CSE 6331 Homework 1 (5 questions)

Due: Thursday, January 18, by 11:59pm

1. (16 pts) Express each of the following functions in the simplest form of Θ notation, such as $\Theta(1)$, $\Theta(\lg^b n)$, $\Theta(n^a)$, $\Theta(n^a \lg^b n)$, $\Theta(a^{bn})$ *etc*. for some constants a, b.

   (a) $f(n) = \log_2(3^{n+2} + 5n^3 + 2^{100})$

   (b) $f(n) = n^{0.2} \times \lg(4n^5 + 3n^3) + 4n^{0.3}$

   (c) $f(n) = 3\log_4(4n - 10) \times \log_3 n + \log_3 (6n^2 - 8n)$

   (d) $f\left(n\right) = 15^n - 10^n + n^{100}$

   (e) $f(n) = 2\left(n+4\right)\log_3\left(2n^3 + 1\right) - 5n + \sqrt{2n}$

   (f) $f(n) = 9 \times 2^{\log_2\left(n^2 + 2n\right)}$

   (g) $f(n) = 9 \times 4^{\log_2\left(n^2 + 2n\right)}$

   (h) $f(n) = 2^{3n^2 + 4n + \log_3 n + 40}$

2. (24 pts) Rank the following 17 functions by order of growth; that is, find a permutation $(j_1,\ j_2, \ldots,\ j_{17})$ of (1, 2, …, 17) such that, as sets

$$O\left(f_{j_1}(n)\right) \subseteq O\left(f_{j_2}(n)\right) \subseteq O\left(f_{j_3}(n)\right) \subseteq \cdots \subseteq O\left(f_{j_{17}}(n)\right).$$

Then in your list, underline consecutive functions whose *O*-sets are equal.

For example, if you think

$O\left(f_5(n)\right) \subseteq O\left(f_1(n)\right) = O\left(f_6(n)\right) \subseteq O\left(f_{10}(n)\right) \subseteq O\left(f_3(n)\right) = O\left(f_9(n)\right)$ then answer

$5, \underline{1, 6}, 10, \underline{3, 9}$ or $f_5, \underline{f_1, f_6}, f_{10}, \underline{f_3, f_9}$. No need to provide a proof or justification.

$f_1(n) = n \lg n$	$f_2(n) = 2^{n+9}$	$f_3(n) = \sqrt{2n^2 \lg n + 3n}$
$f_4(n) = 2^{\lg n}$	$f_5(n) = \lg(n!)$	$f_6(n) = n\sqrt{\ln n}$
$f_7(n) = 5^{900}$	$f_8(n) = 3^{100} \cdot 2^n$	$f_9(n) = 3^n / 2^{100}$
$f_{10}(n) = n \cdot 2^n$	$f_{11}(n) = n^{0.7}$	$f_{12}(n) = \lg(6n+7) \times \lg(5n^{0.3} + 21)$
$f_{13}(n) = \sqrt{n^3 - 2n^2}$	$f_{14}(n) = 3^{2n}$	$f_{15}(n) = \log_6((2n+4)(3n+2)(5n+6))$
$f_{16}(n) = 2^{\ln n}$	$f_{17}(n) = 2^{3n}$	

3. (30 pts) Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, ω, or Θ of B. Assume that $k \geq 1$, $\varepsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of yes or no for each box. You do not need to submit a proof or justification.

	A	B	O	o	Ω	ω	Θ
$a.$	$\lg^k n$	n^{ε}					
$b.$	n^k	c^n					
$c.$	$\sqrt{n}$	$n^{\sin n}$					
$d.$	2^n	$2^{n/2}$					
$e.$	$n^{\lg c}$	$c^{\lg n}$					
$f.$	$\lg(n!)$	$\lg(n^n)$					

4. (15 pts) Let $f(n)$ be a positive function defined for all positive integers n. Prove or disprove the following statement: If $f(n) = \Theta(n^2)$, then $f(n)$ is asymptotically nondecreasing (i.e., $f(n) \leq f(n+1)$ for all sufficiently large integers n).
(Note: to disprove a statement, you need to give a counterexample.)

5. (15 pts) Let $f(n)$ be a positive function defined for all positive integers n. Prove or disprove the following statement: If $f(n) = O(g(n))$, then $2^{f(n)} = O\left(2^{g(n)}\right)$.
(Note: to disprove a statement, you need to give a counterexample.)