

# Optimisation and Control of Distributed Energy Resources in a Microgrid

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## South Australia

### Variable renewable energy generation and storage

- SA is a world-leading jurisdiction for penetration of variable renewable energy generation and storage
  - In fiscal year 2016–17, 48.4% of the electricity generated in SA came from variable renewable energy sources — 39.2% wind and 9.2% rooftop PV
  - Largest battery in the world (100MW/129MWh) is coupled to the Hornsdale wind farm in the state's mid-north
  - SA government has announced a policy to create the world's largest virtual power plant — 50,000 households equipped with solar panels and a battery
- SA has the highest ratio of rooftop PV generation to operational consumption in the National Electricity Market, and likely for any major grid in the world
  - Installed capacity of 800MW on more than 30% of the state's 760,000 households
  - Australian Energy Market Operator forecasts that rooftop PV generation in SA will nearly double by 2026–27
  - First region in the NEM in which rooftop PV generation resulted in operational minimum demand shifting from overnight to the middle of the day (2012–13)
  - AEMO forecasts negative minimum demand in 2025–26\*

\*In 2016–17 operational minimum demand fell to 800MW, and maximum demand reached 3,017MW

## Research motivation

- Mathematics in Industry Study Group brings together mathematicians, scientists and engineers from universities, government and the private sector to tackle complex technical problems facing Australian and New Zealand businesses and industries
- SA Power Networks submitted to MISG (2017) a project to study *electricity pricing and control mechanisms for microgrids*
- This research extends the work of MISG by applying an advanced control technique for multivariate control problems using real-world data supplied by SAPN



## Distributed energy resources in a microgrid

### Optimisation and control

- Suppose that:
  - Microgrid has a thin gateway connection to the main grid
  - Each household is equipped with rooftop solar panels and a residential battery
- Model power imported from/ exported to the main grid as a function of DER in the microgrid
- Implement controller to optimise DER in the microgrid
- Perform virtual trials (i.e., equation-based simulations using real-world data) to examine:
  - Objective function and process constraints that minimise the cost of power imported from the grid while reducing operational maximum demand
  - Effect of electricity tariff structure on the operational load profile
  - Savings achieved by optimising at the microgrid level rather than the individual household level

# Notation

## Single-period setting

|     |                 |  |
|-----|-----------------|--|
| Let | $m$             | be the number of households in the microgrid                                     |
|     | $n$             | number of time intervals in the prediction and control horizons                  |
|     | $N$             | number of time intervals in the simulation horizon                               |
|     | $\delta$        | conversion factor from power (kW) to energy (kWh)                                |
|     | $\eta$          | one-way battery charge/discharge efficiency                                      |
|     | $\mathbf{c}(t)$ | vector of weighting coefficients (e.g., tariffs) for time interval $t$           |
|     | $p_i(t)$        | power imported from the grid by household $i$ during time interval $t^\dagger$   |
|     | $b_i(t)$        | battery charge control signal for household $i$ resolved at time $t^\ddagger$    |
|     | $d_i(t)$        | battery discharge control signal for household $i$ resolved at time $t^\ddagger$ |
|     | $l_i(t)$        | load generated by household $i$ forecast at time $t^\S$                          |
|     | $g_i(t)$        | power generated by rooftop PV of household $i$ forecast at time $t^\S$           |
|     | $e_i(t)$        | state of charge (SOC) of the battery of household $i$ at time $t$                |

$^\dagger p_i(t) < 0$  implies power exported to the grid

$^\ddagger$  Control signals resolved at time  $t$  apply during time interval  $t+1$

$^\S$  Control signals set to forecasts produced at time  $t$  are valid for time interval  $t+1$

# State-space model predictive control (MPC)

## Single-period state-space model

- MPC discretises prediction, control and simulation horizons into half-hourly time intervals (i.e.,  $\delta = 0.50$ )
- Suppose that discrete times  $t-1$  and  $t$ , respectively, translate to clock times  $\varsigma$  and  $\tau$ ; then “at time  $t$ ” refers to clock time  $\tau$ , and “during time interval  $t$ ” refers to clock time interval  $(\varsigma, \tau]$
- State-space model of DER in a microgrid represents power imported from/exported to the main grid as a function of load, rooftop PV generation, and power charging, or discharged from, the battery

$$\begin{bmatrix} p_i(t+1) \\ e_i(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_i(t) \\ e_i(t) \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 & -1 \\ \delta\eta & -\delta/\eta & 0 & 0 \end{bmatrix} \begin{bmatrix} b_i(t) \\ d_i(t) \\ l_i(t) \\ g_i(t) \end{bmatrix}$$

$\mathbf{y}_i(t+1) \quad \quad \quad \mathbf{A} \quad \quad \quad \mathbf{x}_i(t) \quad \quad \quad \quad \quad \quad \quad \mathbf{B} \quad \quad \quad \mathbf{u}_i(t)$

for  $i = 1, \dots, m$  and  $t = 1, \dots, N$

# State-space model predictive control (MPC)

## Multi-period prediction and control horizons

- Set receding prediction/control horizons to 8 hours ( $n = 16$ ), and let

$$\vec{y}_{i,t+1} = [y_i(t+1)^T \quad y_i(t+2)^T \quad \dots \quad y_i(t+n)^T]^T,$$
$$\vec{u}_{i,t} = [u_i(t)^T \quad u_i(t+1)^T \quad \dots \quad u_i(t+n-1)^T]^T$$

- Recursively applying the single-period state-space model

$$y_i(t+1) = Ax_i(t) + Bu_i(t)$$

over the  $n$ -period horizon yields

$$\vec{y}_{i,t+1} = Kx_i(t) + L\vec{u}_{i,t},$$

where

$$K = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^n \end{bmatrix} \text{ and } L = \begin{bmatrix} B & 0 & 0 & \dots & 0 \\ AB & B & 0 & \dots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ A^{n-1}B & A^{n-2}B & \dots & AB & B \end{bmatrix}$$

for  $i = 1, \dots, m$  and  $t = 1, \dots, N$

## MPC controller

### Multi-period performance index

- Performance index is weighted to reflect cost of power imported from the grid
- Define the multi-period performance index as

$$f = \left\| \sqrt{\Lambda} (Kx_i(t) + L\vec{u}_{i,t}) \right\|_2^2,$$

where  $\Lambda = \text{diag}([c(t+1)^T, c(t+2)^T, \dots, c(t+n)^T])$  is a positive semi-definite diagonal weighting matrix reflecting the electricity tariff structure

- Optimisation of performance index is subject to process constraints:
  - Rooftop PV generation and load are set to solar power and demand forecasts
  - Upper and lower bounds on SOC of the battery
  - Energy capacity accounts for its decay over the lifetime of the battery
  - Charge/discharge rates cannot exceed rated power (continuous) of the battery
  - Linear complementarity of battery charge and discharge control signals

# MPC controller

## Mixed integer quadratic programming

- Expanding the performance index, dropping the constant terms and imposing process constraints, the quadratic program is written in standard form

$$\begin{aligned} \underset{\vec{u}_{i,t}}{\operatorname{argmin}} \quad & \frac{1}{2} \vec{u}_{i,t}^T (L^T \Lambda L) \vec{u}_{i,t} + (K \mathbf{x}_i(t))^T \Lambda L \vec{u}_{i,t} \\ \text{subject to} \quad & \underline{b}_i \leq b_i(t+k-1) \leq \overline{b}_i, \\ & \underline{d}_i \leq d_i(t+k-1) \leq \overline{d}_i, \\ & \underline{e}_i \leq e_i(t+k-1) + \delta \eta b_i(t+k-1) - \frac{\delta}{\eta} d_i(t+k-1) \leq \overline{e}_i, \\ & b_i(t+k-1) = 0 \quad \text{or} \quad d_i(t+k-1) = 0, \\ & k = 1, \dots, n \end{aligned}$$

for  $i = 1, \dots, m$  and  $t = 1, \dots, N$

- Introducing binary variable to ensure linear complementarity of battery charge and discharge control signals transforms the optimisation problem into a mixed integer quadratic program (MIQP)

# Optimisation and control

## Mixed integer linear programming

- Let  $p_i(t) \geq 0$  be power imported from the grid by household  $i$  during time interval  $t$ , and  $q_i(t) \geq 0$  power exported to the grid by household  $i$  during time interval  $t$
- Power balance equation is given by

$$\begin{aligned} p_i(t+k) = & b_i(t+k-1) - d_i(t+k-1) + \\ & l_i(t+k-1) - g_i(t+k-1) + q_i(t+k), \\ & k = 1, \dots, n \end{aligned}$$

for  $i = 1, \dots, m$  and  $t = 1, \dots, N$

- Introduce additional binary variable to ensure linear complementarity of power imported from and power exported to the grid

# Optimisation and control

## Mixed integer linear programming

$$\begin{aligned} & \text{minimise} && \sum_{k=1}^n c(t+k)p_i(t+k) \\ & \text{subject to} && \underline{b}_i \leq b_i(t+k-1) \leq \overline{b}_i, \\ & && \underline{d}_i \leq d_i(t+k-1) \leq \overline{d}_i, \\ & && \underline{e}_i \leq e_i(t+k-1) + \delta \eta b_i(t+k-1) - \frac{\delta}{\eta} d_i(t+k-1) \leq \overline{e}_i, \\ & && b_i(t+k-1) = 0 \quad \text{or} \quad d_i(t+k-1) = 0, \\ & && p_i(t+k) = 0 \quad \text{or} \quad q_i(t+k) = 0, \\ & && \hat{l}_i(t+k) \leq l_i(t+k-1) \leq \hat{l}_i(t+k), \\ & && \hat{g}_i(t+k) \leq g_i(t+k-1) \leq \hat{g}_i(t+k), \\ & && k = 1, \dots, n \\ & \text{for} && i = 1, \dots, m \text{ and } t = 1, \dots, N \end{aligned}$$

## Data, variables and parameters

- Real-world data from Salisbury trial:
  - Actual half-hourly time series of rooftop PV generation and load for 75 households over 16-week simulation horizon (04/02/2017–26/05/2017)
  - Assume perfect foresight
  - No household has more than 0.5% of 5,378 half-hourly intervals with missing data — filled with zeroes
- Suppose that each household has installed a Tesla Powerwall 2.0 DC battery:
  - 11.5 kWh energy storage capacity — mid-point assuming decay to 70% of its original capacity over its lifetime
  - 5 kW power rating (continuous charge and discharge)
  - 80% discharge cycle — SOC maintained between 10% and 90%
  - 88% round-trip efficiency when coupled to a solar inverter (i.e.,  $\eta = \sqrt{0.88}$ )
- Power imported from the grid is subject to time-of-day (TOD) tariff (\$/kWh)

|                | Off-peak | Shoulder<br>12:00–16:00 | Peak<br>16:00–21:00 |
|----------------|----------|-------------------------|---------------------|
| April–October  | 0.24     | 0.36                    | 0.36                |
| November–March | 0.24     | 0.36                    | 0.48                |

while power exported to the grid earns a feed-in tariff of \$0.08/kWh

# Virtual trials

- MPC controller determines battery charge/discharge control signals that minimise cost of power imported from the grid subject to process constraints
- Evaluate optimisation algorithms, control techniques and simulation parameters by their effect on:
  - Net cost of electricity for households
  - Operational peak demand (i.e., network upgrades)
  - Battery charge/discharge cycles (i.e., life of the battery)
  - Simulation runtime
- Virtual trials compare:
  - No battery energy storage versus Tesla Powerwall 2.0 DC installed
  - Single-period (half-hour, tariff independent) versus multi-period (8 hours, TOD tariff) control horizon
  - Optimisation at the household level versus the microgrid level<sup>¶</sup>
  - MILP versus MIQP algorithm

<sup>¶</sup> Microgrid level optimisation simply aggregates rooftop PV generation, load and energy storage capacity across households in the microgrid for each half-hourly interval

# Virtual trials

## Empirical research findings

### 1. Battery energy storage reduces net cost of electricity substantially

|   | Net cost of electricity (\$) | Operational peak demand (kW) | Charge/discharge cycles |
|---|------------------------------|------------------------------|-------------------------|
| MIQP, household, single-period, no BESS | 28,086                       | 242.8                        | N/A                     |
| MIQP, household, single-period, BESS    | 16,017                       | 183.8                        | 74.3                    |
| % change                                | -43.0                        | -24.3                        | N/A                     |

### 2. Further cost savings are achieved by optimising at the microgrid level relative to the individual household level

|                                | Net cost of electricity (\$) | Operational peak demand (kW) | Charge/discharge cycles |
|--------------------------------|------------------------------|------------------------------|-------------------------|
| MIQP, household, single-period | 16,017                       | 183.8                        | 74.3                    |
| MIQP, microgrid, single-period | 11,956                       | 242.8                        | 82.1                    |
| % change                       | -25.4                        | 32.1                         | 10.6                    |

## Virtual trials

### Empirical research findings

3. Peak operational demand is markedly lower for a multi-period control horizon employing MIQP relative to a single-period control horizon

|                                | Net cost of electricity (\$) | Operational peak demand (kW) | Charge/discharge cycles |
|--------------------------------|------------------------------|------------------------------|-------------------------|
| MIQP, microgrid, single-period | 11,956                       | 242.8                        | 82.1                    |
| MIQP, microgrid, multi-period  | 12,285                       | 152.7                        | 89.4                    |
| % change                       | 2.7                          | -37.1                        | 8.8                     |

4. Differences between MIQP and MILP algorithms for a single-period control horizon is marginal

|                                | Net cost of electricity (\$) | Operational peak demand (kW) | Charge/discharge cycles |
|--------------------------------|------------------------------|------------------------------|-------------------------|
| MIQP, microgrid, single-period | 11,956                       | 242.8                        | 82.1                    |
| MILP, microgrid, single-period | 11,997                       | 242.8                        | 82.1                    |
| % change                       | 0.3                          | 0.0                          | 0.0                     |

## Virtual trials

### Empirical research findings

5. Operational peak demand for a multi-period control horizon employing MIQP is a fraction of that employing MILP<sup>||</sup>

|                               | Net cost of electricity (\$) | Operational peak demand (kW) | Charge/discharge cycles |
|-------------------------------|------------------------------|------------------------------|-------------------------|
| MILP, microgrid, multi-period | 12,748                       | 434.4                        | 83.6                    |
| MIQP, microgrid, multi-period | 12,285                       | 152.7                        | 89.4                    |
| % change                      | -3.6                         | -64.9                        | 7.0                     |

6. MILP (48 min 10 sec) solves faster than MIQP (2 hr 11 min 29 sec) on an iMac with 2.7 GHz processor and 8 GB memory when optimising DER in a microgrid at the household level over a multi-period control horizon\*\*

<sup>||</sup>QP penalises large power imports from the grid during a given time interval disproportionately more heavily than small imports, while LP penalises large power imports from the grid during a given time interval proportionately equally as small imports

\*\*MPC controller is coded in MATLAB and invokes solvers `cplexmilp()` and `cplexmilp()` from the CPLEX for MATLAB Toolbox



# Actual DER management

Salisbury trial

*75 households in 16-week Salisbury trial  
from 04/02/2017 to 26/05/2017*

|                               |             |
|-------------------------------|-------------|
| Consumption                   | 177,527 kWh |
| Rooftop PV generation         | 152,310 kWh |
| Energy charging battery       | 45,627 kWh  |
| Energy discharge from battery | 42,064 kWh  |
| Operational peak demand       | 217.7 kW    |
| Energy imported from the grid | 79,020 kWh  |
| Energy exported to the grid   | 49,945 kWh  |
| Net cost of electricity       | \$19,316    |

## Other applied mathematics research on clean energy

Dependable supply of wind power with battery energy storage

- Tilt Renewables has announced that it will connect a solar farm (44MW) and utility-scale battery (21MW/26MWh) to its Snowtown wind farm (369MW) in the mid-north of South Australia
- Collaboration with Tilt Renewables is applying optimisation and control techniques to firm-up wind power dispatch using battery energy storage
- Conjecture that if wind farms were to dependably supply power scheduled during pre-dispatch, then wholesale electricity prices would be less volatile and, on average, lower

