Annual Sharpe Ratio

Question: Why do we multiply a daily Sharpe ratio by $\sqrt{252}$ in order to get an annual Sharpe ratio?
Answer:

In this class, we assume that each day's returns are independent and identically distributed (iid) random variables. As discussed in the "Calculations for Investment Analysis.pdf" document, if we are working in log returns and we want to know the return over multiple periods, we just sum the returns over each of the individual periods between two points in time. Therefore, in other words, if we want to get the return over an entire year, when working with log returns (but not regular returns), all we need to do is to sum the 252 daily returns over the course of the year. That is:

$$r_{yearly} = \sum_{i=1}^{252} r_{daily}^{i} \tag{1}$$

For the Sharpe ratio, we're interested in excess returns. It is relatively easy, and we leave it as an exercise for you, to check that all of the calculations we perform here work regardless of whether we are using returns or excess returns.

The Sharpe ratio is the mean excess return divided by its standard deviation. So, if we want a daily Sharpe ratio, it is:

$$\text{Sharpe}_{daily} = \frac{\mathbf{E}\left[r_{daily}\right]}{\text{SD}(r_{daily})}$$

If we want an annual Sharpe ratio, it is:

$$\text{Sharpe}_{annual} = \frac{\mathbf{E}\left[r_{annual}\right]}{\text{SD}(r_{annual})}$$

So, the question is, how do we calculate $[r_{annual}]$ and $\text{SD}(r_{annual})$?

Suppose we have 10 years of data. One option would be to take each of the ten consecutive one-year periods in that data and calculate the return for each. We could then take a mean and standard deviation of those ten observations. But, from a statistical perspective (for reasons that go beyond this course) that is not a very efficient way to calculate an annual mean or standard deviation.

Instead, it is much more efficient to use the assumption that the daily returns are iid random variables and use the fact that the annual return is just a sum of those iid random variables. The properties of sums of iid random variables is a pretty basic topic in statistics and probably familiar to most of you in this course. In particular, given Equation 1 we have:

$$\mathbf{E}\left[r_{yearly}\right] = \mathbf{E}\left[\sum_{i=1}^{252} r_{daily}^{i}\right] = 252\mathbf{E}\left[r_{daily}^{i}\right]$$

Similarly, we have:

$$\text{Var}\left[r_{yearly}\right] = \text{Var}\left[\sum_{i=1}^{252} r_{daily}^{i}\right] = 252\text{Var}\left[r_{daily}^{i}\right]$$

(in the last step we used the fact that the daily returns are iid and thus the variance of their sum is just the sum of their variances - i.e. all covariances between them are zero).

Thus, we have

$$\text{SD}\left(r_{yearly}\right) = \sqrt{\text{Var}\left[r_{yearly}\right]} = \sqrt{252\text{Var}\left[r_{daily}^{i}\right]} = \sqrt{252}SD\left(r_{daily}\right)$$

Thus, we see:

$$\text{Sharpe}_{yearly} = \frac{\mathbf{E}\left[r_{yearly}\right]}{\text{SD}\left(r_{yearly}\right)} = \frac{252\mathbf{E}\left[r_{daily}^{i}\right]}{\sqrt{252}SD\left(r_{daily}\right)} = \left(\frac{252}{\sqrt{252}}\right)\frac{\mathbf{E}\left[r_{daily}\right]}{\text{SD}(r_{daily})} = \sqrt{252}\text{Sharpe}_{daily}$$

Additional Complexities

What is presented here is just the most basic of mathematical treatments. It glosses over many practical complexities that can arise with real returns. We will not concern ourselves with these in this course. The procedure above does indeed work quite well, and it is plenty for our purposes. But, for those who are interested in going in greater depth on this topic, we recommend the article 'The Statistics of Sharpe Ratios" by Andrew Lo, available here.