●day ●month

例6.4 企管三甲 A106270024 廖德皓

依 $E(x_i)=\mu \quad V(x_i)=\sigma^2=E(x_i^2)-\mu^2$

則 $E(\bar{x})=\mu \quad V(\bar{x})=\frac{\sigma^2}{n}=E(\bar{x}^2)-\mu^2$

$E(\hat{\theta}_1)=E\left[\frac{\sum_{i=1}^{n}(x_i-\bar{x})}{n}\right]=\frac{1}{n}\left(\sum_{i=1}^{n}x_i^2-n\bar{x}^2\right)$

$=\frac{1}{n}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)$

$=\sigma^2+\mu^2-\frac{\sigma^2}{n}-\mu^2$

$=\sigma^2-\frac{\sigma^2}{n}$

$=\frac{n-1}{n}\sigma^2$ => $E(\hat{\theta}_1)$未滿足σ^2之不偏估計量 => 偏誤估計量

$E(\hat{\theta}_2)=E\left[\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}\right]=\frac{1}{n-1}E\left[\sum_{i=1}^{n}(x_i-\bar{x})^2\right]$

$=\frac{1}{n-1}(n\sigma^2+n\mu^2-\sigma^2-n\mu^2)$

$=\left(\frac{n\sigma^2}{n-1}+\frac{n\mu^2}{n-1}-\frac{\sigma^2}{n-1}-\frac{n\mu^2}{n-1}\right)$

$=\frac{n\sigma^2}{n-1}-\frac{\sigma^2}{n-1}$

$=\sigma^2$ => $E(\hat{\theta}_2)$滿足σ^2之不偏估計量 => 不偏估計量