# Leveraged Return Calculations

### Basic Setup

Suppose we make a 4:1 leveraged investment in the S&P 500. How do we calculate the returns on that? To approach this, we use the following (assuming 4:1 leverage):

$$\text{Leveraged Return} = 4 * \text{SP 500 Return} - 3 * \text{Borrowing Costs}$$

Here, both the SP 500 Return and the Borrowing costs are expressed as returns.[1] Why is this the case?

### Returns and Leveraged Returns - Zero Borrowing Costs

Suppose that you have \$1 to invest over a period of time. And, suppose that over that period of time, the S&P 500 goes up 10%. If you just invest your \$1, you will get a 10% return. But, what happens if you borrow money? First, for simplicity, let's assume that borrowing is free. You borrow \$3 in addition to your \$1. You now have \$4 to invest in the S&P 500. You do so, and the S&P 500 goes up by 10% again. So, the \$4 you invested is now worth \$4.40. But, you have to pay back the \$3 that you borrowed. So, after you pay that back, you are left with \$1.40. So, your return here is now 40%.[2]

More generally, if you are leveraged N:1 (i.e. you have N total dollars to invest for every \$1 of your own money), and if $r$ is the return on an asset, then your leveraged returns will be:

$$\text{Leveraged Returns} = N * r$$

Here, since we had $r = 10\%$ and $N = 4$, we get our leveraged returns as $4 * 10\% = 40\%$.

### Leveraged Returns with Borrowing Costs

In real life though, we probably won't be able to borrow money for free. We'll need to pay some interest on it. Suppose during this period we need to pay 5% interest in order to borrow the money. Again, assume that we have \$1 and borrow \$3 so that we can invest \$4 total in the S&P 500, which then goes up by 10%. So, we still get \$4.40 from that investment. But, now, instead of just paying back the \$3 we borrowed, we also need to pay back the interest on that \$3. I.e. we pay back \$3 + 5% of \$3 = 3.15. Now therefore, after repaying our loan, we are left with \$4.4 - \$3.15 = \$1.25. So, our actual leveraged return in this instance, accounting for borrowing costs, is 25%, since we started with \$1 and ended up with \$1.25.

More generally then, if you are leveraged N:1, and you have a borrowing cost of $b$ and a return of $r$ then your leveraged return will be:

$$\text{Leveraged Returns} = N * r - (N - 1) * b$$

I.e. in this case, the leveraged return = 4 * 10% - 3 * 5% = 40% - 15% = 25% which is what we came to before. We use the $N - 1$ in the expression since we only need to pay interest rates on the money we borrow, which will be less than the total amount we invest.

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[1]In principal these can be either regular or log returns, as long as you are consistent. For our purposes in this class we always use log returns unless noted otherwise.

[2]Sounds great, right? But, keep in mind, this is the optimistic scenario. Leverage will also amplify your losses!!!

## Rebalancing a Leveraged Position

Suppose you start off leveraged 4:1 as discussed here - e.g. you have $100,000 to invest and you borrow $300,000. As soon as the assets you invest in start gaining or losing money, your leverage ratio will change. (See Week 4's Weekly Strategy Writeup for more on this). So, to maintain a 4:1 leverage ratio as your assets gain/lose money, you must *rebalance* your portfolio through some combination of buying/selling the asset you invest in or adding/pulling out money from your investment account. *Rebalancing* is a technique that comes up frequently in finance in many settings, not just for leverage. The calculations here therefore assume that you are doing this rebalancing in order to maintain a consistent leverage ratio of 4:1.