Aditi Garg
Algorithms
HW#2

(1.)

```
Func6(n)
  s ← 0
  for i ← 6 to n^2 do
    j ← 3
    while (j < 2i^2) do
      s ← s + i - j;
      j ← (1.5) * j
    end
  end
  return (s)
```

$j = 3$

$j = 1.5 \times 3$

$j = (1.5)^2 \times 3$

$j = (1.5)^3 \times 3$

$\vdots$

$(1.5)^k \times 3$

Loop breaks when, $(1.5)^k \times 3 \geq 2i^2$

$k \log 1.5 \geq \log 2i^2 - \log 3$

$k \log 1.5 \geq \log 2 + 2\log i - \log 3$

$k \simeq \Theta \log(i)$

// Integrating

No. of iterations $= \sum_{n=6}^{i^2} \Theta(\log i) + \Theta(1)$

$\Theta(n^2)\ \Theta(\log 6 + \log 7 + \log 8 - - + \log n^2)$

$\boxed{\Theta(n^2 \log n)}$ - Ans

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(2.)

```
Func 7(n)
  S ← 0
  i ← n^3
  while (i ≥ 1) do
    for j ← 1 to i do
      S ← S + i - j
    end
    i ← i - √n
  end
  return (S)
```

$\Theta(1) + \Theta(i)$

$i \geq 1, \quad i = i - \sqrt{n}$

i goes from n^3

$n^3 - \sqrt{n}$

$n^3 - 2\sqrt{n}$

$n^3 - 3\sqrt{n}$

⋮

$n^3 - k\sqrt{n} < 1$

$\frac{n^3 - 1}{n^{1/2}} < k$

$k \simeq \Theta(n^{2.5})$

No. of iterations $= \Theta(1) + \Theta(i)$

$= \Theta(n^{2.5})$

for $i = n^3, n^3 - \sqrt{n}, n^3 - 2\sqrt{n} \ldots n^3 - k\sqrt{n} = 0$

$n^3 + n^3 - \sqrt{n} + n^3 - 2\sqrt{n} \ldots + n^3 - k\sqrt{n}, \quad (k = n^{2.5})$

$kn^3 - \sqrt{n}(1 + 2 + \ldots k) = \Theta(kn^3)$

$= \boxed{\Theta(n^{5.5})}$ - Ans

(3.)

```
Func 8(n)
  S ← 0
  L ← n
  while (i < 4n^3) do
    j ← n^3
    while (j ≥ 10) do
      s ← s + i - j
      j ← ⌊j/4⌋
    end
    L ← 2 * i
  end
  return (S)
```

Inner loop :→ $j = n^3$

(Iterations for j) $= \frac{n^3}{4}$

$= \frac{n^3}{(4)^2}$

$= \vdots$

$\frac{n^3}{4^k} < 10$

$n^3 < 10 \times 4^k$

$\Theta(\log_4 n^3) \simeq k$

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Outer Loop :→ (i) Loop (Iterations for i)
Total no. of iterations :→ n

$2 \times n$

$2 \times (2 \times n)$

$\vdots$

$2^k \times n \geq 4n^3$ (loop breaks)

$2^k \geq \frac{4n^3}{n}$

$\boxed{\Theta(\log_2 4n^3 - \log_2 n) \times \Theta(\log_4 n^3)}$ → Iterations for Loop j

(4.)

```
Func 9(n)
  s ← 0
  i ← n²
while (i ≥ 1) do
  j ← 1
while (j ≤ i) do
  s ← s+i-j
  j ← j×2
end
  i ← i-1
end
return (s)
```

Inner Loop (Iterations for j)

$j = 1$

$j = 2 \times 1$

$= (2 \times 1) \times 2$

$= (2 \times 1 \times 2) \times 2$

$= \vdots$

$(2^k)(1) > i$

$k \simeq \Theta(\log i)$

Outer Loop
No. of iterations $= \sum_{i=1}^{n^2} \log i$

$= \log 1 + \log 2 + \log 3 \ldots \log n^2$

$\Rightarrow \boxed{\Theta(n^2 \log n)}$ — Ans

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⑤
```
Func1o(n)
s ← 0
i ← 1
while (i < 7n) do
    j ← n^2
    while (j > 6)
        s ← s + 9 - j
        j ← j - i
    end
    i = 5 * i
end
return(s)
```

Inner loop
Iterations for j),

$j = n^2$, $j > 6$; $j = j - i$

$= n^2 - i$

$\vdots$

$n^2 - k i > 6$

$k \simeq \Theta\left(\frac{6n^2}{i}\right)$

// Ignoring the constant

$k \simeq \Theta\left(\frac{n^2}{i}\right)$

Outer Loop :- $i = 1$, $i < 7n$, $i = 5 * i$

$\Theta(1) \Rightarrow \Theta(\log n)$

$i = 5 * i$

$$\Theta\left(\frac{n^2}{i}\right) = \Theta\left(n^2 + \frac{n^2}{5} + \frac{n^2}{5^2} + \cdots\right) = \Theta\left(n^2 \sum_{0}^{k} \frac{n^2}{5^k}\right)$$

$= \boxed{\Theta(n^2)}$ - Ans