Optimisation and Control of Distributed Energy Resources in a Microgrid

Silvio Tarca[a]

[a] School of Mathematical Sciences and
ARC Centre of Excellence for Mathematical & Statistical Frontiers
The University of Adelaide, South Australia 5005

silvio.tarca@adelaide.edu.au

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Distributed energy resources in a microgrid

Optimisation and control

- Suppose that:
  - Microgrid has a thin gateway connection to the main grid
  - Each household has rooftop solar panels and a residential battery
- Model power imported from/ exported to the main grid as a function of DER in the microgrid
- Implement controller to optimise DER in the microgrid
- Perform virtual trials (i.e., equation-based simulations using real-world data) to examine:
  - Objective function and process constraints that minimise the cost of power imported from the grid while reducing maximum operational demand
  - Effect of electricity tariff structure on the operational load profile
  - Savings achieved by optimising at the microgrid level rather than the individual household level

Notation

Single-period setting

Let	m	be the number of households in the microgrid
	n	number of time intervals in the prediction and control horizons
	N	number of time intervals in the simulation horizon
	δ	conversion factor from power (kW) to energy (kWh)
	η	one-way battery charge/discharge efficiency
	$c(t)$	vector of weighting coefficients (e.g., tariffs) for time interval t
	$p_i(t)$	power imported from the grid by household i during time interval t^*
	$b_i(t)$	battery charge control signal for household i resolved at time $t^\dagger$
	$d_i(t)$	battery discharge control signal for household i resolved at time $t^\dagger$
	$l_i(t)$	load generated by household i forecast at time $t^\ddagger$
	$g_i(t)$	power generated by rooftop PV of household i forecast at time $t^\ddagger$
	$e_i(t)$	state of charge (SOC) of the battery of household i at time t

$^*p_i(t) < 0$ implies power exported to the grid

† Control signals resolved at time t apply during time interval $t+1$

‡ Control signals set to forecasts produced at time t are valid for time interval $t+1$

State-space model predictive control (MPC)

Single-period state-space model

- MPC discretises prediction, control and simulation horizons into half-hourly time intervals (i.e., $\delta = 0.50$)
- Suppose that discrete times $t-1$ and t, respectively, translate to clock times ς and τ; then "at time t" refers to clock time τ, and "during time interval t" refers to clock time interval $(\varsigma, \tau]$
- State-space model of DER in a microgrid represents power imported from/ exported to the main grid as a function of load, rooftop PV generation, and power charging, or discharged from, the battery

$$\underbrace{\begin{bmatrix} p_i(t+1) \\ e_i(t+1) \end{bmatrix}}_{\boldsymbol{y}_i(t+1)} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} p_i(t) \\ e_i(t) \end{bmatrix}}_{\boldsymbol{x}_i(t)} + \underbrace{\begin{bmatrix} 1 & -1 & 1 & -1 \\ \delta\eta & -\delta/\eta & 0 & 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} b_i(t) \\ d_i(t) \\ l_i(t) \\ g_i(t) \end{bmatrix}}_{\boldsymbol{u}_i(t)}$$

for $i = 1, \ldots, m$ and $t = 1, \ldots, N$

State-space model predictive control (MPC)

Single-period performance index

- Performance index is weighted to reflect cost of power imported from the grid
- Let $\Lambda = \operatorname{diag}\big(c(t+1)\big)$ be a positive semi-definite diagonal weighting matrix reflecting the electricity tariff structure
- Define the single period performance index as

$$\begin{aligned} f &= \left\| \sqrt{\Lambda}\, \boldsymbol{y}_i(t+1) \right\|_2^2 \\ &= \left\| \sqrt{\Lambda}\, \big(A\boldsymbol{x}_i(t) + B\boldsymbol{u}_i(t)\big) \right\|_2^2 \end{aligned}$$

  for $i = 1, \ldots, m$ and $t = 1, \ldots, N$
- Optimisation of performance index is subject to process constraints:
  - Rooftop PV generation and load are set to solar power and demand forecasts
  - Charge/discharge rates cannot exceed rated power (continuous) of the battery
  - Energy capacity accounts for its decay over the lifetime of the battery
  - Upper and lower bounds on SOC restrict battery to partial discharging, which prolongs battery life
  - Linear complementarity of battery charge and discharge control signals

MPC controller

Single-period mixed integer quadratic programming

- Expanding the performance index, dropping the constant terms and imposing process constraints, the quadratic program is written in standard form

$$\begin{aligned} \underset{\boldsymbol{u}_i(t)}{\operatorname{argmin}} \quad & \frac{1}{2}\boldsymbol{u}_i(t)^T \left(B^T \Lambda B\right) \boldsymbol{u}_i(t) + \big(A\boldsymbol{x}_i(t)\big)^T \Lambda B \boldsymbol{u}_i(t) \\ \text{subject to} \quad & \underline{b_i} \le b_i(t) \le \overline{b_i}, \\ & \underline{d_i} \le d_i(t) \le \overline{d_i}, \\ & \underline{e}_i \le e_i(t) + \delta\eta b_i(t) - \frac{\delta}{\eta} d_i(t) \le \overline{e}_i, \\ & b_i(t) = 0 \quad \text{or} \quad d_i(t) = 0 \end{aligned}$$

  for $i = 1, \ldots, m$ and $t = 1, \ldots, N$
- Introducing binary variable to ensure linear complementarity transforms the optimisation into a mixed linear quadratic program (MIQP)

State-space model predictive control (MPC)

Multi-period prediction and control horizons

- Receding prediction and control horizons are 8 hours in duration ($n = 16$)
- Let

$$\vec{\boldsymbol{y}}_{i,t+1} = \begin{bmatrix} \boldsymbol{y}_i(t+1) & \boldsymbol{y}_i(t+2) & \dots & \boldsymbol{y}_i(t+n) \end{bmatrix}^T,$$
$$\vec{\boldsymbol{u}}_{i,t} = \begin{bmatrix} \boldsymbol{u}_i(t) & \boldsymbol{u}_i(t+1) & \dots & \boldsymbol{u}_i(t+n-1) \end{bmatrix}^T$$

- Recursively applying the single-period model over the n-period horizon yields

$$\vec{\boldsymbol{y}}_{i,t+1} = K\boldsymbol{x}_i(t) + L\vec{\boldsymbol{u}}_{i,t},$$

where

$$K = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^n \end{bmatrix} \text{ and } L = \begin{bmatrix} B & 0 & 0 & \dots & 0 \\ AB & B & 0 & \dots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ A^{n-1}B & A^{n-2}B & \dots & AB & B \end{bmatrix}$$

for $i = 1, \dots, m$ and $t = 1, \dots, N$

MPC controller

Multi-period mixed integer quadratic programming

- Define the multi-period performance index as

$$f = \left\| \sqrt{\Lambda} \left(K\boldsymbol{x}_i(t) + L\vec{\boldsymbol{u}}_{i,t} \right) \right\|_2^2,$$

where $\Lambda = \operatorname{diag}\big(c(t+1), c(t+2), \dots, c(t+n)\big)$

- Expanding the performance index, dropping the constant terms and imposing process constraints, the quadratic program is written in standard form

$$\begin{aligned}
\underset{\vec{\boldsymbol{u}}_{i,t}}{\operatorname{argmin}} \quad & \frac{1}{2}\vec{\boldsymbol{u}}_{i,t}^T \left(L^T \Lambda L\right) \vec{\boldsymbol{u}}_{i,t} + \left(K\boldsymbol{x}_i(t)\right)^T \Lambda L \vec{\boldsymbol{u}}_{i,t} \\
\text{subject to} \quad & \underline{b_i} \le b_i(t+k-1) \le \overline{b_i}, \\
& \underline{d_i} \le d_i(t+k-1) \le \overline{d_i}, \\
& \underline{e}_i \le e_i(t+k-1) + \delta\eta b_i(t+k-1) - \frac{\delta}{\eta} d_i(t+k-1) \le \overline{e}_i, \\
& b_i(t+k-1) = 0 \quad \text{or} \quad d_i(t+k-1) = 0, \\
& k = 1, \dots, n
\end{aligned}$$

for $i = 1, \dots, m$ and $t = 1, \dots, N$

Optimisation and control

Mixed integer linear programming

- Let $p_i(t) \geq 0$ be power imported from the grid by household i during time interval t, and $q_i(t) \geq 0$ power exported to the grid by household i during time interval t
- Power balance equation is given by

$$\begin{aligned} p_i(t+k) = b_i(t+k-1) - d_i(t+k-1) + &\\ l_i(t+k-1) - g_i(t+k-1) + q_i(t+k)&, \\ k = 1, \ldots, n& \end{aligned}$$

  for $i = 1, \ldots, m$ and $t = 1, \ldots, N$
- Introduce additional binary variable to ensure linear complementarity of power imported from and power exported to the grid

Optimisation and control

Mixed integer linear programming

$$\begin{aligned}
\text{minimise} \quad & \sum_{k=1}^{n} c(t+k) p_i(t+k) \\
\text{subject to} \quad & \underline{b_i} \leq b_i(t+k-1) \leq \overline{b_i}, \\
& \underline{d_i} \leq d_i(t+k-1) \leq \overline{d_i}, \\
& \underline{e}_i \leq e_i(t+k-1) + \delta\eta b_i(t+k-1) - \frac{\delta}{\eta} d_i(t+k-1) \leq \overline{e}_i, \\
& b_i(t+k-1) = 0 \quad \text{or} \quad d_i(t+k-1) = 0, \\
& p_i(t+k) = 0 \quad \text{or} \quad q_i(t+k) = 0, \\
& \hat{l}_i(t+k) \leq l_i(t+k-1) \leq \hat{l}_i(t+k), \\
& \hat{g}_i(t+k) \leq g_i(t+k-1) \leq \hat{g}_i(t+k), \\
& k = 1, \ldots, n \\
\text{for} \quad & i = 1, \ldots, m \text{ and } t = 1, \ldots, N
\end{aligned}$$

Data, variables and parameters

- Real-world data from Salisbury trial:
  - Actual half-hourly time series of rooftop PV generation and load for 75 households over 16-week simulation horizon (04/02/2017–26/05/2017)
  - Assume perfect foresight
  - No household has more than 0.5% of 5,378 half-hourly intervals with missing data — filled with zeroes
- Suppose that each household has installed a Tesla Powerwall 2.0 DC battery:
  - 11.5 kWh energy storage capacity — mid-point assuming decay to 70% of its original capacity over its lifetime
  - 5 kW power rating (continuous charge and discharge)
  - 80% discharge cycle — SOC maintained between 10% and 90%
  - 88% round-trip efficiency when coupled to a solar inverter (i.e., $\eta = \sqrt{0.88}$)
- Power imported from the grid is subject to time-of-day (TOD) tariff ($/kWh)

	Off-peak	Shoulder 12:00–16:00	Peak 16:00–21:00
April–October	0.24	0.36	0.36
November–March	0.24	0.36	0.48

while power exported to the grid earns a feed-in tariff of $0.08/kWh

Virtual trials

- MPC controller determines battery charge/discharge control signals that minimise cost of power imported from the grid subject to process constraints
- Evaluate optimisation algorithms, control techniques and simulation parameters by their effect on:
  - Net cost of electricity for households
  - Peak operational demand (i.e., network upgrades)
  - Battery charge/discharge cycles (i.e., life of the battery)
  - Simulation runtime
- Virtual trials compare:
  - No battery energy storage versus Tesla Powerwall 2.0 DC installed
  - Single-period (half-hour, tariff independent) versus multi-period (8 hours, TOD tariff) control horizon
  - Optimisation at the household level versus the microgrid level§
  - Different vectors of weighting coefficients applied over a multi-period control horizon versus weighting based on TOD tariff
  - MILP versus MIQP algorithm

§Microgrid level opitimisation simply aggregates rooftop PV generation, load and energy storage capacity across households in the microgrid for each half-hourly interval

Virtual trials

Empirical research findings

1. Battery energy storage reduces net cost of electricity substantially

	Net cost of electricity ($)	Peak operational demand (kW)	Charge/discharge cycles
MIQP, household, single-period, no BESS	28,086	242.8	N/A
MIQP, household, single-period, BESS	16,017	183.8	74.3
% change	-43.0	-24.3	N/A

2. Further cost savings are achieved by optimising at the microgrid level relative to the individual household level

	Net cost of electricity ($)	Peak operational demand (kW)	Charge/discharge cycles
MIQP, household, single-period	16,017	183.8	74.3
MIQP, microgrid, single-period	11,956	242.8	82.1
% change	-25.4	32.1	10.6

Virtual trials

Empirical research findings

3. Peak operational demand is markedly lower for a multi-period control horizon employing MIQP relative to a single-period control horizon

	Net cost of electricity ($)	Peak operational demand (kW)	Charge/discharge cycles
MIQP, microgrid, single-period	11,956	242.8	82.1
MIQP, microgrid, multi-period	12,285	152.7	89.4
% change	2.7	-37.1	8.8

4. Differences between MIQP and MILP algorithms for a single-period control horizon is marginal

	Net cost of electricity ($)	Peak operational demand (kW)	Charge/discharge cycles
MIQP, microgrid, single-period	11,956	242.8	82.1
MILP, microgrid, single-period	11,997	242.8	82.1
% change	0.3	0.0	0.0

Virtual trials

Empirical research findings

5. Peak operational demand for a multi-period control horizon employing MIQP is a fraction of that employing MILP[¶]

	Net cost of electricity ($)	Peak operational demand (kW)	Charge/discharge cycles
MILP, microgrid, multi-period	12,748	434.4	83.6
MIQP, microgrid, multi-period	12,285	152.7	89.4
% change	-3.6	-64.9	7.0

6. MILP (48 min 10 sec) solves faster than MIQP (2 hr 11 min 29 sec) on an iMac with 2.7 GHz processor and 8 GB memory when optimising DER in a microgrid at the household level over a multi-period control horizon[‖]

[¶] QP penalises large power imports from the grid during a given time interval disproportionately more heavily than small imports, while LP penalises large power imports from the grid during a given time interval proportionately equally as small imports

[‖] MPC controller is coded in MATLAB and invokes solvers `cplexmilp()` and `cplexmilp()` from the CPLEX for MATLAB Toolbox

Actual DER management

Salisbury trial

75 households in 16-week Salisbury trial from 04/02/2017 to 26/05/2017	
Consumption	177,527 kWh
Rooftop PV generation	152,310 kWh
Energy charging battery	45,627 kWh
Energy discharge from battery	42,064 kWh
Peak operational demand	217.7 kW
Energy imported from the grid	79,020 kWh
Energy exported to the grid	49,945 kWh
Net cost of electricity	$19,316