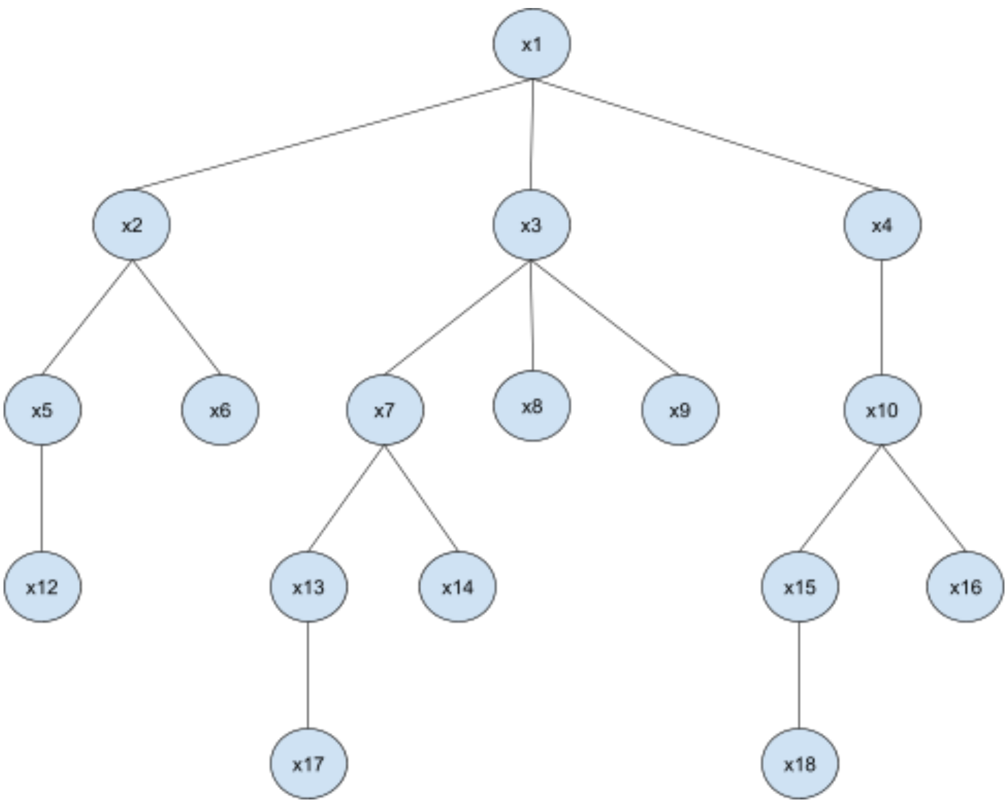
# Basic mathematical concepts

Names: Ana Paula Katsuda Zalce, A01025303

1. Let X be a finite set. Give a recursive definition of the set of subsets of X (the Power set of X). Use union as the operator on the definition

2. Prove by induction on $n$ that $\sum_{i=0}^{n} n = \frac{n(n+1)}{2}$

3. Using the tree below, give the values of each of the items:
    a. the depth of the tree
    b. the ancestors of $x_{18}$
    c. the internal nodes of the tree
    d. the length of the path from $x_3$ to $x_{14}$
    e. the vertex that is the parent of $x_{16}$
    f. the vertices children of $x_7$



# BASIC MATHEMATICAL CONCEPTS

**Andreína Sanánez Rico, A01024927**
**Ana Paula Katsuda Zalce, A01025303**

## 1

Power set → $P(x)$

Basis → $\{\} \in P(x)$ El conjunto vacío pertenece a $P(x)$ en cualquier caso

Recursive step: Definiendo $X = Y \cup \{z\}$, es posible obtener $P(x) = P(Y) \cup \{W \cup \{z\} \mid W \in P(Y)\}$

Closure: $n \in P(X)$ solo si puede ser obtenida de la base usando un conjunto finito de los conjuntos recursivos

Un ejemplo de lo anterior se ve de la siguiente manera

- Considerando $X = \{1,2,3\}$, se sigue el paso recursivo de manera:

$x = \{1,2\} \cup \{3\}$ $P(x) = \{\{\},\{1\},\{2\},\{1,2\}\} \cup \{\{3\},\{1,3\},\{2,3\},\{1,2,3\}\}$

$x = \{1\} \cup \{2\}$ $P(x) = \{\{\},\{1\}\} \cup \{\{2\},\{1,2\}\} = \{\{\},\{1\},\{2\},\{1,2\}\}$

$x = \{\{\}\} \cup \{1\}$ $P(x) = \{\{\},\{1\}\}$

## 2

$$\sum_{i=0}^{n} n = \frac{n(n+1)}{2}$$

Basis → siendo $n = \frac{n(1+n)}{2} = \frac{0(1+0)}{2} = \frac{0}{2} = 0$

Inductive hypothesis: los primeros números hasta K suman $\frac{K(K+1)}{2}$ $\therefore$ $1+2+3+\ldots+k = \frac{K(K+1)}{2}$

Inductive step: para $K+1$, $\frac{(K+1)((K+1)+1)}{2}$ dado que $1+2+3+\ldots+K+(K+1) = \frac{K(K+1)}{2} + (K+1)$. Resolviendo, $\frac{K(K+1)}{2} + \frac{2(K+1)}{2}$

$$= \frac{K(K+1)+2(K+1)}{2}$$

$$= \frac{(K+1)(K+2)}{2}$$

$$= \frac{(K+1)(1+(K+1))}{2}$$

## 3

a) Depth: 4

b) Ancestors of $x_{18}$: $x_{15}, x_{10}, x_4, x_1$

c) Internal nodes: $x_1, x_2, x_3, x_4, x_5, x_7, x_{10}, x_{13}, x_{15}$

d) length from $x_3$ to $x_{14}$: 2

e) Vertex that is the parent of $x_{16}$: $x_{10}$

f) Vertices children of $x_7$: $x_{13}, x_{14}$