Logarithmic and Regular Returns

Similarity Between Logarithmic and Regular Returns

Note that by Taylor's rule,[1] we have the helpful rule of thumb:

$$\text{for } x \text{ near } 0 : \log(1+x) \approx x$$

Now, let r' represent the "regular" (non-logarithmic) return and let r represent the logarithmic return. Suppose our security has price S_0 at time 0 and S_1 at time 1. From the "Calculations for Investment Analysis.pdf" file, we have:

$$r' := \frac{S_1 - S_0}{S_0} = \frac{S_1}{S_0} - 1$$

$$r := \log(S_1) - \log(S_0) = \log\left(\frac{S_1}{S_0}\right)$$

Now, note that since $r' = \frac{S_1}{S_0} - 1$, we have: $\frac{S_1}{S_0} = r' + 1$. Thus, by Taylor:

$$\text{logarithmic return} = r = \log\left(\frac{S_1}{S_0}\right) = \log\left(r' + 1\right) \approx r' = \text{regular return} \tag{1}$$

for r' close to 0.

Thus, as long as our returns are relatively close to zero, then there will be only a very small difference between the logarithmic and regular returns. In particular, note that when working with returns over small periods of time (e.g. a day, a week, etc.), the return over that period of time will tend to be quite small. Also, certain types of returns, such as risk-free returns, tend to be relatively small, even over longer periods of time. But, if you are dealing with returns that are larger in magnitude, treating log returns and regular returns as the same can lead to significant errors.

Converting ˆIRX Interest Rate Data to Daily Log Returns

The daily values for the ˆIRX risk-free interest rate data that you download from Yahoo do not represent daily prices, as they do for almost all of the other data you download from Yahoo. Instead, the daily values represent annualized, regular (not log) returns. In calculating our Sharpe ratios, however, we want daily values for the risk-free interest rate's log return. Here is how to convert from the one to the other.

First, recall from above that we can get a precise (not approximate) conversion between regular returns and log returns via the formula:

$$\log \text{ return} = \log\left(1 + \text{regular return}\right)$$

When we download the interest rate information on ˆIRX, it is initially in the form of annual regular returns. Also, the returns are expressed as percents rather than decimals - i.e. 2% instead of 0.02. Thus, to convert these to the daily log returns we want, we will:

1. Divide the ˆIRX daily values by 100, to change them from percents to decimals

2. Take the log of 1 + the values to convert them to annual log returns

3. Divide the annual log returns by 252 to convert them to daily values

In other words:

$$\text{Daily Log Risk Free Returns} = \frac{\log\left(\frac{\text{IRX Download}}{100} + 1\right)}{252}$$

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[1] You can check this yourself by looking at the Taylor expansion for $f(x) = \log(1+x)$