Aditi Garg

HW-1

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① $f(n) = \log_2(3^{n+2} + 5n^3 + 2^{100})$

$= \log_2(3^{n+2})$ // Ignoring the less dominant terms

$= n+2 \log_2(3)$ $\boxed{\log a^b = b \log a}$

$= \Theta(n)$

② $f(n) = n^{0.2} \times \lg(4n^5 + 3n^3) + 4n^{0.3}$

$= n^{0.2} \times \lg(4n^5) + 4n^{0.3}$

$= \Theta(n^{0.3})$

③ $f(n) = 3\log_4(4n-10) \times \log_3 n + \log_3(6n^2 - 8n)$

$= \log_4(4n) \times \log_3 n + \log_3(6n^2)$ // Ignoring constants

$= \frac{\log_2 n}{\log_2 4} \times \frac{\log_2 n}{\log_2 3} + \frac{\log_2 n}{\log_2 3}$

$\Rightarrow \log^2 n + \log_2 n$

$\Rightarrow \Theta(\log^2 n)$

④ $f(n) = 15^n - 10^n + n^{100}$

$= \Theta(15^n)$ // taking the most dominant term

(e) $f(n) = 2(n+4)\log_3(2n^3+1) - 5n + \sqrt{2}\, n$

$= n\log_3(2n^3) - 5n + \sqrt{2}\, n$

$= n\log_3(n) - 5n + \sqrt{2}\, n$

$\Rightarrow \Theta(n \log n)$

(f) $f(n) = 9 \times 2^{\log_2(n^2+2n)}$ $a^{\log b} = b^{\log a}$

$= (n^2+2n)^{\log_2 2}$ $\log_2 2 = 1$

$\Rightarrow \Theta(n^2)$

(g) $f(n) = 9 \times 4^{\log_2(n^2+2n)}$

$= (n^2+2n)^{\log_2 4}$

$= \Theta(n^2)^2 \Rightarrow \Theta(n^4)$

(h) $f(n) = 2^{3n^2+4n+\log_3 n+40}$

$= \Theta(2^{3n^2+4n+\log_3 n})$ // Ignoring constants

Q2 $f_7, f_{15}, f_{12}, f_{16}, f_{11}$, [illegible], $f_4, f_3, f_6, f_1, f_5, f_{13}, f_2, f_8, f_{10}, f_9, f_{17}, f_{14}$

Q3

A	B	O	o	Ω	ω	Θ
$\lg^k n$	n^ϵ	Yes	Yes	No	No	No
n^k	c^n	Yes	Yes	No	No	No
$\sqrt{n}$	$n^{\sin n}$	No	No	No	No	No
2^n	$2^{n/2}$	No	No	Yes	Yes	No
$n^{\lg c}$	$c^{\lg n}$	Yes	No	Yes	No	Yes
$\lg(n!)$	$\lg(n^n)$	Yes	No	Yes	No	Yes

Q4 False

$$f(n) = \begin{cases} n^2 & n \text{ is even} \\ \left(\frac{n-1}{2}\right)^2 & n \text{ is odd} \end{cases}$$

$f(n) = \Theta(n^2)$

For $n+1$, $f(n+1) = \left(\frac{n+1-1}{2}\right)^2 \Rightarrow \frac{n^2}{4}$ (n is odd)

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$\Rightarrow f(n+1) < f(n)$

$\therefore f(n)$ is not asymptotically non decreasing

(5.) Take, $f(n) = k\log_2 n \quad (k > 1)$

$g(n) = \log_2(n)$

$f(n) = O\,g(n)$ i.e $\boxed{f(n) \le c\,g(n) \quad n \ge n_0}$

Now

$2^{f(n)} = 2^{k\log_2 n} \Rightarrow 2^{\log_2 n^k}$

$= n^k$

$2^{g(n)} = 2^{\log_2(n)} \Rightarrow n$

$2^{f(n)} > 2^{g(n)}$

Hence, $2^{f(n)} \neq O(g(n))$