

3/9 #HW

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生產函數

邊際產量

MRTS

規模報酬

產量彈性

生產力彈性

替代彈性

5 替代彈性 (a) $F(K, L) = K^{\frac{1}{2}} L^{\frac{1}{2}}$ (b) $F(K, L) = 2K + L$ $MRTS = \frac{MP_L}{MP_K}$

$$(a) \epsilon = \frac{d \ln(K/L)}{d \ln(K/L)} = 1 \quad MRTS = \frac{K}{L}$$

$$\epsilon = \frac{d \ln(K/L)}{d \ln MRTS}$$

$$(b) \epsilon = \frac{d \ln(K/L)}{d \ln(\frac{1}{2})} = \infty \quad MRTS = \frac{2}{1}$$

6. Cobb-Douglas 生產函數: $Q = f(L, K) = L^{\alpha} K^{\beta}$

① 產出彈性: MP_L 和 AP_L 為

MP_K 和 AP_K

$$MP_L = \frac{\partial Q}{\partial L} = \alpha L^{\alpha-1} K^{\beta}$$

$$MP_K = \frac{\partial Q}{\partial K} = \beta L^{\alpha} K^{\beta-1}$$

$$AP_L = \frac{Q}{L} = \frac{L^{\alpha} K^{\beta}}{L} = L^{\alpha-1} K^{\beta}$$

$$AP_K = \frac{Q}{K} = \frac{L^{\alpha} K^{\beta}}{K} = L^{\alpha} K^{\beta-1}$$

(a) 勞動產出彈性:

$$\epsilon^L = \frac{MP_L}{AP_L} = \frac{\alpha L^{\alpha-1} K^{\beta}}{L^{\alpha-1} K^{\beta}} = \alpha$$

(b) 資本產出彈性:

$$\epsilon^K = \frac{MP_K}{AP_K} = \frac{\beta L^{\alpha} K^{\beta-1}}{L^{\alpha} K^{\beta-1}} = \beta$$

② 生產力彈性:

勞動 and 資本要素同時增加 ϕ 倍對生產函數影響

$$Q = f(\phi L, \phi K) = \phi^{\alpha+\beta} L^{\alpha} K^{\beta}$$

(a) 生產力彈性:

$$\epsilon^{\phi} = \frac{\frac{dQ}{d\phi}}{\frac{Q}{\phi}} = \frac{\frac{dQ}{d\phi}}{\frac{Q}{\phi}} = \frac{(\alpha+\beta)\phi^{\alpha+\beta-1} L^{\alpha} K^{\beta}}{\phi^{\alpha+\beta} L^{\alpha} K^{\beta}} = \alpha + \beta$$

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或是 $Q = Q^L + Q^K = \alpha + \beta$

② 替代彈性:

$$\rightarrow \text{MRTS 是實際技術替代率} \quad \text{MRTS} = \frac{MPL}{MPK} = \frac{\alpha L^{\alpha-1} K^{\beta}}{\beta L^{\alpha} K^{\beta-1}} = \frac{\alpha}{\beta} \times \frac{K}{L}$$

$$\Rightarrow \epsilon^{LK} = \frac{d \ln(\frac{K}{L})}{d \ln(\text{MRTS})} = \frac{d \ln(\frac{K}{L})}{d \ln(\frac{\alpha}{\beta}) + d \ln(\frac{K}{L})} = 1$$

因 α 和 β 為固定常數，並不隨資本勞動比的變動而變動，故上式可化簡
可以發現 Cobb-Douglas 形成生產函數，其替代彈性恆為一，並不因 α 和 β 的變動而改變。

⑧ $Q = 3K + 2L$, K 資本、 L 勞動、 Q 產出 (A) 函數呈現固定規模報酬。

假設生產函數：若 K 和 L 同時增 n 倍

$$f(nL, nK) = 2(nL) + 3(nK)$$

$$= n^2 Q \Rightarrow \text{CRS}$$

(B) 函數呈現資本與勞動的實際生產力

(C) 函數呈現固定的技術替代率

選 (A) 固定

請選正確敘述：

$$\text{MRTS} = \frac{MPL}{MPK} = \frac{3K}{2L}$$

⑨ 生產函數規模報酬 (A) $Q = (L^{\alpha} + K^{\alpha})^{\beta}$ (B) $\ln Q = 5 + 0.5 \ln L + 0.2 \ln K$

$$(C) Q = [\text{Min}(aL, bK)]^{\alpha}$$

$$(A) F(\lambda L, \lambda K) = ((\lambda L)^{\alpha} + (\lambda K)^{\alpha})^{\beta} = \lambda^{\alpha\beta} (L^{\alpha} + K^{\alpha})^{\beta} = \lambda^{\alpha\beta} Q \Rightarrow \begin{matrix} \alpha\beta > 1, \text{IRS} \\ \alpha\beta = 1, \text{CRS} \\ \alpha\beta < 1, \text{DRS} \end{matrix}$$

$$(B) \text{左右取} e, Q = e^5 L^{0.5} K^{0.2} \Rightarrow \text{DRS}$$

$$(C) F(\lambda L, \lambda K) = [\text{Min}(a\lambda L, b\lambda K)]^{\alpha} = \lambda^{\alpha} Q \Rightarrow \begin{matrix} \alpha > 1, \text{IRS} \\ \alpha = 1, \text{CRS} \\ \alpha < 1, \text{DRS} \end{matrix}$$