test_poisson

November 30, 2018

As shown in this github issue: https://github.com/scikit-learn/scikit-learn/issues/5975

I've had many arguements that for poisson regression Counts/Exposures is equivalent to handling the offsets in the optimization loop explicitly. This isn't true as shown later on in this notebook.

The notebook also shows how to do poisson regression with offsets in tensorflow. Pretty baller right.

```
In [1]: import numpy as np
        import pandas as pd
```

```
In [180]: np.random.seed(333)
          n = 1000000
          df = pd.DataFrame(
              np.random.normal(np.tile(np.arange(10), n), np.tile(np.arange(1, 11) * 2, n), (n
          wts = np.array([-0.1, -0.2, 0, 0, 0.1, 0.2, 0.3, 0.0, 0.25, 0.5])
          df['y'] = (df.values * wts).sum(axis=1)
          df['rate'] = np.exp(df['y'])
```

```
In [181]: df.describe()
```

Out[181]:

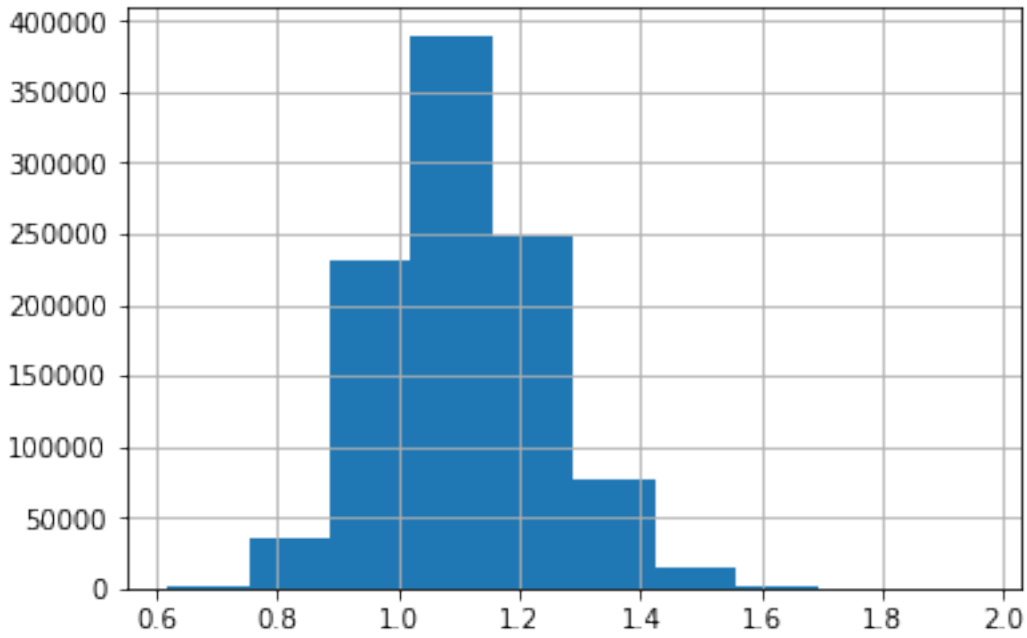
	0	1	2	3	\
count	1000000.000000	1000000.000000	1000000.000000	1000000.000000	
mean	-0.000003	0.009928	0.020052	0.029953	
std	0.020028	0.040016	0.060064	0.080018	
min	-0.105630	-0.209110	-0.309568	-0.352887	
25%	-0.013521	-0.017128	-0.020421	-0.024039	
50%	-0.000023	0.009892	0.020028	0.029904	
75%	0.013515	0.036934	0.060464	0.083954	
max	0.095576	0.200082	0.317362	0.410525	

	4	5	6	7	\
count	1000000.000000	1000000.000000	1000000.000000	1000000.000000	
mean	0.039883	0.050059	0.060012	0.070168	
std	0.099852	0.120065	0.140026	0.159790	
min	-0.421594	-0.500034	-0.681308	-0.690213	
25%	-0.027489	-0.030903	-0.034518	-0.037456	
50%	0.039912	0.050219	0.060004	0.070210	
75%	0.107181	0.131026	0.154161	0.178102	

```
max          0.560201        0.682567        0.761380        0.846154

                    8               9               y            rate
count  1000000.000000  1000000.000000  1000000.000000  1000000.000000
mean         0.080178        0.090096        0.095111        1.107802
std          0.180064        0.199852        0.120557        0.134044
min         -0.801777       -0.907063       -0.481539        0.617832
25%         -0.041322       -0.044670        0.013792        1.013888
50%          0.079852        0.090129        0.095101        1.099770
75%          0.201702        0.224866        0.176557        1.193102
max          0.957091        1.058712        0.673366        1.960826
```

In [182]:
```
%matplotlib inline
df.rate.hist()
```

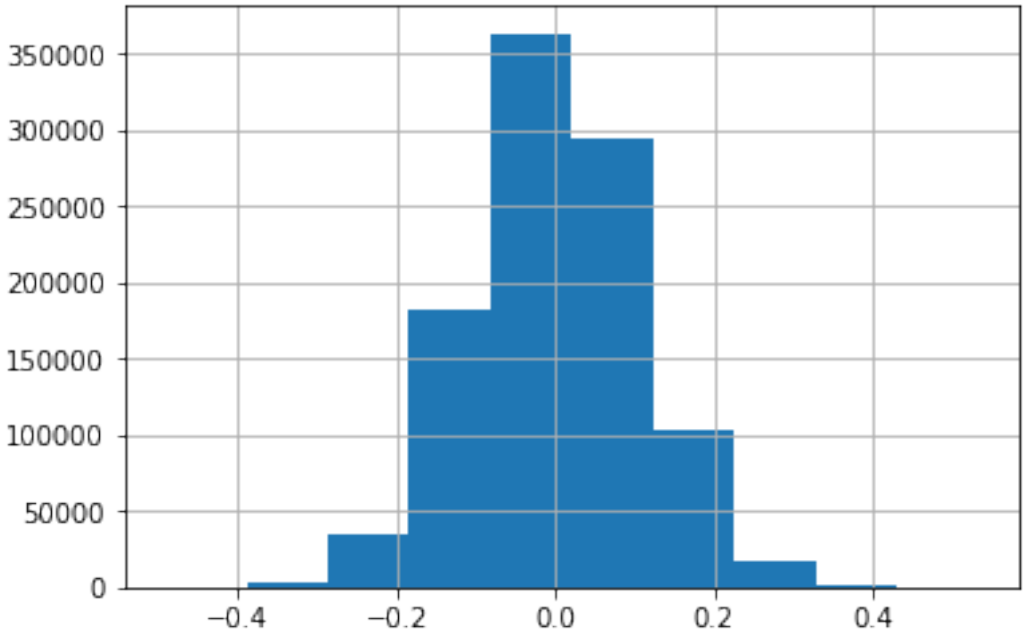
Out[182]: <matplotlib.axes._subplots.AxesSubplot at 0x7f27c1823048>



In [183]:
```
df['offset'] = np.random.randint(1, 10, n)
```

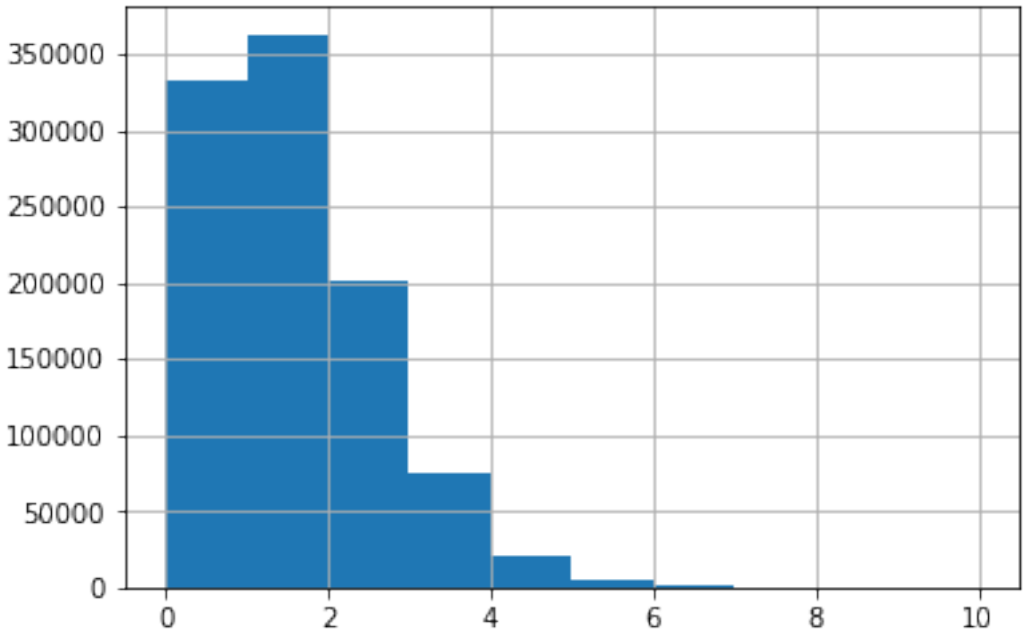
In [184]:
```
(np.random.poisson(df['rate'], (10, n)).T.mean(axis=1) - df.rate).hist()
```

Out[184]: <matplotlib.axes._subplots.AxesSubplot at 0x7f27c17e1d68>




```
In [185]: df['cnt'] = np.random.poisson(df['rate'])
          df.cnt.hist()
```

```
Out[185]: <matplotlib.axes._subplots.AxesSubplot at 0x7f27c1993358>
```



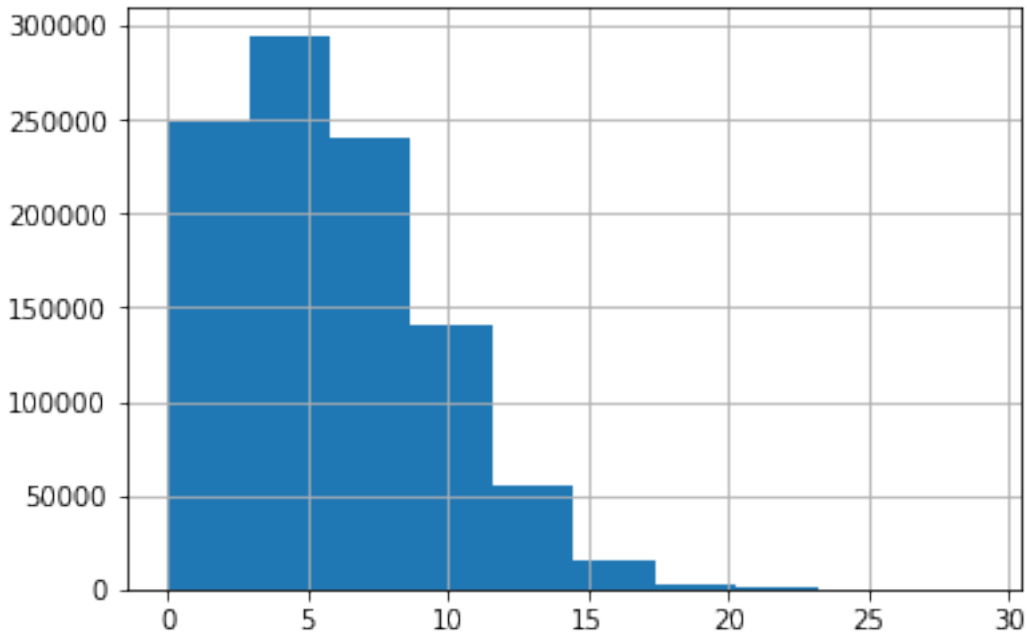
In [187]:
```
import statsmodels.api as sm


mod = sm.GLM(df['cnt'], df[np.arange(10)], family=sm.families.Poisson())

mod = mod.fit()

mod.summary()
```

Out[187]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                 Generalized Linear Model Regression Results
==============================================================================
Dep. Variable:                    cnt   No. Observations:              1000000
Model:                            GLM   Df Residuals:                   999990
Model Family:                 Poisson   Df Model:                            9
Link Function:                    log   Scale:                           1.0
Method:                          IRLS   Log-Likelihood:            -1.3606e+06
Date:                Mon, 12 Feb 2018   Deviance:                   1.1522e+06
Time:                        21:18:44   Pearson chi2:                 1.00e+06
No. Iterations:                     5
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
0             -0.1093      0.047     -2.303      0.021      -0.202      -0.016
1             -0.2031      0.023     -8.646      0.000      -0.249      -0.157
2              0.0097      0.015      0.626      0.531      -0.021       0.040
3              0.0008      0.012      0.069      0.945      -0.022       0.023
4              0.0944      0.009     10.253      0.000       0.076       0.112
5              0.1912      0.008     25.097      0.000       0.176       0.206
6              0.3053      0.006     47.020      0.000       0.293       0.318
7             -0.0041      0.006     -0.716      0.474      -0.015       0.007
8              0.2543      0.005     50.560      0.000       0.244       0.264
9              0.4978      0.004    111.282      0.000       0.489       0.507
==============================================================================
"""
```

In [188]:
```
df['cnt'] = [np.random.poisson(rate, offset).sum() for rate, offset in zip(df['rate']
```

In [189]:
```
df.cnt.hist()
```

Out[189]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x7f27c2181cf8>
```



In [192]:
```
import statsmodels.api as sm
```

```
mod = sm.GLM(df['cnt'], df[np.arange(10)], offset=np.log(df['offset']), family=sm.fa

mod = mod.fit()

mod.summary()
```

Out[192]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                 Generalized Linear Model Regression Results
==============================================================================
Dep. Variable:                    cnt   No. Observations:              1000000
Model:                            GLM   Df Residuals:                   999990
Model Family:                 Poisson   Df Model:                            9
Link Function:                    log   Scale:                           1.0
Method:                          IRLS   Log-Likelihood:            -2.1458e+06
Date:                Mon, 12 Feb 2018   Deviance:                   1.0616e+06
Time:                        21:19:50   Pearson chi2:                 9.98e+05
No. Iterations:                     5
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
0            -0.0757      0.021     -3.569      0.000      -0.117      -0.034
1            -0.2146      0.011    -20.415      0.000      -0.235      -0.194
```

```
2              -0.0060      0.007     -0.870      0.384      -0.020       0.008
3              -0.0071      0.005     -1.369      0.171      -0.017       0.003
4               0.1057      0.004     25.683      0.000       0.098       0.114
5               0.2010      0.003     59.013      0.000       0.194       0.208
6               0.2976      0.003    102.507      0.000       0.292       0.303
7              -0.0033      0.003     -1.301      0.193      -0.008       0.002
8               0.2496      0.002    111.029      0.000       0.245       0.254
9               0.4987      0.002    249.294      0.000       0.495       0.503
==============================================================================
"""
```

In [194]:
```
import statsmodels.api as sm


mod = sm.GLM(df['cnt'] / df['offset'], df[np.arange(10)], family=sm.families.Poisson

mod = mod.fit()

mod.summary()
```

Out[194]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                 Generalized Linear Model Regression Results
==============================================================================
Dep. Variable:                      y   No. Observations:              1000000
Model:                            GLM   Df Residuals:                   999990
Model Family:                 Poisson   Df Model:                            9
Link Function:                    log   Scale:                             1.0
Method:                          IRLS   Log-Likelihood:            -1.1432e+06
Date:                Mon, 12 Feb 2018   Deviance:                   3.4568e+05
Time:                        21:25:05   Pearson chi2:                 3.14e+05
No. Iterations:                     5
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
0              -0.0644      0.047     -1.357      0.175      -0.157       0.029
1              -0.2159      0.023     -9.191      0.000      -0.262      -0.170
2              -0.0064      0.015     -0.411      0.681      -0.037       0.024
3              -0.0102      0.012     -0.883      0.377      -0.033       0.012
4               0.1066      0.009     11.574      0.000       0.089       0.125
5               0.1994      0.008     26.177      0.000       0.184       0.214
6               0.2994      0.006     46.098      0.000       0.287       0.312
7              -0.0018      0.006     -0.317      0.752      -0.013       0.009
8               0.2511      0.005     49.914      0.000       0.241       0.261
9               0.4991      0.004    111.565      0.000       0.490       0.508
==============================================================================
"""
```

In [327]:
```
from pyglmnet import GLM
```

```
          # create an instance of the GLM class
          glm = GLM(distr='poisson')
          glm = glm.fit(df[np.arange(10)].values, df['cnt'].values/df['offset'].values)
          glm
```

```
Out[327]: <
          Distribution | poisson
          alpha | 0.50
          max_iter | 1000.00
          lambda: 0.50 to 0.01
          >
```

```
In [328]: glm.get_params()
```

```
Out[328]: {'Tau': None,
           'alpha': 0.5,
           'distr': 'poisson',
           'eta': 4.0,
           'group': None,
           'learning_rate': 0.2,
           'max_iter': 1000,
           'random_state': 0,
           'reg_lambda': array([ 0.5       ,  0.3237394 ,  0.2096144 ,  0.13572088,  0.0878763
                  0.0568981 ,  0.03684031,  0.02385332,  0.01544452,  0.01      ]),
           'score_metric': 'deviance',
           'tol': 0.001,
           'verbose': False}
```

```
In [313]: import keras

          inl = keras.layers.Input((10,))
          out = keras.layers.Dense(1, use_bias=False)(inl)
          out = keras.layers.Lambda(lambda x: keras.backend.exp(x))(out)
          model = keras.models.Model(inl, out)

          model.compile(keras.optimizers.Adam(1e-3), 'poisson')
          model.summary()
          model.fit(df[np.arange(10)], df['cnt']/df['offset'], verbose=1)
```

```
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
input_32 (InputLayer)        (None, 10)                0
_________________________________________________________________
dense_22 (Dense)             (None, 1)                 10
_________________________________________________________________
lambda_18 (Lambda)           (None, 1)                 0
=================================================================
```

```
Total params: 10
Trainable params: 10
Non-trainable params: 0
_________________________________________________________________
Epoch 1/1
1000000/1000000 [==============================] - 25s 25us/step - loss: 0.9870
```

```
Out[313]: <keras.callbacks.History at 0x7f27b93bb898>
```

```
In [314]: model.get_weights()[0].ravel()
```

```
Out[314]: array([-0.05754524, -0.23257338,  0.00755339, -0.05471236,  0.12637407,
                 0.19810426,  0.2976492 , -0.01716288,  0.26042998,  0.51705188], dtype=float3
```

```
In [310]: import keras

          inl = keras.layers.Input((10,))
          out = keras.layers.Dense(1, use_bias=False)(inl)
          off = keras.layers.Input((1,))
          out = keras.layers.add([out, off])
          out = keras.layers.Lambda(lambda x: keras.backend.exp(x))(out)

          model = keras.models.Model([inl, off], out)

          model.compile(keras.optimizers.Adam(1e-3), 'poisson')
          model.summary()
          model.fit([df[np.arange(10)], np.log(df['offset'])], df['cnt'], verbose=1)
```

```
_________________________________________________________________________________________________
Layer (type)                 Output Shape              Param #     Connected to
=================================================================================================
input_29 (InputLayer)        (None, 10)                0
_________________________________________________________________________________________________
dense_20 (Dense)             (None, 1)                 10          input_29[0][0]
_________________________________________________________________________________________________
input_30 (InputLayer)        (None, 1)                 0
_________________________________________________________________________________________________
add_3 (Add)                  (None, 1)                 0           dense_20[0][0]
                                                                   input_30[0][0]
_________________________________________________________________________________________________
lambda_16 (Lambda)           (None, 1)                 0           add_3[0][0]
=================================================================================================
Total params: 10
Trainable params: 10
Non-trainable params: 0
_________________________________________________________________________________________________
Epoch 1/1
1000000/1000000 [==============================] - 25s 25us/step - loss: -4.7883
```

```
Out[310]: <keras.callbacks.History at 0x7f27afa61e10>

In [311]: model.get_weights()[0].ravel()

Out[311]: array([-0.09250503, -0.19198173, -0.04260972, -0.02864283,  0.11696044,
                  0.20959049,  0.29357442, -0.02965078,  0.23306984,  0.48882231], dtype=float3

In [ ]: # now with unequal number of exposures we need to use the offsets correctly to get the

In [341]: offsets = np.arange(1, 101)
          import itertools
          offsets=np.array(list(itertools.chain.from_iterable([np.repeat(i, off) for i, off in
          X=np.array([x/100 for x in offsets])[:, np.newaxis]
          X.shape, offsets.shape

Out[341]: ((5050, 1), (5050,))

In [342]: y=np.random.poisson(np.exp(X*1.2+.33)).ravel()
          y.shape

Out[342]: (5050,)

In [343]: mod = sm.GLM(y, sm.add_constant(X), family=sm.families.Poisson())

          mod = mod.fit()

          mod.summary()

Out[343]: <class 'statsmodels.iolib.summary.Summary'>
          """
                           Generalized Linear Model Regression Results
          ==============================================================================
          Dep. Variable:                      y   No. Observations:                 5050
          Model:                            GLM   Df Residuals:                     5048
          Model Family:                 Poisson   Df Model:                           1
          Link Function:                    log   Scale:                          1.0
          Method:                          IRLS   Log-Likelihood:                -9733.6
          Date:                Mon, 12 Feb 2018   Deviance:                       5327.1
          Time:                        22:54:05   Pearson chi2:                 4.88e+03
          No. Iterations:                     4
          ==============================================================================
                           coef    std err          z      P>|z|      [0.025      0.975]
          ------------------------------------------------------------------------------
          const          0.3279      0.028     11.874      0.000       0.274       0.382
          x1             1.2006      0.037     32.733      0.000       1.129       1.272
          ==============================================================================
          """

In [345]: Xg=pd.DataFrame(X).groupby(offsets).mean().values
          yg=pd.DataFrame(y).groupby(offsets).sum().values.ravel()
          Xg.shape, yg.shape
```

```
Out[345]: ((100, 1), (100,))

In [346]: offsetsg = pd.DataFrame(offsets).groupby(offsets).size().values.ravel()

In [347]: mod = sm.GLM(yg, sm.add_constant(Xg), offset=np.log(offsetsg), family=sm.families.Po

          mod = mod.fit()

          mod.summary()

Out[347]: <class 'statsmodels.iolib.summary.Summary'>
          """
                           Generalized Linear Model Regression Results
          ==============================================================================
          Dep. Variable:                      y   No. Observations:                  100
          Model:                            GLM   Df Residuals:                       98
          Model Family:                 Poisson   Df Model:                           1
          Link Function:                    log   Scale:                          1.0
          Method:                          IRLS   Log-Likelihood:                -368.64
          Date:                Mon, 12 Feb 2018   Deviance:                       97.737
          Time:                        22:56:15   Pearson chi2:                     98.3
          No. Iterations:                     5
          ==============================================================================
                           coef    std err          z      P>|z|      [0.025      0.975]
          ------------------------------------------------------------------------------
          const          0.3279      0.028     11.874      0.000       0.274       0.382
          x1             1.2006      0.037     32.733      0.000       1.129       1.272
          ==============================================================================
          """

In [348]: mod = sm.GLM(yg/offsetsg, sm.add_constant(Xg), family=sm.families.Poisson())

          mod = mod.fit()
          mod.summary()

Out[348]: <class 'statsmodels.iolib.summary.Summary'>
          """
                           Generalized Linear Model Regression Results
          ==============================================================================
          Dep. Variable:                      y   No. Observations:                  100
          Model:                            GLM   Df Residuals:                       98
          Model Family:                 Poisson   Df Model:                           1
          Link Function:                    log   Scale:                          1.0
          Method:                          IRLS   Log-Likelihood:                -144.13
          Date:                Mon, 12 Feb 2018   Deviance:                      5.7707
          Time:                        22:57:02   Pearson chi2:                      6.43
          No. Iterations:                     4
          ==============================================================================
                           coef    std err          z      P>|z|      [0.025      0.975]
```

```
------------------------------------------------------------------------------
const          0.3547      0.143      2.485      0.013       0.075       0.634
x1             1.1635      0.219      5.320      0.000       0.735       1.592
==============================================================================
"""
```

In [349]:
```
# wow amazing dividing by exposures doesn't work!!!! Guess you actually have use the
```

In [350]:
```
# bonus let's try tensorflow
```

In [465]:
```
import keras

inl = keras.layers.Input((2,))
out = keras.layers.Dense(1, use_bias=False)(inl)
off = keras.layers.Input((1,))
out = keras.layers.add([out, off])
out = keras.layers.Lambda(lambda x: keras.backend.exp(x))(out)

model = keras.models.Model([inl, off], out)

model.compile(keras.optimizers.SGD(1e-19), 'poisson')
model.summary()
model.fit([sm.add_constant(Xg), np.log(offsetsg)], yg, verbose=1, epochs=1)
```

```
_________________________________________________________________________________________________
Layer (type)                 Output Shape              Param #     Connected to
=================================================================================================
input_145 (InputLayer)       (None, 2)                 0
_________________________________________________________________________________________________
dense_79 (Dense)             (None, 1)                 2           input_145[0][0]
_________________________________________________________________________________________________
input_146 (InputLayer)       (None, 1)                 0
_________________________________________________________________________________________________
add_60 (Add)                 (None, 1)                 0           dense_79[0][0]
                                                                   input_146[0][0]
_________________________________________________________________________________________________
lambda_75 (Lambda)           (None, 1)                 0           add_60[0][0]
=================================================================================================
Total params: 2
Trainable params: 2
Non-trainable params: 0
_________________________________________________________________________________________________
Epoch 1/1
100/100 [==============================] - 1s 6ms/step - loss: -499.4098
```

Out[465]:
```
<keras.callbacks.History at 0x7f27a15dfe80>
```

In [466]:
```
model.get_weights()[0]
```

```
Out[466]: array([[-1.10750854],
                 [ 0.13667548]], dtype=float32)
```