Introduction to Bayesian Inference

Social Statistics Research Away Day

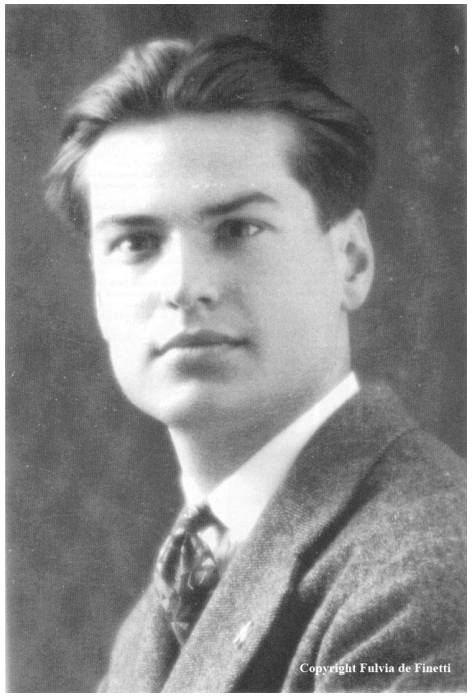
16/05/2024

Definition of probability

- Subjective

“Probability does not exist”

Bruno de Finetti (1906 – 1985)



Bayes Theorem & Posterior distribution

$$P(A|B)= \frac{P(B|A)P(A)}{P(B)}$$

$$P(\text{Unknown}|\text{Data})= \frac{P(\text{Data}|\text{Unknown})P(\text{Unknown})}{P(\text{Data})}$$

$$P(\text{Unknown}|\text{Data}) \propto P(\text{Data}|\text{Unknown})\, P(\text{Unknown})$$

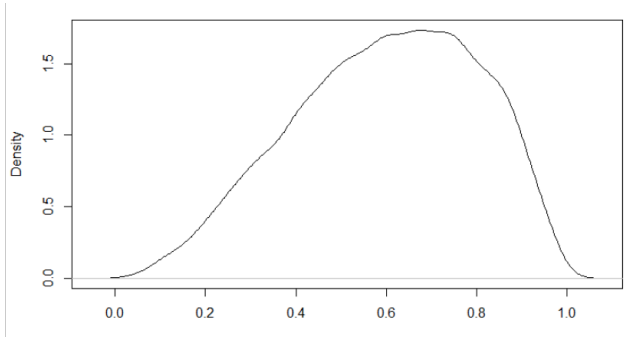
posterior

likelihood
(model for data)

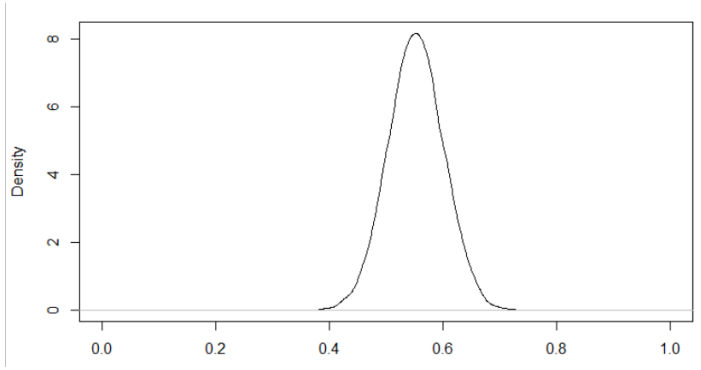
prior

Binomial distribution

- Prior for θ:
  - θ ~ Beta(a=3, b=2)
  - E(θ) = 0.6, SD(θ) = 0.2
- n = 100, x = 55 (e.g. boys born)

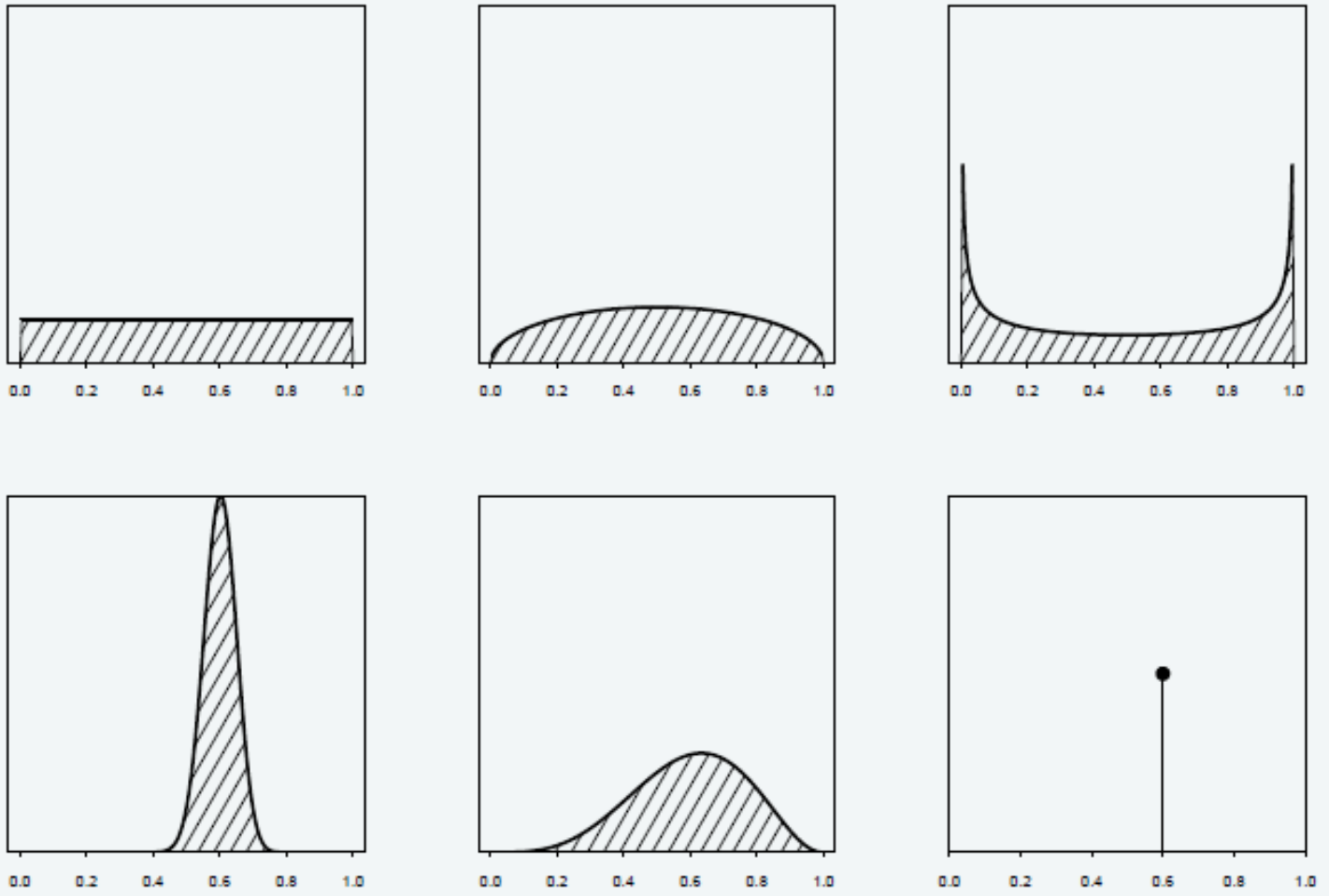


- Posterior:
  - θ|x ~ Beta(a+x , b+n-x) ➔ Beta(3+55 , 2+100-55)
  - E(θ|x) = 0.552, SD(θ|x) = 0.049



Binomial distribution

- Different types of priors:
  weakly informative (top) and informative (bottom)



Frequentist vs Bayesian

- **Frequentist**:
- “Evaluates the accuracy of an estimate of an unknown value in terms of how different this estimate could be”
- unknown value (parameter, prediction) is fixed

- **Bayesian:**
- “Updates beliefs about the true unknown value (parameter of prediction)”
- Beliefs are expressed using probabilities
- → unknown value has a probability distribution