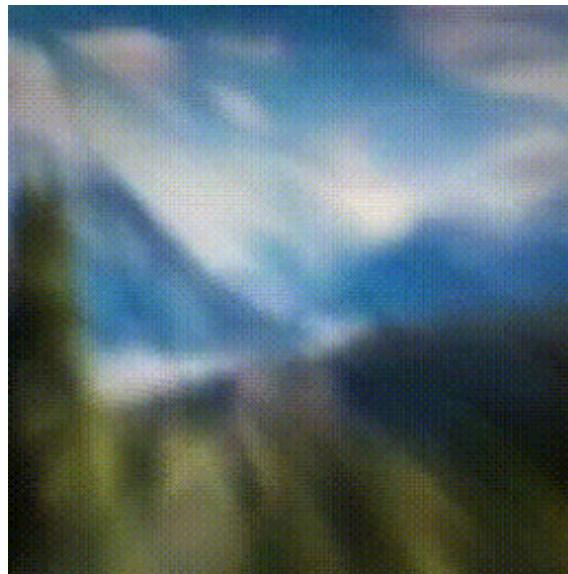


What if we train a simple fully-connected network (MLP) to do this instead?

# Naive approach fails!



Ground truth image



Neural network output fit  
with gradient descent

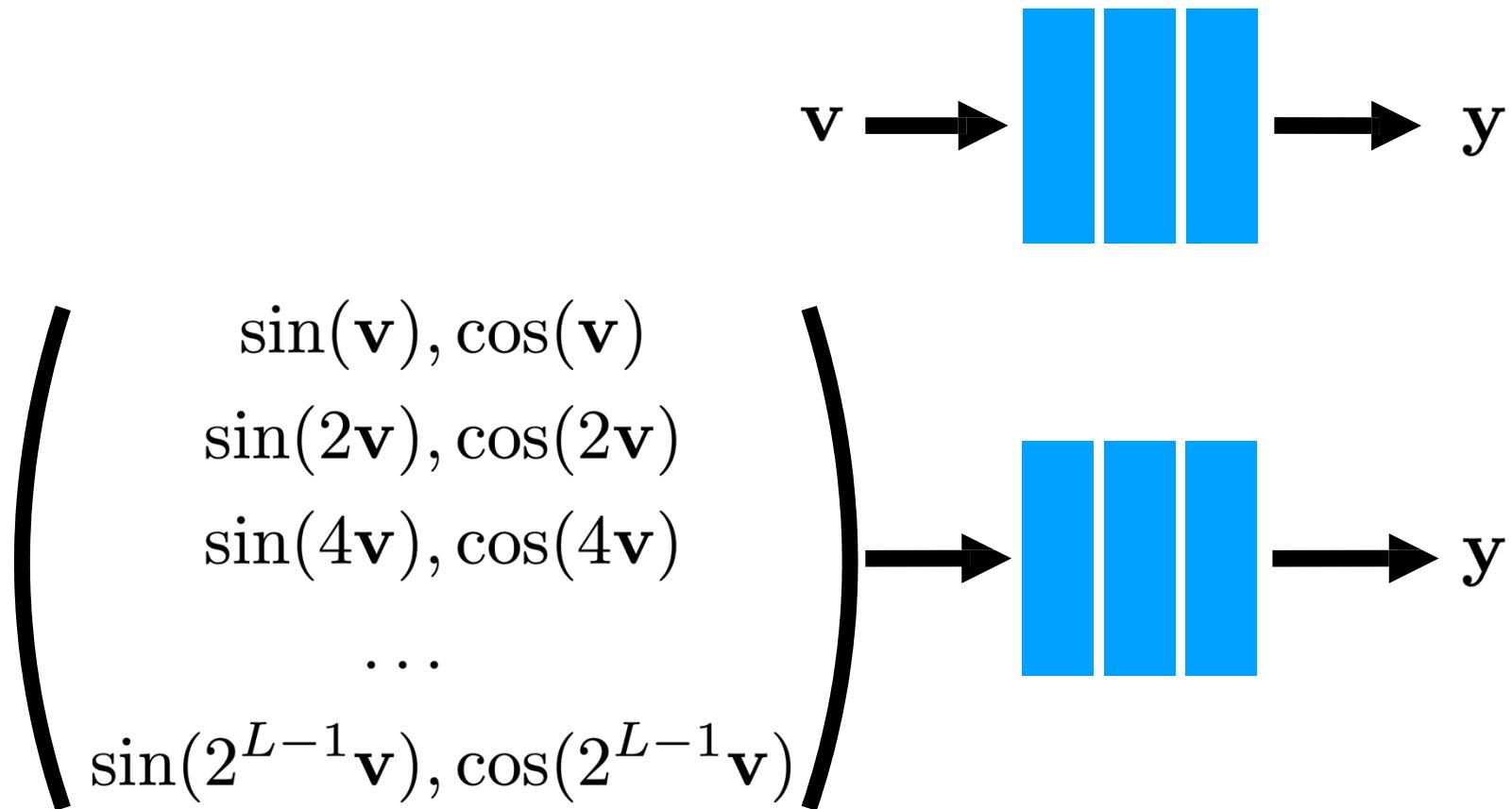
## Problem:

“Standard” coordinate-based MLPs cannot represent  
high frequency functions

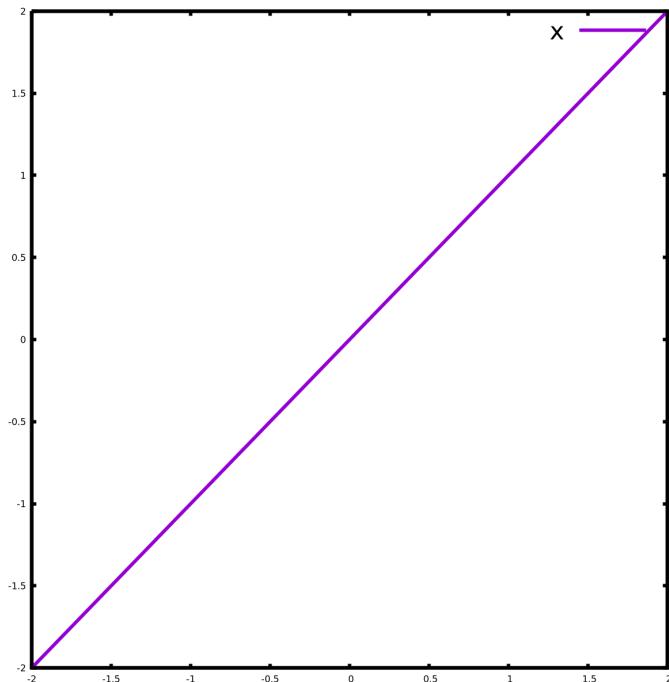
## Solution:

Pass input coordinates through a  
high frequency mapping first

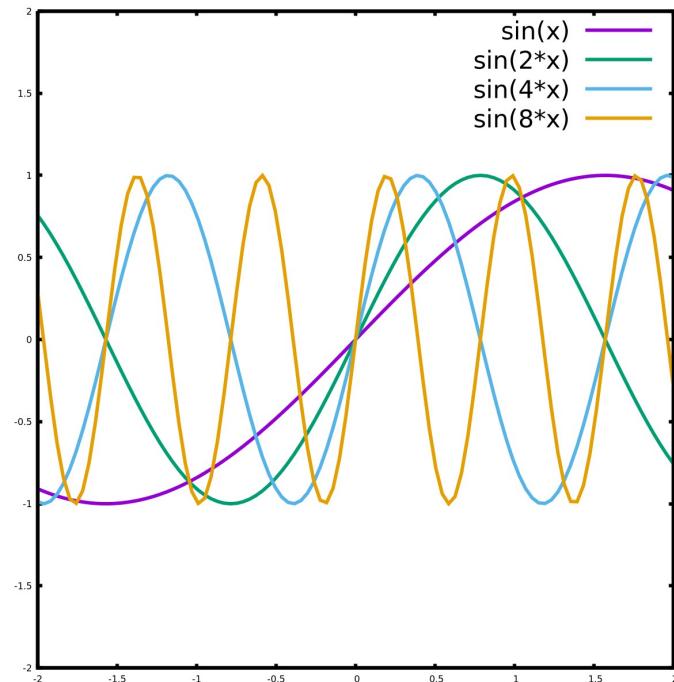
## Example mapping: “positional encoding”



# Positional encoding

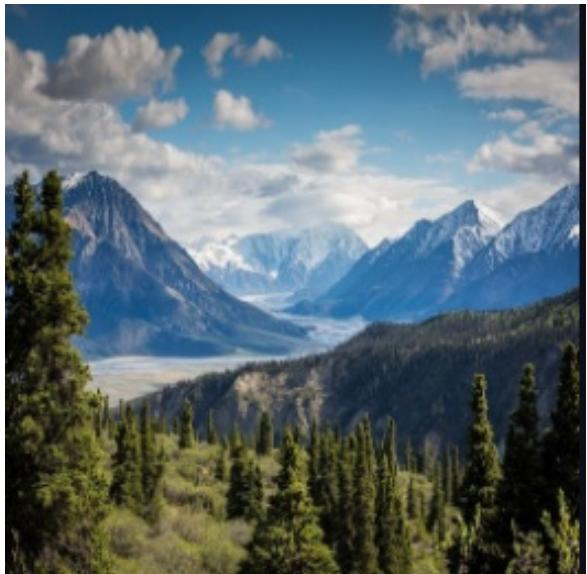


Raw encoding of a number  $\mathbf{x}$

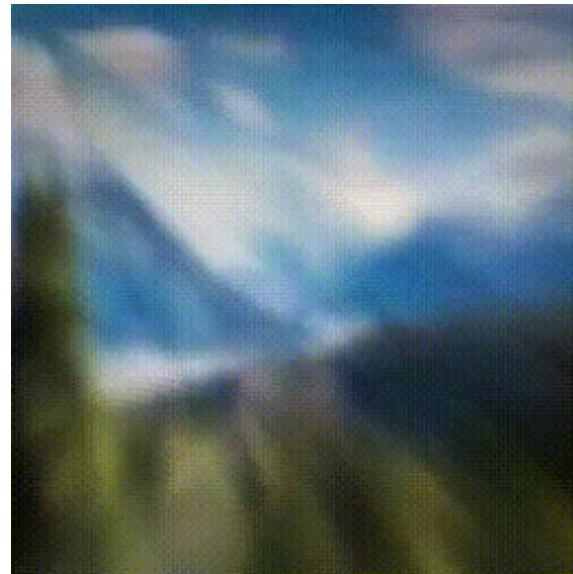


"Positional encoding" of a number  $\mathbf{x}$

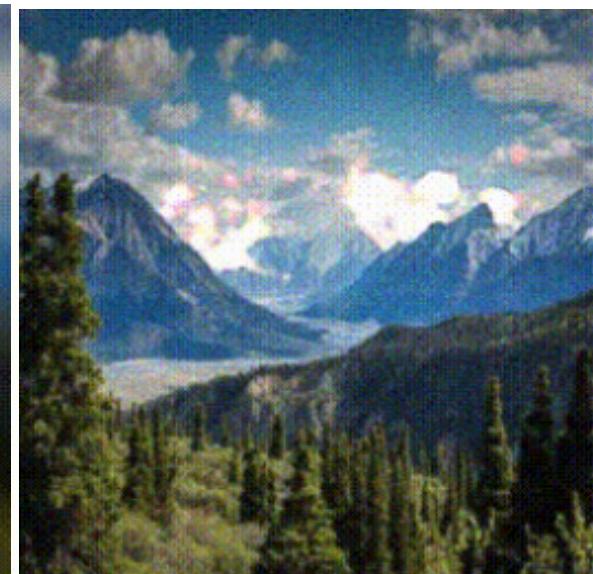
# Problem solved!



Ground truth image



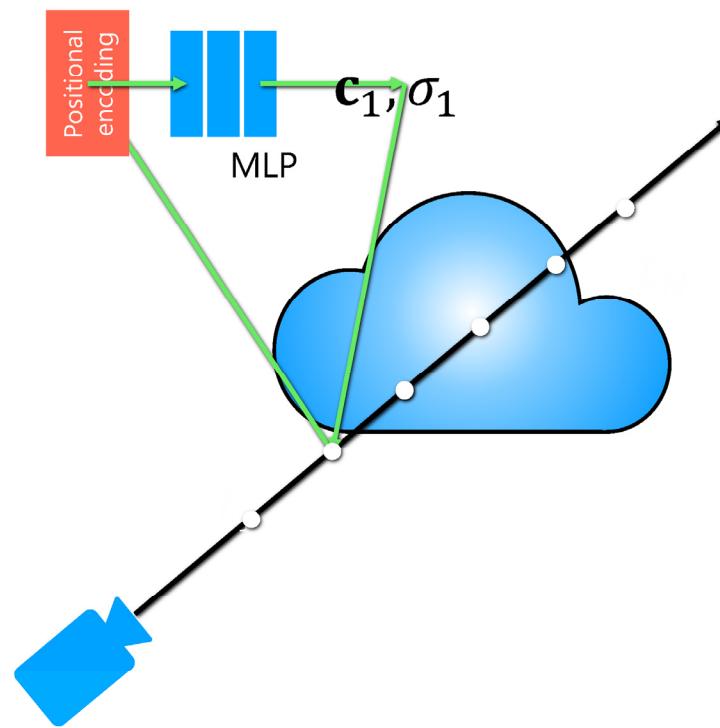
Neural network output without  
high frequency mapping



Neural network output with  
high frequency mapping

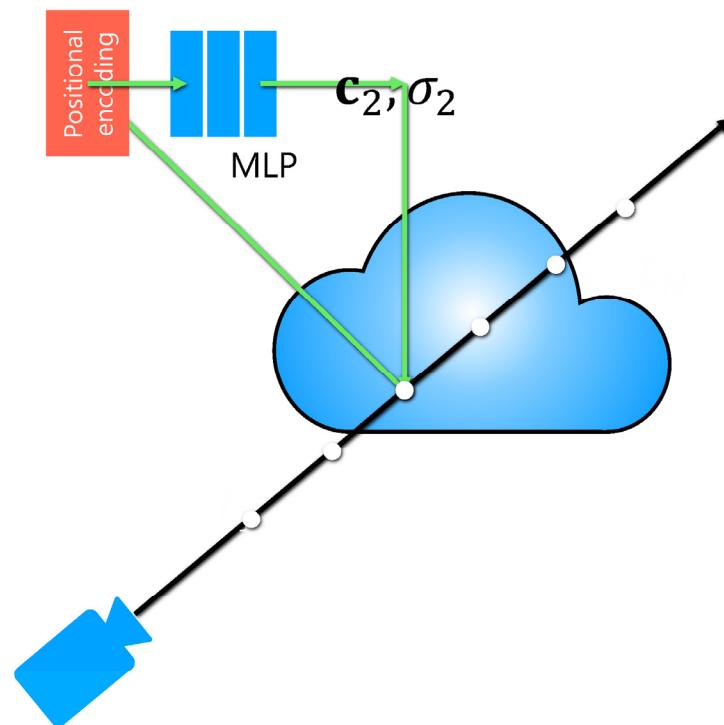
$\text{NeRF} = \text{volume rendering} +$   
 $\text{coordinate-based network}$

How do we store the values of  $\mathbf{c}, \sigma$  at each point in space



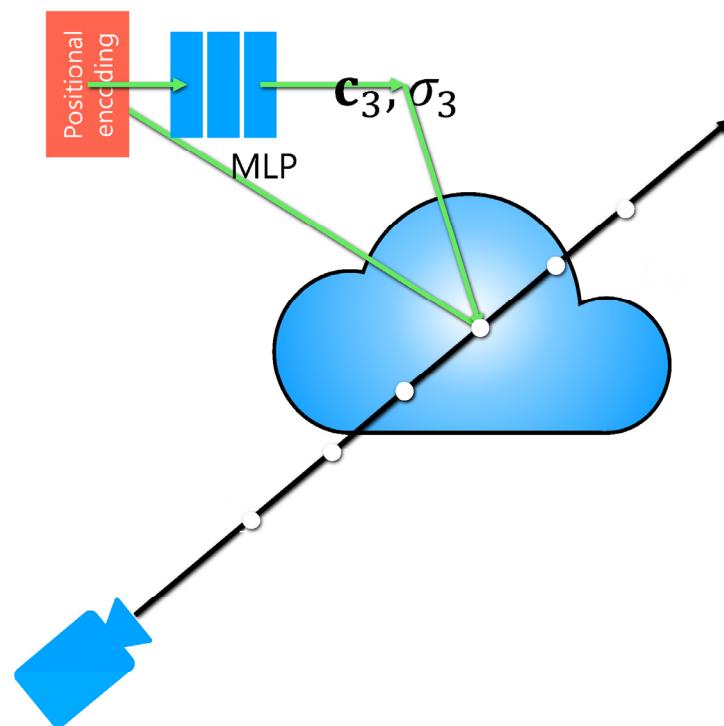
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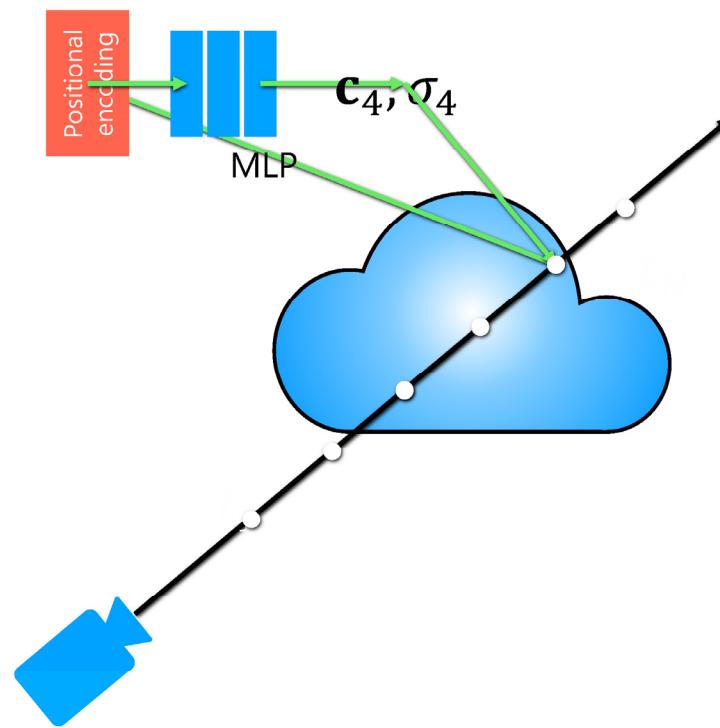
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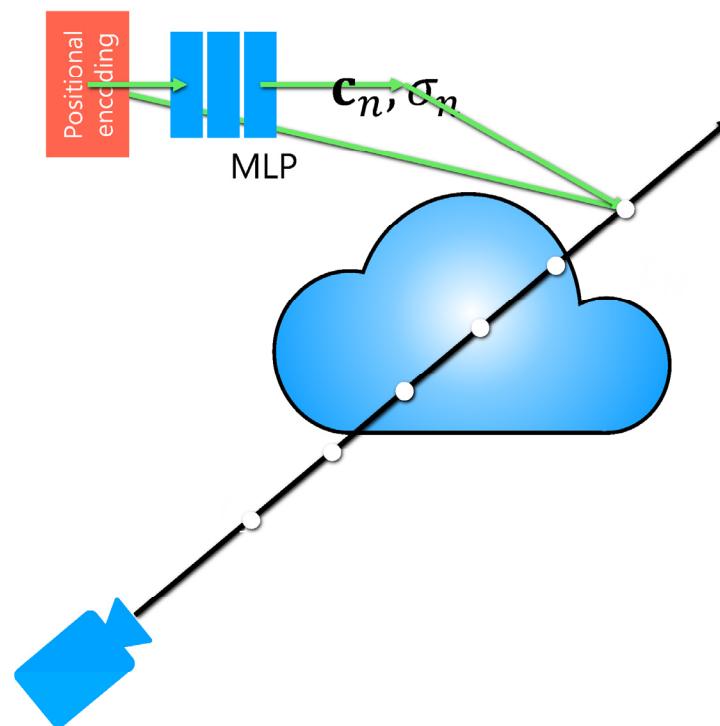
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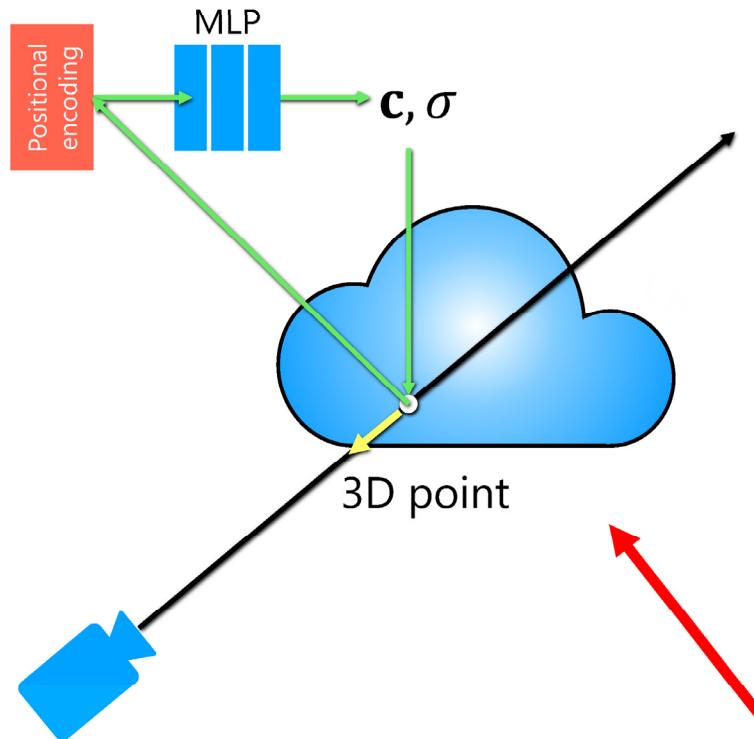


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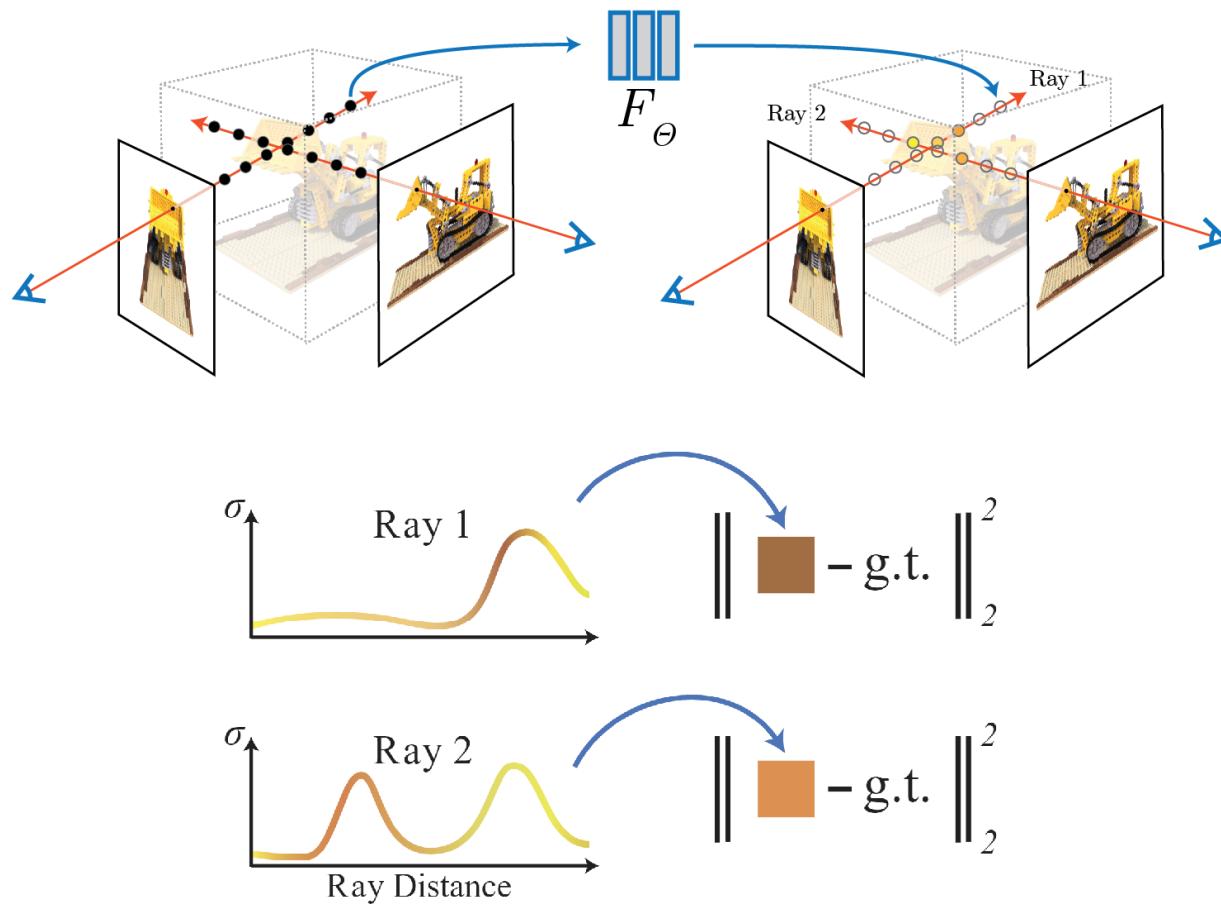


# Extension: view-dependent field



Include the ray direction in the input to the MLP → allows for capturing and rendering view-dependent effects (e.g., shiny surfaces)

# Putting it all together



NeRF: Representing Scenes as  
Neural Radiance Fields for View  
Synthesis, Mildenhall et al. ECCV20

# Results

# NeRF: Representing Scenes as Neural Radiance Fields for View Synthesis

Ben Mildenhall\*  
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Pratul P. Srinivasan\*  
UC Berkeley

Matthew Tancik\*  
UC Berkeley

Jonathan T. Barron  
Google Research

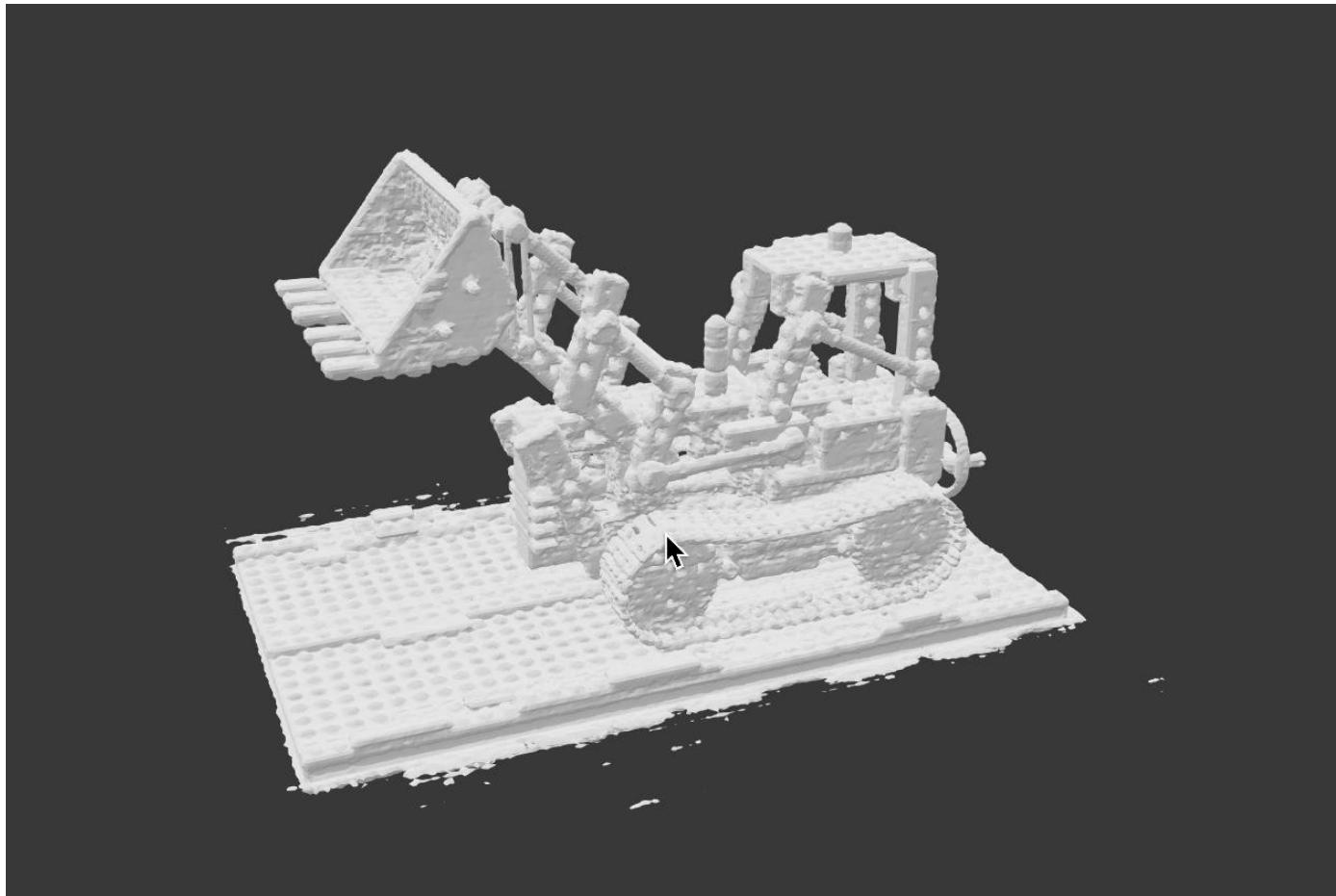
Ravi Ramamoorthi  
UC San Diego

Ren Ng  
UC Berkeley

\* Denotes Equal Contribution

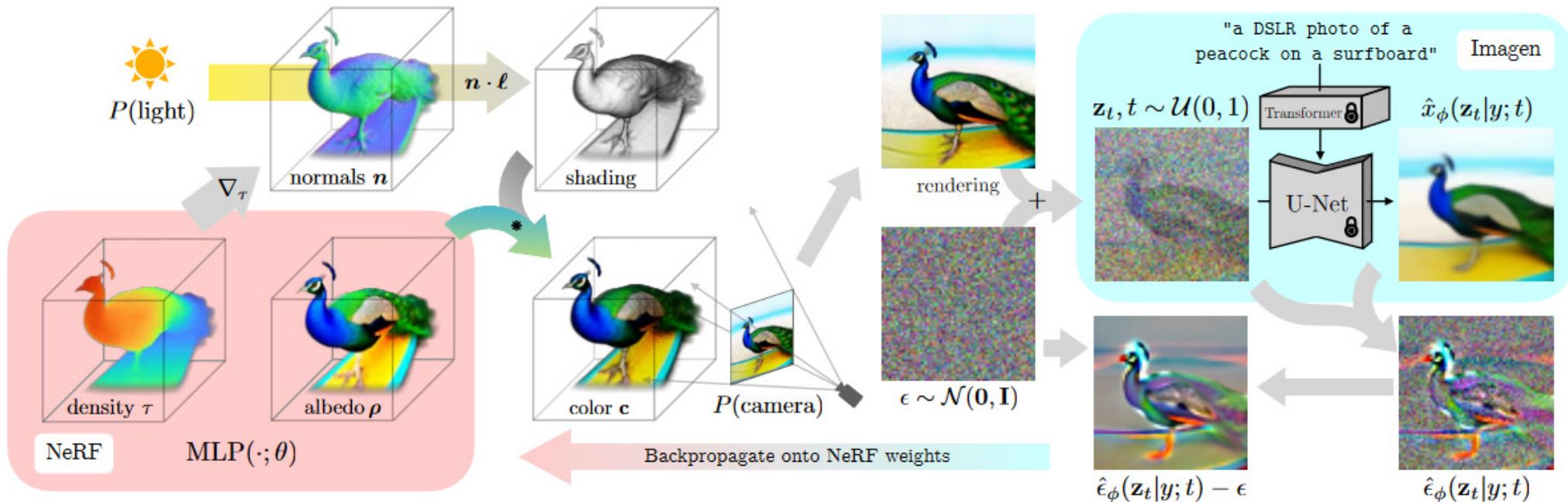


NeRF encodes detailed scene geometry



# DreamFusion!

<https://dreamfusion3d.github.io/>



Train NeRF scene representation  
guided by text-conditioned diffusion model

# What we covered

1. Image and Shape Representations
2. Introduction to Neural Networks
3. 3D Deep Learning: the multi-view approach
4. 3D Deep Learning: the voxel-based approach
5. 3D Deep Learning: the point-based approach
6. 3D Deep Learning: the mesh-based approach
7. 3D Generative Models: GANs, VAEs, diffusion
8. 3D Generative models: Generating views, voxels, octrees, points, implicits, meshes
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**Many open research challenges / questions:**

**What is the right 3D representation to generate for each task?**

**Generate interactive 3D models (not static) – neural 4D reconstruction/animation**

**Zero-shot learning – test on categories not seen during training**

**Leverage text + 2D + 3D supervision, self-supervision**

# Tasks to finish

- Submit assignment 5 [**this Fri, NO EXTENSIONS**]
- Presentation for 574 [**May 25, 5pm, NO EXTENSIONS**]
- Project for 674 [**May 25, 5pm, NO EXTENSIONS**]