A Quantitative Study of Double-way Capital Flow Management

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Abstract

Emerging markets with substantial financial openness regularly confront volatile capital inflows, leading to adverse economic outcomes. Traditional discourse focuses on managing liabilities and inflows, often sidelining the asset side of the external balance sheets. This paper presents a novel perspective on capital flow management by distinguishing between outflows and inflows as key external balance adjusters. Using a DSGE model, we highlight an emerging market's social planner's bond pricing role. Through a focused study on Brazil, we advocate for a holistic management approach to both sides of the external balance sheet. We also explore policy instruments, such as ad-valorem taxes on asset purchases, that can reproduce efficient allocations proposed by a social planner, revealing their nuanced implications based on specific economic conditions.

1 Introduction

Emerging markets with high financial openness are frequently susceptible to volatile capital inflows. Throughout history, abrupt interruptions in international funds to these markets have precipitated a cascade of adverse outcomes: from real exchange rate depreciation and bank failures to sovereign defaults and overarching economic contractions. In response to these challenges, capital flow management policies have been extensively implemented in the real world and rigorously examined in academic literature.

While conventional wisdom on capital flow management primarily emphasizes restricting the magnitude of liabilities and capital inflows, they often overlook the asset side of the ledger. In reality, emerging markets' external balance sheet encompasses liabilities and assets. Concomitant with sudden stops in capital inflows, often spurred by external financial shocks, emerging markets also witness retrenchment of their foreign capital (Forbes and Warnock 2012). Such movements can substantially counterbalance decreased inflows' negative ramifications and even achieve a positive net wealth transfer to emerging markets (Gourinchas and Rey 2022). Yet, the prevailing scholarship on capital flow management primarily dwells on total or net capital inflows, sidelining capital outflows as a vital mechanism for external balance adjustments.

This paper revisits the topic of optimal capital flow management, emphasizing the distinction between capital outflows and inflows as channels of external balance adjustment. Grounded in a welfare-based DSGE model, we argue that the optimal management of aggregate capital flows is intricately tied to the strategic pricing of bonds, wherein an emerging market nation assumes the monopolist role. Given the long-term nature of these bonds, the social planner must navigate both the current bond issuances and potential future bond redemptions to and from foreign entities. It is typical for the portfolio curated by the social planner, in pursuit of the optimal bond price, to diverge from household diversification decisions. The degree and direction of this divergence are shaped by the particular characteristics of the balance sheet at equilibrium. Initially, we leverage a simplified model to underscore the inherently quantitative essence of optimal two-way capital management. This sets the stage for our deep dive into Brazil's case, where we quantify the optimal strategies for double-way capital flow management and the ensuing welfare gains.

In our model, a small open economy issues long-term bonds to international financiers to support its consumption and short-term bond holdings. The external financing conditions depend on the fluctuating wealth of these financiers. In wealthy periods, long-term bond prices are set by risk-averse financiers' discount factors. In lean times, the price reflects the financiers' wealth relative to their bond holdings, potentially triggering a firesale of long-term bonds. To counteract this, households hold short-term bonds during financial easing periods. In the event of a firesale, these short-term bonds can be used to repurchase long-term bonds at a substantially lower cost.

Since financial tightness grants economies monopolistic power over their bond issuance, a laissez-faire economy with decentralized decision-making may be inefficient for the open economy. This is because individual households cannot internalize their portfolios' impact on long-term bond prices. This lack of coordination manifests in two ways: first, they cannot collectively bargain to reduce redemption prices during external financial constraints, and second, they fail to work together to elevate bond-selling prices in times of external financial abundance. Recognizing these coordination gaps, a social planner can enhance the overall welfare by effectively leveraging the economy's inherent monopoly power on behalf of private agents.

The nature of the bond being long-term means a preemptive social planner cannot simultaneously boost its price during financially unconstrained times and reduce the future firesale price during financial tightenings. The planner faces a trade-off between these conflicting goals. The starting position of the long-term bond is crucial in determining this trade-off. In a simplified version of our model where the planner must maintain zero consumption before potential financial tightening, we identify a threshold for the initial bond position guiding the planner's decisions. If the bond position exceeds this threshold, the planner expands the external balance sheet by increasing both long-term borrowings and short-term savings. Otherwise, reducing the external balance sheet becomes more favorable.

The abovementioned threshold is inherently an equilibrium attribute, making optimal capital flow management primarily a quantitative issue. The decision to expand or contract the overall balance sheet size or adjust liabilities versus assets hinges on the model's foundational parameters. To navigate this complexity, we turn to a welfare-based analysis, selecting Brazil, a prominent emerging market with notable financial openness, as our benchmark. Through meticulous calibration and estimation, we ensure that the model's simulated moments resonate with real-world data. Our analysis indicates that Brazil stands to benefit by concurrently reducing its long-term borrowings and short-term savings. This welfare gain amounts to 0.2 percent of the consumers' permanent consumption in laissez-faire, a figure echoing similar studies. Notably, this welfare boost is reflected in a reduced duration of subdued consumption following financial tightening events.

We delve deeper to understand why Brazil's social planner might opt to reduce the external balance sheet. There are two fundamental motivations for this decision. Firstly, net capital flow management compels the planner to decrease both consumption and long-term debt earmarked for consumption. Secondly, even when keeping current consumption constant, the planner is still inclined to lessen the volume of long-term debt funded by short-term bonds. This stems from a stronger desire to improve future redemption conditions along the equilibrium path, surpassing the urge to enhance the current borrowing situation. Collectively, these factors push the planner towards a stance of reduced assets accompanied by lesser liabilities.

We finally explore policy tools that can emulate the efficient allocation directed by the social planner. Specifically, we examine ad-valorem taxes on asset purchases, a central focus in capital flow management studies. Our calibrated model demonstrates that these taxes, both in sign and size, are contingent on the state. For high initial long-term bond positions, where efficient capital inflow is positive, the planner subsidizes short-term bond acquisitions while taxing net long-term bond issuances. Conversely, with negative efficient capital inflows, the planner taxes short-term bond acquisitions and promotes net bond issuance. Quantitatively, when capital inflow is positive, the short-term bond holding subsidy is 0.17% of its price, while the long-term bond issuance tax is 0.22% of its price. If inflow is negative, both the tax for short-term bond holding and the subsidy for long-term bond issuance stand at roughly 1.5% of their respective prices.

The outline of this paper is organized as follows: Section 2 offers a review of pertinent literature, emphasizing our unique contributions. Section 3 presents the foundational model setup and elaborates on the laissez-faire and constrained efficient allocations. Section 4 introduces a streamlined version of the model, underscoring our core argument that the optimal bidirectional capital flow management is fundamentally quantitative. Section 5 delves into parameter determination and discusses the results from our GMM estimation. Our primary quantitative findings, anchored in the Brazilian context, are showcased in Section 6. Finally, Section 7 provides concluding remarks.

2 Literature Review

This paper relates to two strands of the literature: one is the global financial cycle and gross capital flows, and ther other one is capital flow management policies.

Our research resonates with the ongoing discussions on gross capital flows within the global financial cycle (GFC). As articulated by Miranda-Agrippino et al. (2020), the GFC is characterized by a pronounced co-movement in risky asset prices, capital flows, leverage, and various financial aggregates on a global scale. A single global factor, rooted in global risky asset prices, accounts for a significant portion of the variation in these asset prices. Notably, the US monetary policy emerges as a critical driver of the GFC.

Beyond asset prices, capital flows serve as another significant dimension of the GFC. Davis et al. (2021) identifies two global factors accounting for over 40% of annual capital flow variations, a perspective bolstered by Miranda-Agrippino and Rey (2022) on a quarterly basis. Notably, the first factor aligns closely with the global financial factor from asset prices, while the second resonates with commodity indices and international trade. Focusing on emerging markets, Cerutti et al. (2019) observes that common factors in capital flows are most pronounced in gross inflows to emerging markets, with their sensitivity largely influenced by lender characteristics rather than their domestic fundamentals. The private capital flow transmission mechanism, as Obstfeld et al. (2019) suggests, is pivotal for relaying global financial shocks to emerging markets' domestic financial dynamics. In the context of extreme capital flow events, Forbes and Warnock (2012) emphasizes the overriding influence of global financial risk and contends that while capital controls might not prevent sudden stops, they can bolster resistance against them. Chari et al. (2020) illustrates the profound impact of global risk aversion shifts on capital inflows to emerging markets and their stock and bond valuations. An ensemble of research papers spotlight the instrumental role global financial mediators assume, be it through global banking channels (Bruno and Shin 2015; Ivashina et al. 2015; Bräuning and Ivashina 2020; Avdjiev et al. 2022) or via non-traditional finance vehicles like mutual funds (Gelos 2011; Raddatz and Schmukler 2012; Bertaut et al. 2021; Chari et al. (2022)). Collectively, these empirical insights act as the foundation for our evaluative study centered on the shocks within global capital flows towards emerging markets.

Numerous theories try to explain global capital flows. Notable works in this arena include those by Tille and Van Wincoop 2010, Devereux and Sutherland 2011, Bruno and Shin 2015, Bräuning and Ivashina 2020, Jiang et al. 2020, Akinci et al. 2022, and Kekre and Lenel 2021. These studies all adopt a global lens, endogenizing the intricacies of the global financial cycle. They delve into multi-faceted dimensions such as financial risk, production uncertainty, and preferences for safety and liquidity. Despite their insights, there's a discernible gap: these studies often need to offer a robust quantitative exploration of capital flows directed towards non-financial center countries, like emerging markets. This leaves many questions about the policies and well-being of these non-centered countries needing to be answered, and that's what this paper aims to address.

In our exploration, we spotlight a curated selection of theoretical studies that address both inflows and outflows in economies outside the core financial centers. When unraveling the reasons behind open economies, especially emerging markets, taking on external liabilities to support their external asset holdings, the literature branches into two key themes: firesales and default. Caballero and Simsek (2020) postulates a model where liquidity shocks and fire sales shape the dynamics of gross capital flows. In such a scenario, an external shock forces the affected economy to pull back foreign assets to redeem their domestic assets at a cheaper cost. Jeanne and Sandri (2023) delves into an open economy's strategy of deploying long-term debt to fund its domestic operational capital. This economy, as posited, maintains an upbeat liquidity stance to safeguard against unpredicted fire sales targeting its long-term debt. Distinctively, their model advocates for the concurrent augmentation of long-term borrowings and short-term reserves rather than the traditional method of curtailing both, suggesting a novel, welfare-enhancing approach to capital flow management.

Turning our gaze to defaults, we find open economies, particularly their governments, using defaultable bonds as vehicles to amass safe bonds in the real world. Pioneering this discourse, Jeanne and Ranciere (2011) champions emerging markets' strategy to leverag debt whose payoff is state-contigent to bolster liquidity, ensuring consumption remains stable amidst unforeseen upheavals. Concurrently, Alfaro and Kanczuk (2009) probes whether the favorable payoff resulting from short-term debt default – coupled with the official reserves financed by this debt pre-default – aligns with real-world data. Their findings are striking: the official reserves an economy is inclined to maintain stand at almost zero. Building on this, Bianchi et al. (2018) offers an insightful perspective, underlining the insurance facet of long-term defaultable debt financing short-term official reserves. They contend that this insurance, acting as a bulwark against rollover risk, can explain a substantial portion of the official reserves emerging market central banks hold. Moreover, when considering the macro-stability advantages of debt-financed reserves, the optimal holdings of such reserves can escalate further, as elucidated by Bianchi and Sosa-Padilla (2020).

Our quantitative model pivots on the "firesale" perspective to analyze aggregate capital flows in open economies. We favor this perspective due to the significant equity-based capital influx into emerging markets, with over 50% in countries like Brazil. Likewise, the market prices of emerging long-term bonds are also profoundly influenced by international capital market liquidity. Reinforcing this, the IMF's 2022 financial stability report highlighted the impact of open-ended funds' withdrawals on emerging market bond spreads. One of our contributions to this literature is presenting a quantitative model that accurately captures the dynamics of capital flows, gross positions, and asset pricing in emerging markets, especially regarding volatility and correlation. In addition, while sharing a similar modeling framework with Jeanne and Sandri (2023), our study reveals that optimal capital flow management is indeed a quantitative question.

Our study also intersects with the literature on capital flow management in emerging markets. Over the past two decades, many theoretical works have shed light on the potential welfare benefits of capital controls. A unifying thread across these works is that capital flows can introduce externalities private agents overlook. As Erten et al. (2021) notes, key externalities underscored include pecuniary externalities linked to financial instability (e.g., Caballero and Krishnamurthy 2003, Lorenzoni 2008, and others), and aggregate demand externalities tied to unemployment (e.g., Korinek and Simsek 2016, Farhi and Werning 2016). Regardless of the type, the general policy recommendation is to counteract the "overborrowing" tendencies during financially lax periods to reduce vulnerability to future financial constrictions.

Pecuniary externalities, in particular, garner more focus, stemming from the widely held view that emerging markets possess less mature financial and banking systems than advanced economies. In this domain, Bianchi (2011) stands out, offering a compelling comparison between laissez-faire and constrained efficiency in open economies with fluctuating collateral constraints. This work underscores how strategic regulations can drastically reduce the likelihood and impact of balance sheet crises. Yet, a common shortcoming across these models is their exclusive focus on net inflows, bypassing the nuanced dynamics of inflows and outflows. Moreover, they tend to position financial frictions primarily on the emerging market side, sidelining the role of foreign lenders.

Our model stands distinct from prior models in several significant ways. Firstly, while previous studies mostly focus on capital inflows, we emphasize capital outflows equally. Understanding these aspects is crucial for comprehensively assessing a country's financial posture. Secondly, our model also incorporates financial frictions on the foreign lenders' side instead of just the domestic borrowers' side, motivated by the frictional international fund intermediation by large financial intermediaries like global banks extensively discussed in the literature. This approach better reflects the real-world intricacies of financial transactions. Lastly, contrary to conventional thought, we posit that the primary inefficiency is about something other than unregulated emerging markets overlooking significant financial pitfalls. Instead, we see a need for a coordinated effort when these markets price their bonds for international sale. Thus, while our conclusions might resemble those of past studies, especially when considering cases like Brazil, the underlying rationale is notably different.

3 Model

This model depicts an emerging market's strategy in issuing illiquid long-term bonds to sustain consumption and short-term bonds to weather the time-varying global financial conditions. Risk-averse foreign financiers price long-term emerging market bonds with stochastic discount factors during financial easing periods and with their wealth divided by holdings in financially constrained episodes. The model underscores the EMs' adaptability to the global financial climate through dual-direction capital flows and highlights the role of a social planner in navigating bidirectional capital flow more efficiently.

3.1 Households

We consider the emerging market (Henceforce EM) as a small open economy comprising homogeneous households that consume a single good. The objective of the representative household is to maximize its expected lifetime utility as given below:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

Here, C_t denotes the consumption of the single good at period t, and β is the subjective discount factor. The economy engages in global trade through two types of assets: long-term and short-period bonds. The former, which resembles the bonds discussed in Bianchi et al. (2018), is issued at time t and promises $\delta(1-\delta)^{j-1}$ units of goods at $t+j$ indefinitely. In contrast, the latter promises one unit of goods at $t+1$. The short-term bond's price is $\frac{1}{R_f}$, where R_f represents the constant global risk-free rate. The long-term bond's price is denoted by q_t.

The household's budget constraint for a given period t is represented as follows:

$$\bar{y} + a_t + \delta b_t = c_t + \frac{a_{t+1}}{R_f} + q_t\big(b_{t+1} - (1-\delta)b_t\big)$$

Households finance their consumption c_t, risk-free bond acquisition a_{t+1}, and long-term bond purchases $q_t\big(b_{t+1} - (1-\delta)b_t\big)$, utilizing their fixed endowment income $\bar{y}$, returns from risk-free bonds a_t, and long-term bond coupons δb_t. We posit that EMs cannot short the short-term bonds and cannot maintain a net long position on long-term bonds, i.e., $b_{t+1} \leq 0$ and $a_{t+1} \geq 0$ for all t. Importantly, we assume the condition $\beta R_f < 1$ in our model, ensuring that the equilibrium position of EM households concerning long-term bonds remains perpetually negative.

Additionally, we impose a supplementary constraint on b_{t+1}, namely $b_{t+1} \geq \underline{B}$. This limitation prevents the economy from indefinitely rolling over its long-term debt.

3.2 Foreign Financiers

Our study considers foreign financiers as the exclusive traders of long-term bonds issued by the emerging market. These financiers, homogeneous and perfectly competitive, hold the EM long-term bonds for one period before selling them off and exiting the market.

The financiers, born at period t, have finite wealth W_t and stochastic discount factors $SDF_{t,t+1}$. It is always ensured that bond price, q_t, is the lesser of two values: one derived from the stochastic discount factor $SDF_{t,t+1}$, and the other determined by financiers' wealth W_t divided by their holdings of EM bonds, b^f_{t+1}.

$$q_t = \min\{E_t\big[SDF_{t,t+1}\big(\delta + (1-\delta)q_{t+1}\big)\big], \frac{W_t}{b^f_{t+1}}\} \tag{1}$$

The financiers' wealth, fluctuating between high and low values (W_H and W_L respectively), serves as a barometer for the global financial condition. This is tracked by an exogenous Markov-switching process with a transition matrix:

$$Pr(W_{t+1}|W_t) \sim \begin{bmatrix} \pi_{HH} & 1-\pi_{HH} \\ 1-\pi_{LL} & \pi_{LL} \end{bmatrix}$$

This matrix represents the probabilities of transition between high and low-wealth states. The stochastic discount factor scheme is as follows:

$$SDF_{t,t+1} = \begin{cases} \dfrac{1}{R_f + (1-\pi_{HH})(R^\kappa - 1)R_f}, & \text{if } W_t = W_H \text{ \& } W_{t+1} = W_H, \\ \dfrac{R^\kappa}{R_f + (1-\pi_{HH})(R^\kappa - 1)R_f}, & \text{if } W_t = W_H \text{ \& } W_{t+1} = W_L, \\ \dfrac{1}{R_f + \pi_{LL}(R^\kappa - 1)R_f}, & \text{if } W_t = W_L \text{ \& } W_{t+1} = W_H, \\ \dfrac{R^\kappa}{R_f + \pi_{LL}(R^\kappa - 1)R_f}, & \text{if } W_t = W_L \text{ \& } W_{t+1} = W_L. \end{cases}$$

The financiers' stochastic discount factor is characterized by risk aversion and alignment with the international risk-free rate. Financiers value future payoffs under low wealth more than those under high, indicating risk-averse nature. The effective risk-aversion is effectively captured by $R^\kappa > 1$: the higher R^κ, the more risk averse these financiers. Regardless of the R^κ value, the financiers' required return aligns with the international risk-free rate if the future payoff remains constant irrespective of the future financial condition.

When examining financiers' wealth constraints, the endogeneity of long-term bond prices becomes evident. Consider a frictionless international fund market, characterized by a sufficiently high W_L such that $W_L > -\bar{q}\underline{B}$, the term $\bar{q} = \frac{\delta}{\delta + r_f}$ signifies the discounted value of all future coupons of a unit of the long-term bond using the international risk-free rate. Meanwhile, $\underline{B}$ stands for the minimum bond level retained by EM households when the bond price remains constant at $\bar{q}$. In this scenario, financiers face no wealth constraints, enabling them to price EM bonds at their intrinsic value, $\bar{q}$. However, when W_L is constrained, i.e., $W_L < \bar{q}\underline{B}$, financiers will be detered from pegging the EM bonds at $\bar{q}$. As a result, competitive market dynamics can cause the bond price to drop beneath $\bar{q}$, even if individual financiers have no market power. This distinction elucidates our characterization of W_H as unconstrained financial conditions and W_L as constrained conditions. Importantly, the equilibrium bond price q might descend below $\bar{q}$ even in unconstrained environments due to expectations of future financial states and the long-term nature of the EM bonds.

3.3 Decentralized Equilibrium

This subsection examines the decentralized equilibrium, commonly termed as *laissez-faire*, without any central planner's involvement. Our attention is chiefly directed towards the *Markov Perfect Equilibrium*. In this context, the decision-making rules of EM households are entirely based on state variables relevant to payoff, represented by $s \equiv (W, A, B)$. We employ a subscript "LF" to designate allocations in the laissez-faire framework and use a superscript "LF" for the long-term bond price. The decentralized equilibrium is described as follows:

1. Laws of motion for gross positions $A'_{LF}(s)$ and $B'_{LF}(s)$
2. An aggregate consumption function $C_{LF}(s)$
3. A long-term bond price function $q^{LF}(s)$

These components must satisfy:

- **Budget Constraint:**

$$\bar{y} + A + \delta B = C_{LF}(s) + \frac{A'_{LF}(s)}{R_f} + q^{LF}(s)\big(B'_{LF}(s) - (1-\delta)B\big).$$

- **Euler Equations:** Given $q^{LF}(s)$, $A'_{LF}(s)$, $B'_{LF}(s)$ and $C_{LF}(s)$ must solve:

$$\frac{u'(C_{LF}(s))}{R_f} = \beta E_{W'|W} u'(C_{LF}(s')) + \mu^A_{LF}(s),$$
$$u'(C_{LF}(s))q^{LF}(s) = \beta E_{W'|W}\left[u'(C_{LF}(s'))\left(\delta + (1-\delta)q^{LF}(s')\right)\right] + \mu^B_{LF}(s) - \gamma_{LF}(s).$$

where $s' = (W', A'_{LF}(s), B'_{LF}(s))$ and $u'(\cdot) = \cdot^{-\sigma}$ represents the marginal utility of consumption. The terms $\mu^A_{LF}(s)$ and $\mu^B_{LF}(s)$ denote the Lagrange multipliers corresponding to the constraints $A'_{LF}(s) \geq 0$ and $B'_{LF}(s) \geq \underline{B}$, respectively. Conversely, $\gamma(s)$ is associated with the constraint $B'_{LF}(s) \leq 0$.

- **Bond Pricing Rule:** Given $A'_{LF}(s)$ and $B'_{LF}(s)$, $q^{LF}(s)$ must satisfy:

$$q^{LF}(s) = \min\left\{E_{W'|W}\left[SDF_{W,W'}\left(\delta + (1-\delta)q^{LF}\left(W', A'_{LF}(s), B'_{LF}(s)\right)\right)\right], -\frac{W}{B'_{LF}(s)}\right\}.$$

where we abuse the notation such that $SDF_{W,W'}$ represents the SDF between the current financial condition W and the future financial condition W'.

3.4 Centralized Equilibrium

In this subsection, we consider a scenario where a benevolent social planner intervenes only under unconstrained external financing conditions. This social planner engages with the same foreign financiers as individual households. We term this equilibrium as the "centralized equilibrium" and its corresponding allocation as the "constrained efficient allocation". We employ a subscript "SP" to designate allocations under the social planner's supervision and use a superscript "SP" for the long-term bond price.The centralized equilibrium is defined as follows:

1. Portfolio rules $A'_{SP}(s)$ and $B'_{SP}(s)$,
2. An aggregate consumption function $C_{SP}(s)$
3. A value function $V_{SP}(s)$,
4. A long-term bond price function $q^{SP}(s)$.

when $W = W_H$: these components must satisfy:

- **Bellman Equation:** Given $q^{SP}(s)$, $A'_{SP}(s)$, $B'_{SP}(s)$ and $V_{SP}(s)$ must solve:

$$V_{SP}(W_H, A, B) \equiv \max_{\substack{A'_{SP}\geq 0\\ \underline{B}\leq B'_{SP}\leq 0}} \left\{u\left(\bar{y} + A + \delta B - Q\big(W_H, A'_{SP}, B'_{SP}\big)\big(B'_{SP} - (1-\delta)B\big) - \frac{A'_{SP}}{R_f}\right)\right.$$
$$\left. + \beta E_{W'|W_H} V\big(W', A'_{SP}, B'_{SP}\big)\right\}$$

subject to

$$Q(W_H, A'_{SP}, B'_{SP}) \equiv E_{W'|W_H}\left[SDF_{W_H,W'}\big(\delta + (1-\delta)q^{SP}(W', A'_{SP}, B'_{SP})\big)\right]$$

- **Bond Pricing Rule:** Given $A'_{SP}(s)$ and $B'_{SP}(s)$, $q^{SP}(s)$ must satisfy:

$$q^{SP}(W_H, A, B) = Q\big(W_H, A'_{SP}(W_H, A, B), B'_{SP}(W_H, A, B)\big).$$

when $W = W_L$: these components must satisfy:

- **Budget Constraint:**

$$\bar{y} + A + \delta B = C_{SP}(s) + \frac{A'_{SP}(s)}{R_f} + q^{SP}(s)\big(B'_{SP}(s) - (1-\delta)B\big).$$

- **Euler Equations:** Given $q^{SP}(s)$, $A'_{SP}(s)$, $B'_{SP}(s)$ and $C_{SP}(s)$ must solve:

$$\frac{u'(C_{SP}(s))}{R_f} = \beta E_{W'|W} u'(C_{SP}(s')) + \mu^A_{SP}(s),$$
$$u'(C_{SP}(s))q^{SP}(s) = \beta E_{W'|W}\left[u'(C_{SP}(s'))\left(\delta + (1-\delta)q^{SP}(s')\right)\right] + \mu^B_{SP}(s) - \gamma_{SP}(s).$$

  where $s' = (W', A'_{SP}(s), B'_{SP}(s))$ and $u'(\cdot) = \cdot^{-\sigma}$ represents the marginal utility of consumption. The terms $\mu^A_{SP}(s)$ and $\mu^B_{SP}(s)$ denote the Lagrange multipliers corresponding to the constraints $A'_{SP}(s) \geq 0$ and $B'_{SP}(s) \geq \underline{B}$, respectively. Conversely, $\gamma_{SP}(s)$ is associated with the constraint $B'_{SP}(s) \leq 0$.

- **Bond Pricing Rule:** Given $A'_{SP}(s)$ and $B'_{SP}(s)$, $q^{SP}(s)$ must satisfy:

$$q^{SP}(s) = \min\left\{E_{W'|W}\left[SDF_{W,W'}\left(\delta + (1-\delta)q_{SP}\left(W', A'_{SP}(s), B'_{SP}(s)\right)\right)\right], -\frac{W_L}{B'_{SP}(s)}\right\}.$$

Under W_L, the equilibrium conditions in constrained efficiency mirror those in laissez-faire due to the absence of the social planner. Nevertheless, it's crucial to note that due to the distinction between the long-term bond price functions $q^{SP}(s)$ and $q^{LF}(s)$, the realized allocation will differ between the two scenarios even when the state s remains consistent.

3.5 Laissez-faire versus Constrained Efficiency

In exploring both laissez-faire and constrained efficiency, the nuances lie primarily in the bond pricing and portfolio choices. At the heart of this distinction is the function $Q(W_H, A', B')$, which establishes a direct correlation between the current bond price and the present bond position B'. Notably, this relationship is tactically leveraged by the social planner when orchestrating portfolio decisions for the entire economy.

In the laissez-faire scenario, individual households operating without coordination treat both the current bond price $q^{LF}(W, A, B)$ and future bond price $q^{LF}(W', A', B')$ as exogenous when determining their portfolio strategies. They recognize that their collective choices might influence the bond price. However, in practice, they do not account for this potential impact because each household perceives its influence as minimal, with no ability to sway aggregate outcomes.

On the other hand, a social planner assumes the mantle of the entire economy, wielding the clout of a monopolist. With this broadened perspective, the planner can make strategic aggregate portfolio choices to optimize societal welfare. This proactive approach to maximizing welfare comes into sharp relief when we analyze the First Order Conditions (FOCs) associated with A'.

$$
\begin{aligned}
&\frac{u'(C_{SP}(s))}{R_f} - \beta E_{W'|W_H} u'(C_{SP}(s')) - \mu_{SP}^A(s) \\
&= \underbrace{-u'(C_{SP}(s))\frac{\partial Q(W_H, A', B')}{\partial A'}(B' - (1-\delta)B)}_{\text{marginal welfare gain in constrained efficiency by increasing } A'} \\
&\quad \underbrace{-\beta(1-\pi_{HH})u'\big(C_{SP}(W_L, A', B')\big)\frac{\partial q^{SP}(W_L, A', B')}{\partial A'}(B'' - (1-\delta)B')}_{\text{marginal welfare loss in constrained efficiency by increasing } A'}
\end{aligned}
$$

and with respect to B':

$$
\begin{aligned}
&u'(C_{SP}(s))q^{SP}(s) - \beta E_{W'|W_H}[u'(C_{SP}(s')) * (\delta + (1-\delta)q^{SP}(s'))] - \mu_{SP}^B(s) + \gamma_{SP}(s) \\
&= \underbrace{-u'(C_{SP}(s))\frac{\partial Q(W_H, A', B')}{\partial B'}(B' - (1-\delta)B)}_{\text{marginal welfare gainin constrained efficiency by increasing } B'} \\
&\quad \underbrace{-\beta(1-\pi_{HH})u'\big(C_{SP}(W_L, A', B')\big)\frac{\partial q(W_L, A', B')}{\partial B'}(B'' - (1-\delta)B')}_{\text{marginal welfare loss in constrained efficiency by increasing } B'}
\end{aligned}
$$

Let B' and B'' represent $B'_{SP}(W_H, A, B)$ and $B'_{SP}(W_L, A'_{SP}(W_H, A, B), B'_{SP}(W_H, A, B))$, respectively. While the above First Order Conditions (FOCs) might seem intricate initially, they embed critical economic interpretations. Comparatively, each equation's left-hand side (LHS) can be considered the corresponding Euler equation in a laissez-faire framework where the right-hand side (RHS) would conventionally be zero. However, the RHS remains non-zero in our analysis, reflecting the nuanced welfare effects of modifications in A' and B'.

We first analyze the Euler equation related to B'. An equilibrium feature (obtained after we solve and simulate the model) is that the economy, irrespective of the equilibrium type, tends to witness positive long-term bond inflows under a relaxed financial constraint and negative inflows when constraints tighten. This pattern suggests $(B' - (1-\delta)B)$ is typically negative and $(B'' - (1-\delta)B')$ is positive. The first term on the RHS, $-u'(C)\frac{\partial Q(W_H, A', B')}{\partial B'}(B' - (1-\delta)B)$, illustrates the welfare benefit regarding increasing long-term bonds by one unit. Conversely, the second term (including the negative sign) represents the associated welfare loss. Next, I will explain the reasoning behind these outcomes.

In the understanding of why $-u'(C_{SP}(s))\frac{\partial Q(W_H,A',B')}{\partial B'}(B'-(1-\delta)B)$ reflects welfare gains from an increase in A', it's vital to acknowledge that long-term bond holdings, signified by B', empower the EM with less long-term debt to rollover during a financial tightening, which escalates the firesale bond price and the current bond price. Therefore, a marginal increase in B' will have a positive impact on the current long-term bond price, captured by $\frac{\partial Q(W_H,A',B')}{\partial B'}$. This effect is first scaled by $-(B'-(1-\delta)B)$, the existing bond issuance, as all bonds in the same period share the same price, and then $u'(C_{SP}(s))$, the marginal utility of households.

The expression $-\beta(1-\pi_{HH})u'\big(C_{SP}(W_L,A',B')\big)\frac{\partial q^{SP}(W_L,A',B')}{\partial B'}(B''-(1-\delta)B')$ captures the social planner's dynamic considerations. Bond prices are influenced by bond positions during financial tightening, creating valuation shifts exploitable by the EM, a net bond purchaser. The lower the bond price, the greater the wealth transfer from the past-generation financiers to the EM. Therefore, an increase in B' has a positive impact on the firesale long-term bond price (captured by $\frac{\partial q(W_L,A',B')}{\partial B'}$) but can lead to welfare loss as the EM is buying, not selling long-term bonds during a financial tightening. This effect is scaled by $-(B'-(1-\delta)B)$, the existing bond redemption, the marginal utility $u'(C_{SP}(W_L,A',B'))$ in the next period's firesale stage, the discount factor β, and the probability of a financial tightening occurring in the next period.

Examining these terms reveals the social planner's trade-off concerning long-term bond position adjustment. The planner wishes to raise the long-term bond price, thereby benefiting the EM as a borrower during unconstrained periods, by increasing B'. Conversely, the planner desires a more significant wealth transfer through a reduced firesale price, achievable by reducing B'. Balancing these objectives results in a seemingly ambiguous marginal adjustment of B' compared to private households.

Nonetheless, uncoordinated households tend to inflate bond prices beyond the optimum under constraints. The social planner can rectify this by reducing the long-term bond position ex-ante, thus dampening the firesale price relative to private agents. This term mirrors the first term but factors in the likelihood of financial tightening $(1-\pi_{HH})$ and the subjective discount factor β.

The social planner's problem becomes more complicated with the addition of short-term bonds (A'). In terms of the pricing effect, increasing A' has a similar effect to increasing B'. While a decreased long-term debt level reduces the economy's exposure to the firesale, an increase in A' endows the economy more resources to redeem the long-term bonds during the firesale. Analyzed separately with a fixed B', raising A' elevates the immediate short-term bond price, improving welfare, but simultaneously increasing the future firesale price, harming welfare. Thus, a similar trade-off to the long-term bond arises. However, the analysis becomes intricate when the planner can modify both A' and B'. For instance, if, when scrutinizing A' alone with fixed B', we deduce that the planner leans towards reducing A', valuing wealth transfers during constraints more, it doesn't necessarily mean that the planner will continue to reduce A' when both A' and B' can vary. Why?

This arises because if the social planner aims to reduce the firesale price, decreasing B' is also a feasible option. Yet, simultaneously decreasing both A' and B' may not be ideal, as this could disrupt the intertemporal consumption smoothing, resulting in excessively high current consumption. Therefore, the final A' and B' selection hinges on adjusting which asset most effectively reduces the future firesale price without markedly altering present consumption. The preferred instrument could be either A' or B'. For instance, if A' proves more effective, shrinking the external balance sheet might be advantageous, as a decrease in A' could more than offset an increase in B' to lower the firesale bond price. Consequently, optimal gross capital flow management is a quantitative question requiring a quantitative solution.

To underscore that optimal gross capital flow management hinges on quantitative factors, the next section subsequently delves into a unique case within the infinite-period model. This scenario involves households with a linear utility function, an inclination for perfectly smoothed consumption, and a single instance of financial tightening. The insights gleaned from this model reinforce the notion that the optimal gross capital flow management is a quantitative question.

4 A Simplified Case

We consider a particular instance of the infinite-period model that retains the essence of the comprehensive model while boasting high tractability. The model initiates at period 0, characterized by legacy portfolios (a_0, b_0) and high foreign financiers' wealth W_H. At period 1, a potential financial tightening looms with a probability π. Subsequently, from period 2 onwards, the model stabilizes into a frictionless steady-state, ensuring $q_t = \bar{q}$ for all $t \geq 2$. This effectively reduces our infinite-period model to a two-period structure.

For analytical simplicity, we introduce certain assumptions: households exhibit linear utility functions, captured by $u(c_t) = c_t$, and we set $\beta R_f = 1$. These two conditions jointly suggest that households opt for zero consumption if any asset offers a return surpassing R_f. To decipher this streamlined model, we adopt a backward induction methodology.

4.1 Equilibrium at Period 1

During period 1, the economy encounters two potential external financial scenarios: the unconstrained with higher financiers' W_H and the constrained with lower financiers' wealth W_L. Under the unconstrained scenario, the determination of the optimal problem becomes ambiguous since both the long-term bonds and liquidity yield identical returns. Without sacrificing any general insights, we set the long-term bond choice at the period's end, b_2^N, to adhere to

$$\bar{y} + a_1 + \delta b_1 = \bar{q}\left(b_2^U - (1-\delta)b_1\right), \tag{2}$$

with the subscript U designating the "Unconstrained" state.

On the other hand, under a tightened financial condition, the equilibrium portfolio choice becomes singular. In this state, households gravitate towards zero consumption, exhausting their short-term bonds to acquire the long-term bonds. This situation can be described by

$$\bar{y} + a_1 + \delta b_1 = q_1^C\left(b_2^C - (1-\delta)b_1\right), \tag{3}$$

with the subscript C symbolizing the "Constrained" condition. In tandem, foreign financiers calibrate the pricing of long-term bonds as

$$q_1^C = -\frac{W_L}{b_2^C}. \tag{4}$$

4.2 Equilibrium at Period 0

In this subsection, we initially demonstrate that at period 0, the economy can invariably augment the long-term bond price by enlarging the external balance sheet. This establishes an equivalence between the management of gross positions and the control of the long-term bond price. Subsequently, we separately solve for laissez-faire and the constrained efficiency allocations, asserting that the optimal capital flow management at period 0 is contingent upon the inherited long-term bond level denoted as b_0.

4.2.1 Bond price and External Balance Sheet Size

Regardless of the equilibrium type between laissez faire and constrained efficiency, the following equilibrium conditions always hold at period 0.

$$m_0 = \beta a_1 + q_0\left(b_1 - (1-\delta)b_0\right) \tag{5}$$
$$q_0 = \beta\delta + (1-\pi)(1-\delta)M^U\bar{q} + \pi(1-\delta)M^C q_1^C \tag{6}$$

(5) is the budget constraint of households. Note that the zero consumption choice is an equilibrium result under both laissez-faire and constrained efficiency. (6) is the pricing rule of financiers regarding the long-term bond, where M^U is the SDF between period 0 and period 1's unconstrained state, and M^C is the SDF between period 0 and period 1's constrained state. Financiers' risk-aversion implies $M^C > M^U$. As period 1 corresponds to the unconstrained state, the price is pinned down by the stochastic discount factor instead of the financiers' wealth.

If we combine equations (3) to (6), we can derive an expression of the firesale price q_1^C as follows

$$q_1^C = \bar{q} - \frac{1}{1-\delta} * \frac{\beta(W_L + \bar{y}) + n_0}{(1-\delta)\pi M^C b_0 + (\beta - \pi M^S) b_1} \tag{7}$$

where $n_0 \equiv \bar{y} + a_0 + \bar{q} R_f b_0$ is the country's net foreign assets in period 0. Let us assume $\beta(W_L + \bar{y}) + n_0 < 0$ and $\beta > \pi M^C$. Then it follows from equation (7) that q^C is increasing with b_1, that is, the firesale price of a long-term bond is increasing with the number of long-term bonds issued in period 0. Like in Jeanne and Sandri (2023) this is because the additional long-term bonds finance short-term bonds that can be used to buy back the long-term bonds in period 1. It follows from equation (6) that q_0 is also increasing in b_1.

Since the household does not consume in period 0 or in period 1, period 0-welfare is equal to the expected discounted value of period-2 welfare, $U_0 = \beta^2 E_0 U_2$. Given $a_2 = 0$ period-2 welfare is equal to the PDV of income plus the payment on the long-term bonds held by the household, $U_2 = \bar{y}/(1-\beta) + \bar{q} R_f b_2$. Hence maximizing U_0 is equivalent to maximizing $E_0 b_2$. The difference between the decentralized laissez-faire allocation and the social planner allocation is that under laissez-faire households take the prices q_0 and q^C as given, whereas the social planner takes into account that these prices are endogenous to the representative household's balance sheet. The rest of this subsection compares the two allocations.

4.2.2 Laissez-faire

The period-1 budget constraint implies $b_2 = \frac{y + a_1}{q_1} + b_1 \left(\frac{\delta}{q_1} + 1 - \delta \right)$. Using the period-0 budget constraint (5) to substitute out b_1, and ommitting irrelevant terms gives

$$E_0 b_2 = a_1 E_0 \left[\frac{1}{q_1} - \frac{\beta}{q_0} \left(1 - \delta + \frac{\delta}{q_1} \right) \right] + ...$$

In an equilibrium where households hold a non-zero finite level of short-term bonds a_1 the factor of a_1 in the equation above must be equal to zero, which implies

$$q_0 = \beta \left[\delta + \frac{1-\delta}{E_0 \left(1/q_1 \right)} \right].$$

Using (6) to substitute out q_0 gives a fixed-point equation for q^C. As $q^C < \bar{q}$, we solve for q^C as follows

$$q^C = \frac{M^U}{M^C} \bar{q} \tag{8}$$

In the face of financial tightening, the long-term bond price declines below its intrinsic value $\bar{q}$. This allows us to derive

$$q_0 = \beta\delta + (1-\delta) M^U \bar{q}$$

and subsequently compute the equilibrium household balance sheet, denoted by (a_1, b_1).

4.2.3 Constrained Efficiency

Venture into the scenario where a social planner in period 0 determines a_1 and b_1 with the intent of maximizing $E_0 b_2$. This leads to:

$$\begin{aligned}
E_0 b_2 &= (1-\pi) b_2^U + \pi b_2^C, \\
&= \frac{1-\pi}{\bar{q}}(a_1 + \bar{q} R_f b_1) - \pi \frac{W_L}{q^C} + \cdots, \\
&= \frac{1-\pi}{\bar{q}} \left(\frac{q_0 b_0 (1-\delta)}{(1-\pi) M^U} \right) - \pi \frac{W_L}{q^C} + \cdots, \\
&= -\pi \left[(-b_0) \frac{(1-\delta)^2 q^C}{\bar{q} M^U / M^C} + \frac{W_L}{q^C} \right] + \cdots,
\end{aligned}$$

where the ellipsis $(\cdots)$ signifies constant terms, which we can conveniently overlook due to their independence from q^C. The derivations are facilitated by leveraging equations (2), (4), (3), and (5).

Therefore, the social planner aims to minimize $(-b_0) \frac{(1-\delta)^2 q^C}{\bar{q} M^U / M^C} + \frac{W_L}{q^C}$, leading to the formulation:

$$q^C = \frac{1}{1-\delta} \sqrt{\left(\frac{M^U}{M^C} \bar{q} \right) \left(\frac{W_L}{-b_0} \right)}. \tag{9}$$

Comparing the above with its laissez-faire counterpart, provided by equation (8), it's indeterminate whether the social planner either escalates or reduces q^C relative to laissez-faire. Consequently, the decision to either amplify or diminish the households' balance sheets (compared to the laissez-faire) remains to be determined.

The social planner will only choose to enhance the balance sheet and elevate q^C relative to laissez-faire when:

$$b_0 > -\frac{1}{(1-\delta)^2} \frac{W_L}{\bar{q}} \frac{M^C}{M^U},$$

i.e., if the country does not inherit too much long-term debt from the past. Intuitively, the social planner can expand the country's balance sheet so as to accumulate short-term bonds and buy back the country's long-term debt in a financial tightening, or it can use the short-term bonds that it already has to buy back the long-term debt in period 0. The social planner tends to follow the first strategy when legacy long-term debt is not too high.

4.3 A Discussion of the Results

In a simplified model, we demonstrate that the optimal capital management strategy for a social planner can either involve expanding or contracting the balance sheet. This decision hinges on the magnitude of long-term debt the economy has inherited from the past. Viewing this from a more abstract perspective, the social planner's capital intervention fundamentally aims at a wealth transfer favorable to emerging markets, balancing between different stakeholders' interests.

Besides the emerging markets themselves, three critical stakeholders are involved: financiers born in period -1 holding $-b_0$ units of long-term bonds, financiers born in period 0 with $-b_1$ units of such bonds, and financiers emerging during the financial tightening in period 1 possessing $-b_2^C$ units of these bonds. By elevating bond prices in period 0, the social planner benefits the financiers from period -1 but undermines the interests of financiers emerging during the financial tightening in period 1. Therefore, the decision to raise or lower prices is a matter of determining which wealth transfer yields maximum benefits for the economy.

In Jeanne and Sandri (2023), as the initial inherited long-term bond amount is zero, raising bond prices doesn't transfer benefits to financiers from period -1. Instead, it boosts the economy's expected welfare at the expense of harming the interest of financiers born in period 1. This model concludes that the social planner should aim to maximize the balance sheet to enhance lending conditions in period 0.

However, due to the presence of financiers from period -1 in our model, excessively elevating asset prices in period 0 might over-transfer resources to these financiers. By suppressing long-term bond prices in period 0, there's a potential for resource transfer from the financiers of period -1 back to the economy. As such, our model only unequivocally supports expanding the balance sheet. Instead, the decision largely depends on specific preconditions—in this case, the level of inherited long-term debt.

Considering our model's ambiguity regarding optimal capital flow management, we are keen to discern how real-world emerging markets should manage their external balance sheets. Specifically, we aim to understand whether they should expand, contract, or adjust the ratio of external liabilities to external assets. We undertake a quantitative assessment using Brazil as a representative case study to glean insights into this.

5 Parameters

This section identifies the parameters in our model. We have selected Brazil as the representative numerical instance, motivated by its unique balance sheet attributes and comprehensive records of its external balance sheet. Our data primarily stems from the IMF's BOP-IIP database, supplemented by the FRED Economic

Data and OECD Statistics. We have decently determined our model's parameters through meticulous calibration and estimation, focusing on crucial moments of Brazil's external balance sheet.

5.1 An Ideal Numerical Instance: Brazil

Our choice of Brazil as the model's numerical representative is informed by its distinct balance sheet attributes and the depth of available data. Unlike numerous emerging markets which initiated their gross position data recordings post the 2008 Global Financial Crisis, Brazil has a rich data history dating back to 2001 Q4. This expansive dataset becomes particularly invaluable when applying the GMM to gauge our non-linear model, enhancing the robustness of our estimations.

Brazil's external balance sheet also represents the traits typical of emerging markets. The trajectory of Brazil's gross capital capital flows and returns to its external assets and liabilities during our sample periods is depicted in Figure 1. An intriguing observation is the comovement between Brazil's capital inflows and outflows, characterized by a correlation of approximately 0.8. Notably, the returns on Brazil's external assets consistently display reduced volatility compared to its liabilities. This distinctive feature positions Brazil as an exemplary candidate for our numerical explorations.

5.2 Data Sources

Our study harnesses Brazil's balance sheet data from Q4 2001 to Q4 2022. At the heart of our research is the BOP-IIP database, offering a detailed account of a country's capital inflows, outflows, external liabilities, and assets, both in aggregate and granular breakdowns. Entries in the IIP are marked at current dollar market valuations, with variances propelled by BOP-logged capital movements and valuation adjustments.

Complementing our primary data, we integrate FRED Economic Data and OECD Statistics. The former facilitates inflation adjustment using the U.S. GDP price deflator, essential for computing real capital flows and gross positions in dollars. Concurrently, OECD Statistics offers GDP figures adjusted for purchasing power parity (PPP), invaluable for standardizing Brazil's capital flow metrics and gross positions. For more details regarding data sources and data processing, please refer to the Appendix.

5.3 Parameter Identification

We have refined specific model parameters through calibration, while the others were estimated using the continuously-updating Generalized Method of Moments (CU-GMM). Table 1 presents these parameter values. A comprehensive outline of our calibration and estimation methodologies is available in the Appendix.

Figure 1: Capital Flows and Returns for Brazil

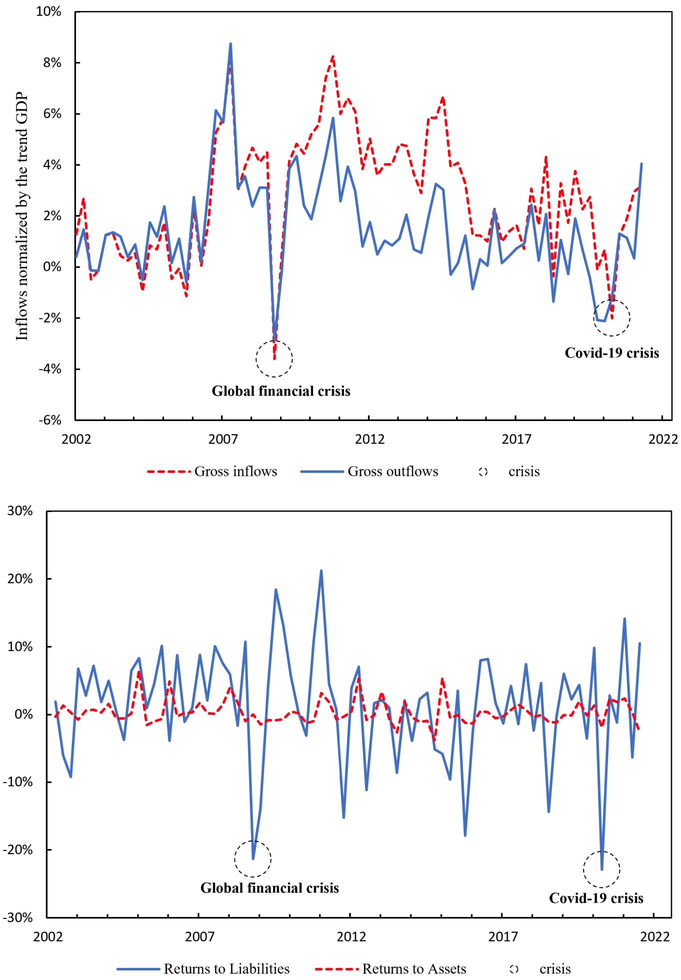


Table 1: Parameter Estimation Results

Parameter	Value	Determination Method	Description
R_f	1.0017	Calibration	International risk-free rate
δ	0.0174	Calibration	Depreciation rate of long-term bonds
σ	4.0	Calibration	Relative risk-aversion
β	0.9859	Estimation	Subjective discount factor for households
R^{κ}	1.3627	Estimation	Risk-aversion measure for financiers
W_L	1.1101	Estimation	Lower bound of financiers' wealth
π_{UU}	0.9664	Estimation	Continuation probability in unconstrained states
π_{CC}	0.5366	Estimation	Continuation probability in constrained states

5.4 Model Fitness

Upon completion of our model's estimation, we delve deeper into its accuracy and relevance. A detailed comparison of the estimated moments against their empirical counterparts, as presented in Table 2, reveals several illuminating observations.

Table 2: Comparison between data moments and model moments

Moment	Theoretical Moments	Empirical Moments	Targeted
std(inflows)	0.0263	0.0235	Yes
std(outflows)	0.0170	0.0191	Yes
std(excess return)	0.0730	0.0798	Yes
std(NFA)	0.1395	0.2194	No
ρ(inflows, outflows)	0.8278	0.7740	Yes
ρ(inflows, excess return)	0.4148	0.1939	Yes
ρ(outflows, excess return)	0.2554	0.4191	Yes
ρ(inflows, NFA)	-0.6393	-0.6806	No
ρ(outflows, NFA)	-0.2709	-0.2918	No
E(excess return)	0.0151	0.0107	Yes
E(liability-GDP-ratio)	1.5785	1.5585	Yes
E(inflow-GDP-ratio)	0.0289	0.0277	No
E(asset-GDP-ratio)	0.0340	0.7984	No

Note: E[X], std[X], and ρ[X, Y] denote the mean of variable X, the standard deviation of variable X, and the correlation between variables X and Y, respectively.

Foremost, the model exhibits remarkable accuracy in mirroring the variability inherent in inflows, outflows, and excess returns. The minor discrepancies in the volatility of inflows and outflows suggest nuanced volatilities not entirely captured by the model, but these deviations remain within academically acceptable boundaries. This level of fidelity in reproducing volatility dynamics stands as a testament to our model's robustness.

Turning to the correlation metrics, while the model astutely captures the positive interplay between inflows and outflows and their negative correlations with the net foreign asset positions, there is an overestimation in the correlation of inflows with excess returns and an underestimation in the correlation of outflows with excess returns.

Our model's estimation of means, particularly the liability-to-GDP ratio, aligns commendably with empirical data. The mean of the inflow-to-GDP ratio is also quite close to its empirical counterpart, but this is primarily due to our calibration of δ, which implicitly matches the average inflow-to-GDP ratio between the model and data. However, the significant misalignment in the asset-to-GDP ratio necessitates further scrutiny.

Overall, the primary limitations of our model manifest predominantly on the asset front. Two prominent discrepancies arise: a notably subdued correlation between outflows and the excess return and a discernibly diminished representation of assets. Several rationales underpin these observed inconsistencies.

In our model, short-term bonds primarily serve to repurchase long-term bonds at depressed prices during financial tigntening episodes. This mechanism, while valuable, may not encapsulate the multifaceted reasons emerging economies accumulate highly liquid safe assets. Real-world motivations extend beyond our model's scope. For instance, the convenience yield theory posits that instruments like U.S. treasuries offer a dual benefit to financial entities in emerging economies: a stable store of value and liquidity on demand.

A more comprehensive model, factoring in this multifunctionality of short-term bonds, would arguably yield a heightened correlation between overall outflows and excess returns. In our current setup, the short-term bond holding often stays at zero until the long-term debt reaches to high enough level, so it doesn't align synchronously with excess returns. Moreover, broadening the roles and functionalities of the short-term bond within the model would inherently elevate the average holdings of such assets within an economy.

Yet, it's crucial to underscore the primary thrust of our research, which revolves around the repurchase utility of short-term bonds for riskier asset classes. While acknowledging its limitations, we remain committed to our existing model framework for our quantitative explorations, given its focused and specialized approach.

6 Quantitative Results: Short-run Capital Flow Management

This section details the primary quantitative results of our DSGE model, focusing on the comparative study of balance sheet sizes under laissez-faire and constrained efficiency allocations in the short run. Results indicate a social planner's propensity to shrink the external balance sheet relative to private households. This allocation, enacted by the social planner, bolsters short-term welfare by enhancing financial crisis resilience and shortening the low consumption episodes.

6.1 Balance Sheet Size Comparison

This section delves into the distinction between the portfolio choices of a social planner and private households from a short-run perspective. Given our focus on the short term, we always assume that the social planner starts from the same inherited short and long-term bonds as private households in laissez-faire. Considering the laissez-faire equilibrium paths account for only a tiny portion of the state space (see Figure 2), we focus on a small subset with a non-zero likelihood of occurrence. Because A and B exhibit an almost perfect correlation along the equilibrium path when A is positive, we can concentrate on the dimension of B without looking at A. This strategic focus on the B dimension of the state vector along the ergodic laissez-faire equilibrium path considerably streamlines our analysis.

Figure 2: The ergodic path of $(A, -B)$ in laissez-faire

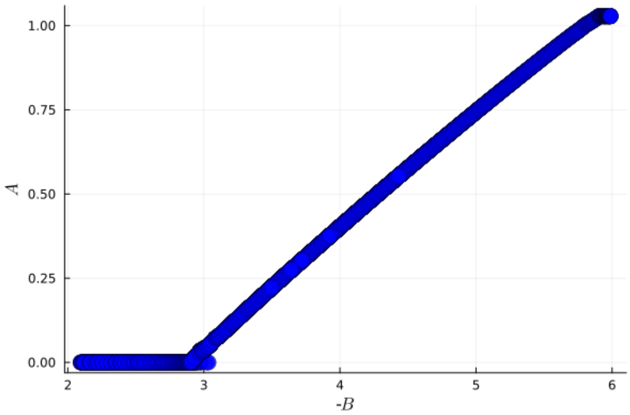


Figure 2 provides insight into the portfolio choices of the social planner relative to private households along the laissez-faire equilibrium path. Specifically, it demonstrates that the social planner typically curtails both long-term borrowings and short-term savings. To elucidate this, we introduce the portfolio differentials for short-term and long-term bonds between the two allocations:

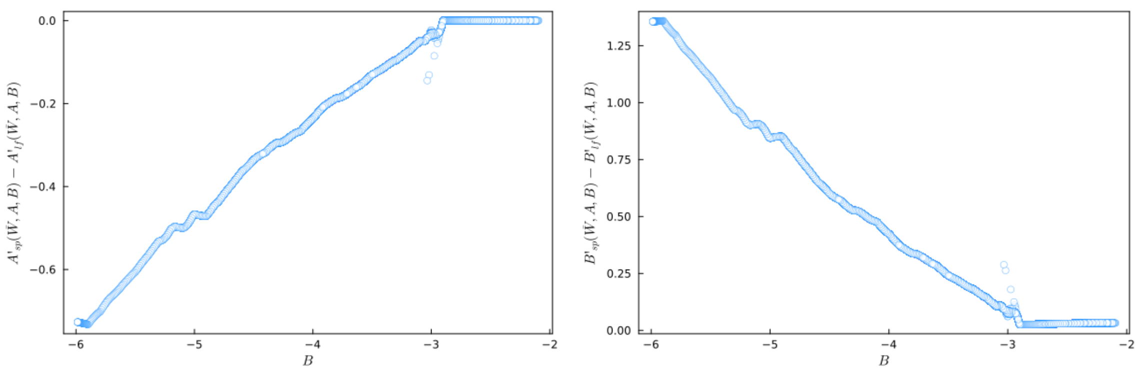
$$\Delta A'(W_H, A, B) = A'_{SP}(W_H, A, B) - A'_{LF}(W_H, A, B)$$

and

$$\Delta B'(W_H, A, B) = B'_{SP}(W_H, A, B) - B'_{LF}(W_H, A, B)$$

where the subscripts “SP” and “LF” denote the ”social planner” and “laissez-faire”, respectively. Figure 3 plots these differentials with respect to the dimension of B. The negative $\Delta A'$ and the positive $\Delta B'$, with mean values of -0.024 and 0.067 respectively, stand out. It is evident that the social planner's strategy to diminish the balance sheet size intensifies with a more extensive initial balance sheet characterized by higher A and B in their absolute values. $\Delta A'(W_H, A, B)$, manifests a robust positive correlation of 0.93 with the inherited long-term bond position, B. Concurrently, $\Delta B'(W_H, A, B)$ is inversely correlated with the inherited bond position B, with a correlation coefficient of -0.91. These results align with the our simplified model's implication that the social planner tends to expands the external balance sheet when inherited long-term debt gets lower.

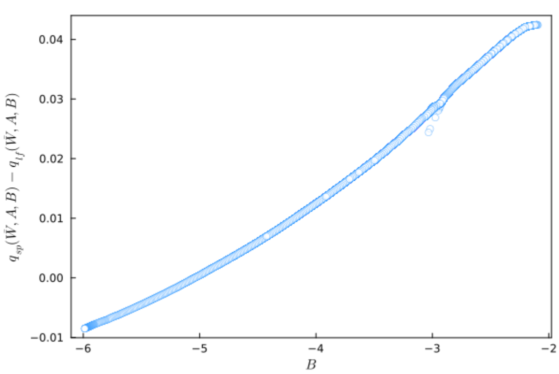
Figure 3: Portfolio differentials $\Delta A'$ and $\Delta B'$



Given the observed contraction of the external balance sheet, interpreting this outcome becomes paramount. Recalling our earlier simplified model, one might conjecture that the reason of a smaller sized external balance sheet chosen by the social planner is that long-term bond positions B fall beneath certain thresholds along the equilibrium path. This would lead the social planner to deem simultaneous reductions in both borrowings and savings as optimal. If this were the case, the simplified model would suggest a diminished long-term bond price under the social planner's allocation. Figure 4 depicts the bond price differential $\Delta q(W_H, A, B)$, as given by $q^{SP}(W_H, A, B) - q^{LF}(W_H, A, B)$.

Contrary to this hypothesis, for a majority of cases where the inherited long-term debt position is not exceedingly large, $\Delta q(W_H, A, B)$ is significantly positive. Thus, relying purely on insights from the simplified model doesn't address this apparent "puzzle". Our subsequent discussion will delve into the rationale behind the social planner's concurrent reduction of long-term bond issuance and short-term bond holdings, seeking to align it with the elevated long-term bond price.

Figure 4: Long-term bond price differential Δq



6.2 Reasoning: Why the Social Planner Shrinks the Balance Sheet

To understand why the social planner both reducing the external balance sheet size and raising the bond price, we need to delve deeper than our prior simplified model. This exploration involves considering net capital flow management. Specifically, while consumption varies over time in the full-fledged model, it remains consistently zero during the initial two periods in its simple version. In this section, we will demonstrate that understanding the puzzle requires examining a synergy between net capital flow management and the strategy of financing short-term bonds with long-term debt.

Our analysis follows a clear sequence. First, we highlight the social planner's inherent tendency to moderate consumption compared to laissez-faire private households, leading to a reduced demand for long-term bond issuance. Subsequently, we introduce a metric that encapsulates the concept of long-term debt financing short-term bonds, which we conveniently term "debt-financed liquidity" for brevity. Finally, we integrate insights from net capital flow management and debt-financed liquidity to elucidate the observed patterns in bond prices and the external balance sheet sizes under the two allocations: laissez-faire and the social planner's perspective.

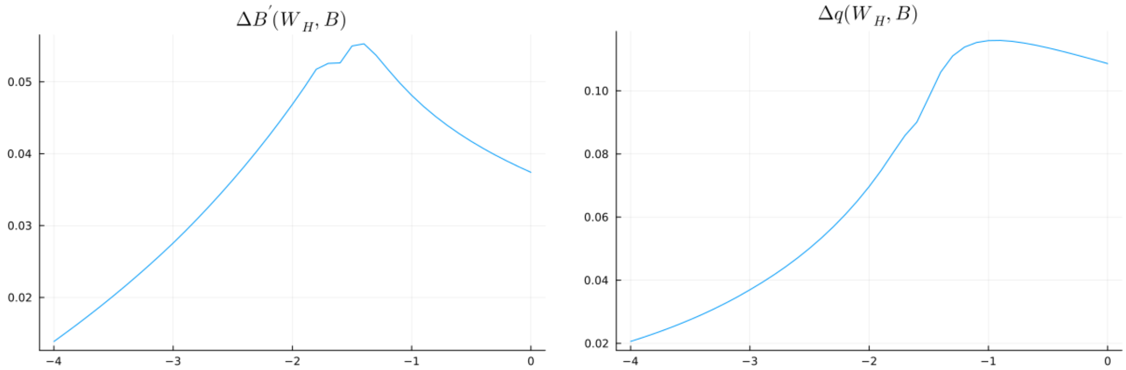
6.2.1 Net Capital Flow Management

We initiate our exploration by examining the net capital flow management strategy employed by the social planner. The decision pertaining to B' bifurcates into two components: one dedicated to consumption, and the other to finance short-term bond holdings. Our simplified model hones in on the latter, leaving the former untouched. To bridge this gap, we envisage a scenario where the economy's sole avenue for inter-periodic trade hinges on long-term bonds. Here, these bonds exclusively finance consumption. Our aim is to discern how the social planner's portfolio choices in this context deviate from those of private households.

Figure 5 contrasts the portfolio choices B' and bond prices q under both laissez-faire and constrained efficient allocations. Specifically, $\Delta B(W_H, B)$ and $\Delta q(W_H, B)$ represent the differences in bond holdings and bond prices between these two allocations, defined as $\Delta B(W_H, B) \equiv B_{SP}(W_H, B) - B_{LF}(W_H, B)$ and $\Delta q(W_H, B) \equiv q_{SP}(W_H, B) - q_{LF}(W_H, B)$ respectively. It is noteworthy that the ergodic path of B under laissez-faire is entirely located between -3 and -2 on the B-axis. The analysis reveals that, along the laissez-faire trajectory, a social planner is inclined to secure higher long-term bond holdings to a significant degree. Such a strategy ensures that an economy, when overseen by a social planner, confronts a lesser long-term debt obligation during financial contractions. This proactive approach reduces susceptibility to the "firesale" risks linked with bond prices. Consequently, the long-term bond price in a planner-administered economy exhibits diminished volatility. It tends to be more resilient than its laissez-faire equivalent for identical current states (W, B). While this aligns with conventional thinking—that emerging markets ought to curb their borrowings preceding sudden stops—our model introduces a novel perspective. Instead of anchoring an endogenous collateral constraint on domestic entities, we assign an exogenous wealth limitation to foreign lenders.

Obviously, the social planner's inclination to reduce net borrowings is crucial to explain what we have observed from the gross capital flow: low long-term debt and higher bond prices in constrained efficient allocation.

Figure 5: Portfolio and price differentials $\Delta B'(W_H, B)$ and $\Delta q(W_H, B)$



6.2.2 Debt-financed Liquidity Management

After examing the component of long-term bonds for sustaining consumption, now we turn our focus to its component that finance the short-term bond holding.

Firstly, it's imperative to highlight a pivotal insight consistent across both the formal and the simplified models: the ability of the social planner to enhance the long-term bond price by augmenting the external balance sheet. As illustrated in Figure 4, the discrepancy in long-term bond prices, denoted as $\Delta q(W_H, A, B) = q^{SP}(W_H, A, B) - q^{LF}(W_H, A, B)$, exhibits a positive correlation with the legacy bond position, B. This echoes the findings of our simplified model where, at elevated values of b_0, the social planner demonstrates a heightened propensity to expand the external balance sheet compared to private households, consequently elevating the price of long-term bonds. The upper echelons of B in Figure 4 distinctly showcase this strengthened bond pricing. Moreover, in scenarios of lower bond positions, specifically between -5 and -6, the social planner might even suppress bond prices in contrast to private households. Therefore, the crux of the matter isn't the capability of the social planner to modulate bond prices but the alignment of such actions with overarching objectives.

The planner's hesitancy in boosting bond prices is driven by an attempt to address coordination issues among households during financial tightenings to maximize monopolistic purchasing advantages. A larger foreign-held bond amount intensifies the planner's motive to reduce bond prices during financial tightening. Since the planner can't intervene directly during these periods, ex-ante actions to reduce firesale bond prices magnify wealth transfer from foreign bondholders to the emerging market. Reducing debt-financed liquidity becomes a strategic choice to lower ex-post bond prices.

To formalize our above argument, we introduce a metric that quantifies the social planner's inclination towards issuing more long-term bonds to hold short-term bonds than private households. This metric is applicable specifically under any state (W_H, A, B) where, in a laissez-faire environment, private households show a preference for positive short-term bond holdings.

Under such states, consider a scenario where the social planner intervenes only at period t (absent for all subsequent periods) and can make only marginal adjustments around private agents' portfolio choices. Another crucial stipulation is that the social planner's consumption must align with the laissez-faire level. In this context, the planner's assessment of private households' debt-financed liquidity is an insightful indicator of the social planner's propensity to adjust the external balance sheet. A value greater than 0 suggests that the social planner would deem it favorable to manage a more extensive external balance sheet than what's chosen by private households and vice versa.

Formally, the problem facing the social planner is outlined as follows:

$$\begin{aligned}
&\max_{B'} u(\bar{C}) + \beta E_{W'|W_H} V_{LF}(W', \tilde{A}', B') \\
&\quad \text{s.t.} \quad \bar{y} + \delta B + A - \frac{\tilde{A}'}{R_f} - Q(W_H, \tilde{A}', B')(B' - (1-\delta)B) - \bar{C} = 0,
\end{aligned}$$

where $\bar{C}$ represents the consumption level observed in laissez-faire under the same state, which the social planner must obey when choosing portfolios. Therefore, the associated short-term bond holding $\tilde{A}'$ is not an independent control variable of B'.

The social planner's assessment of laissez-faire debt-financed liquidity is a function of the state, defined as $MV(W_H, A, B)$. Particularly, $MV(W_H, A, B)$ derives as follows (please see the Appendix for more details of the derivation):

$$MV(W_H, A, B) \equiv -E_{W'|W_H} \left[\frac{\partial V_{LF}(W_H, \tilde{A}', B')}{\partial B'} + \frac{\partial V_{LF}(W_H, \tilde{A}', B')}{\partial A'} \frac{\partial \tilde{A}'}{\partial B'} \right] \Bigg|_{\substack{\tilde{A}' = A'_{LF} \\ B' = B'_{LF}}},$$

where we abuse the notation such that A'_{LF} (B'_{LF}) stands for $A'_{LF}(W_H, A, B)$

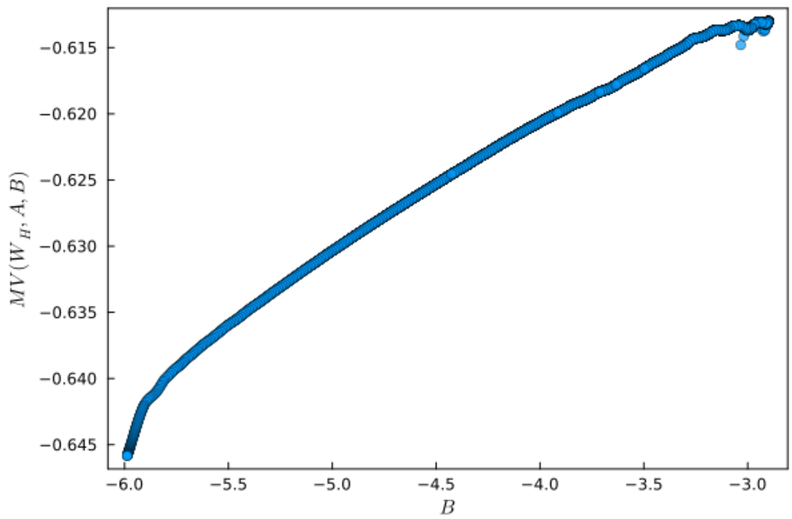
$(B'_{LF}(W_H, A, B))$. It further simplifies to:

$$
\begin{aligned}
&- E_{W'|W}\Bigg\{ u'\left(C_{LF}(W', A', B')\right) \times \\
&\Bigg[\Big(\delta + (1-\delta) q^{LF}(W', A', B')\Big) - \frac{\partial q^{LF}(W', A', B')}{\partial B'} \Big(B'' - (1-\delta)B'\Big) \\
&\quad + \left(1 - \frac{\partial q^{LF}(W', A', B')}{\partial A'} \Big(B'' - (1-\delta)B'\Big)\right) \times \frac{\partial \tilde{A}'}{\partial B'} \Bigg] \Bigg|_{\substack{A'=A'_{LF} \\ B'=B'_{LF}}} \Bigg\}.
\end{aligned}
$$

where $\frac{\partial \tilde{A}'}{\partial B'}$ is the units of short-term bonds the social planner has to increase by holding one more unit of long-term bonds on the margin of (A'_{LF}, B'_{LF}), which is a negative number. $MV(W_H, A, B)$ is visualized against B in Figure 6. The value consistently leans negative, indicating the planner's tendency to reduce both long-term borrowing and short-term saving compared to private households. Interestingly, this value mimics the pattern of $\Delta A'(W_H, A, B)$ in Figure 3, indicating a bond position's influence.

To conclude, the social planner's portfolio choices stem from balancing net capital flow management and debt-financed liquidity strategies. These two forces work in the same direction to suppress the size of the external balance sheet.

Figure 6: Social planner's valuation of debt-fianced liquidity chosen in laissez-faire



6.3 Welfare Improvement

We have previously illustrated the social planner's strategy of curbing the external balance sheet. Let's delve deeper to understand the implications of this strategy, especially in strengthening the economy's defenses against sudden financial tightenings.

Starting with the mechanics: we've carried out extensive simulations to discern the nuances between the equilibrium dynamics of regulated versus unregulated economies during a pivotal shift—financiers' wealth turning from high to low. Specifically, we simulated the decentralized economy for a staggering one million periods. From this vast dataset, we zeroed in on 27200 unique episodes. Each episode spans eleven periods, shifting to a constrained condition in the midpoint. The results have been averaged across these episodes and plotted in Figure 2 for clearer visualization. In this depiction, the transition to a constrained condition is normalized to 0, giving us a window from period -5 to 5.

Taking this analysis further, we've juxtaposed the sudden stop dynamics of both the unregulated and regulated economies. Here's our approach: for each identified financial tightening episode in the unregulated economy, we hypothesize that both economies (decentralized and centralized) commence with identical levels of long-term and short-term bonds at period -5. Using this baseline, we then extrapolate the equilibrium dynamics of the centralized economy based on its constrained efficient allocation. We let the centralized economy experience the same sequence of external financial conditions that the decentralized setup undergoes from period -5 to 5. The comparative results of this exercise are demarcated by orange dashed lines in Figure 7.

Figure 7 underscores the social planner's strategy of contracting the external balance sheet compared to private agents, especially leading up to a financial tightening. This approach effectively mitigates the adverse repercussions of such incidents. While private agents are expanding their balance sheets, indicated by a rise in A' and a decline in B', the social planner takes the opposite stance. In a decentralized economy, this results in heightened capital flow volatility and sustained periods of diminished consumption. Essentially, the planner's oversight serves to stabilize consumption patterns and enhance the value of long-term assets.

Upon observing the social planner's regulatory measures enhancing resilience against abrupt financial disturbances, our next step is quantifying its contribution to overarching welfare. We introduce the variable $\Delta(W_H, A, B)$ to gauge the welfare benefits derived from the social planner's intervention. This metric juxtaposes the utilities of two distinct allocations, translating the variances into consumption equivalents:

$$\Delta(W_H, A, B) = 100 * \left[\left(\frac{V_{SP}(W_H, A, B)}{V_{LF}(W_H, A, B)} \right)^{\frac{1}{1-\sigma}} - 1 \right]$$

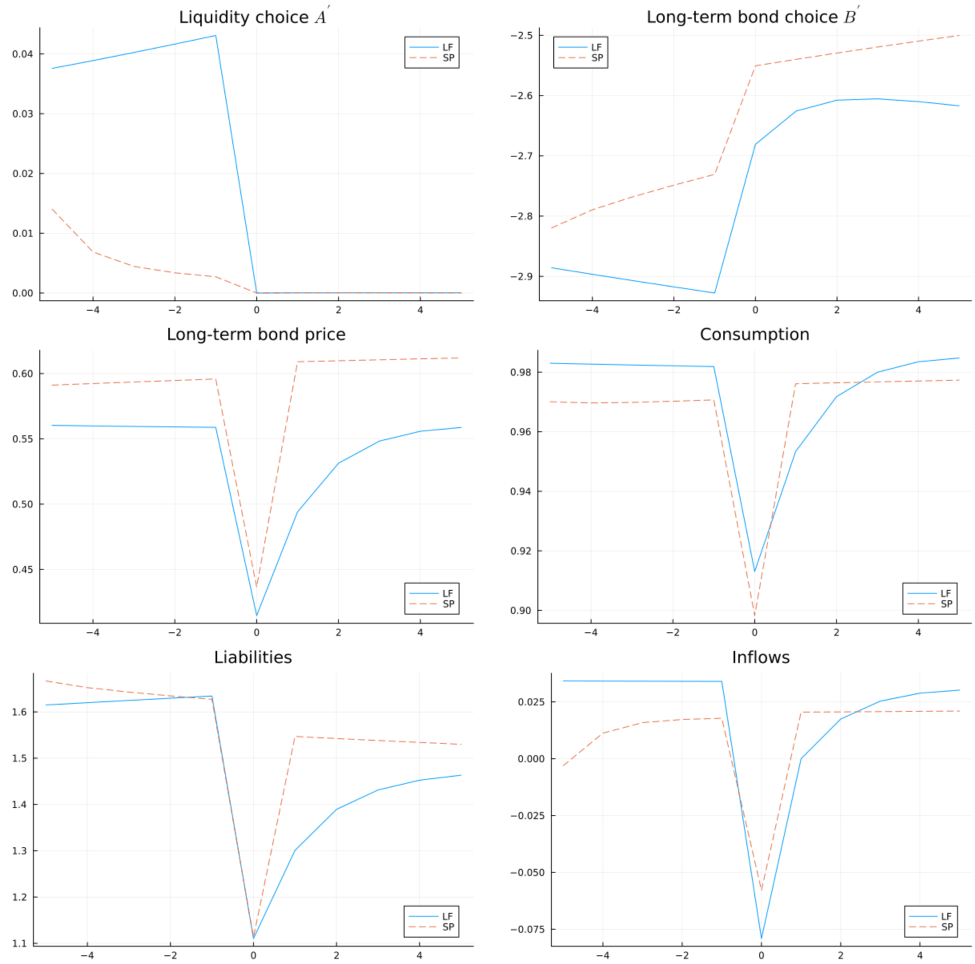
Here, $V_{SP}(W_H, A, B)$ and $V_{LF}(W_H, A, B)$ represent the lifetime welfare of households under the supervision of the social planner and in laissez-faire, respectively, given the state (W_H, A, B). The welfare gain is positive across all states in our model (not shown here); thus, the discretionary social planner's intervention benefits the economy.

To assess the average benefit of capital flow management, we also define a variable $E\Delta(W_H, A, B)$ as follows:

$$E\Delta = E[\Delta(W_H, A, B)]$$

The estimation of unconditional welfare gain is 0.20 percent of the permanent consumption by taking the expectation using the ergodic distribution of the state (W_H, A, B) in the laissez-faire scenario. This finding is comparable to the welfare improvement in the literature (Bianchi 2011) and consistent with the well-known result that business cycle welfare costs are generally small. However, it is noteworthy that the welfare gain reported here should be considered a lower bound. The actual welfare gain could be more significant if other factors were considered, such as the distortion of efficient use of production resources resulting from financial tightenings, the idiosyncratic risks to firms, or the long-term impacts on economic growth.

Figure 7: Dynamics around A Financial Tightening



6.4 Policy Instruments

This subsection investigates the strategic policy instruments employed by the discretionary social planner to restore constrained efficient allocations. Our discussion centers on prevalent state-contingent taxes tailored for reinstating such efficiency. These taxes are levied on an ad-valorem basis. Whenever private households in the open economy opt to acquire a unit of either long-term or short-term bonds, they are obliged to remit $q(W_H, A, B) + \tau_B$ or $\frac{1}{R_f} + \tau_A$ units of goods correspondingly. Subsequently, this tax revenue is reimbursed to the households in a lump-sum fashion.

Drawing a comparison between the social planner's and private households' Euler equations, we can get the taxation for short-term bond procurement that can restore the centralized equilibrium under the state (W_H, A, B) as

$$\begin{aligned}\tau_A(W_H, A, B) = &\frac{\partial Q(W_H, A'_{SP}, B'_{SP})}{\partial A'}(B'_{SP} - (1-\delta)B) \\ &+ \beta(1-\pi_{HH})\frac{u'(c_{SP}(W_L, A'_{SP}, B'_{SP}))}{u'(c_{SP}(W_H, A, B))}\frac{\partial q^{SP}(W_H, A'_{SP}, B'_{SP})}{\partial A'}(B''_{SP} - (1-\delta)B'_{SP})\end{aligned}$$

Conversely, the long-term bond acquisition's optimal tax formulation is:

$$\begin{aligned}\tau_B(W_H, A, B) = &\frac{\partial Q(W_H, A'_{SP}, B'_{SP})}{\partial B'}(B'_{SP} - (1-\delta)B) \\ &+ \beta(1-\pi_{HH})\frac{u'(c_{SP}(W_L, A'_{SP}, B'_{SP}))}{u'(c_{SP}(W_H, A, B))}\frac{\partial q^{SP}(W_H, A'_{SP}, B'_{SP})}{\partial B'}(B''_{SP} - (1-\delta)B'_{SP})\end{aligned}$$

Where:

$$Q(W_H, A', B') \equiv E_{W'|W_H}\left[SDF_{W_H,W'}\left(\delta + (1-\delta)q^{SP}(W', A', B')\right)\right]$$

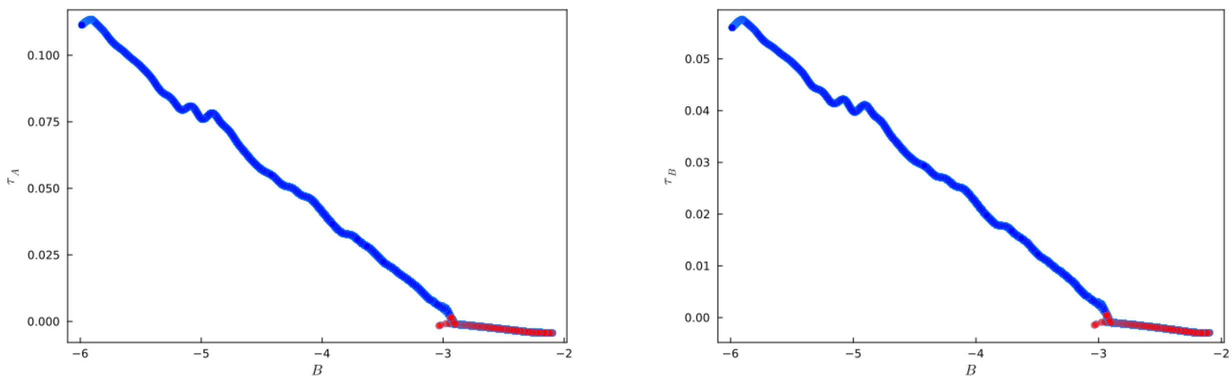
And, within the context of the state (W_H, A, B), A' and B' represent the social planner's decisions regarding short-term and long-term bonds, respectively, while B'' pertains to the long-term bond choices under state (W_L, A', B'). The tax's directional trend, positive or negative, hinges on capital flow nuances of the current and ensuing periods. On the established equilibrium path, the metric $B''_{SP} - (1-\delta)B'_{SP}$ invariably retains a positive value, attributable to the open economy's inclination towards long-term bond redemptions during financial tightenings. Contrastingly, $B'_{SP} - (1-\delta)B$ oscillates between positive and negative values, contingent on the prevailing state.

Figure 8 separately illustrates τ_A and τ_B in relation to the inherited long-term bond position, B. Bubbles are shaded red when the social planner's choice for long-term bonds, B'_{SP}, falls below $(1-\delta)B$, signaling a positive capital inflow in the centralized economy. It's evident that only for high values of B does the social planner opt to be a net issuer of long-term bonds.

In the context of the depicted red region, a negative tax, which effectively serves as a subsidy, correlates closely with periods of positive capital inflows. This correlation underscores the strategy adopted by the social planner, who, in response to positive capital inflows, not only imposes taxes on long-term bond issuance but also incentivizes accumulation in short-term bonds. Notably, the subsidy to the shor-term bond holding should not be interpreted as the social planner's propensity to increase the external balance sheet, but as an approach to decrease the net capital inflow. Quantitatively, though the magnitudes of the taxes are modest, they are still non-trivial: τ_B averages at -0.13 percent, while τ_A is approximately -0.17 percent. When the long-term bond tax is expressed in terms of the tax-price ratio, i.e., $\frac{\tau_B}{q}$, the mean value is -0.22 percent.

On the contrary, within the blue region—where the social planner predominantly opts for the repurchase of long-term bonds, the tax dynamic is reversed. In this scenario, the economy witnesses subsidized long-term borrowing and taxed short-term savings. The rationale behind it is to facilitate a judicious intertemporal transfer of wealth, particularly leveraging the anticipated firesales of bonds. From a quantitative perspective, the tax magnitudes are conspicuous: τ_B has an average value of 0.008, while τ_A is approximately 0.015. Translated into the tax-price ratio for long-term bonds, the mean of $\frac{\tau_B}{q}$ stands at a significant 1.5%.

Figure 8: Ad-valorem taxes of purchasing short and long-term bonds



7 Conclusion

This paper explores optimal capital flow management in emerging markets. Emerging markets issue long-term bonds to support consumption expenditure and short-term bond holding. During financially relaxed periods, marked by elevated foreign financiers' wealth, such economies expand their external balance sheets with concurrent positive inflows and outflows. Conversely, during tighter financial conditions, foreign financiers resort to offloading the long-term bonds issued by the emerging market at depressed prices. This prompts local households to liquidate their short-term bonds and curtail consumption to repurchase these bonds. Despite their monopolistic power over long-term bonds, uncoordinated private households can't leverage this advantage, leading to inefficient dynamic pricing under laissez-faire.

Our paper's primary contributions are twofold. Firstly, we offer a more holistic view of optimal capital flow management by accounting for the capital outflow and the external asset facet — aspects often overlooked in earlier studies. Secondly, our research underscores the quantitative dimensions of optimal capital flow management. It aligns with the perspectives of Obstfeld (2021), emphasizing the potential pitfalls of expanding gross liabilities and assets in a laissez-faire environment.

From a policy angle, our findings endorse the use of ad-valorem taxes on short and long-term bond holdings to achieve constrained-efficient allocations, a strategy prevalent in both academic and practical circles. The magnitude and direction of these taxes, though, are state-dependent. In phases of positive capital inflows and outflows, the government should tax long-term bond issuance while subsidizing short-term bond holdings. However, the opposite strategy should be employed when the direction of gross capital flows reverses. Quantitatively, our model recommends a modest long-term bond issuance tax (about 0.22% of the concurrent bond price) and a short-term bond tax of around 0.17% during positive capital inflows. Conversely, a bond issuance subsidy and short-term bond holding tax, both at 1.5% of their respective bond prices, are deployed during negative capital inflow periods in a constrained-efficient setting.

8 Appendix

8.1 Data Source and Processing

Table 3: Data Sources and Descriptions (2001q4 - 2022q4)

Variable	Description	Source
Baseline Variables		
Gross Inflows	Sum of FDI, portfolio, and other inflows (current \$)	BOP-IIP
Gross Outflows	Sum of FDI, portfolio, reserve, other inflows (current \$)	BOP-IIP
External Liabilities	Liabilities (current \$)	BOP-IIP
External Assets	Assets (current \$)	BOP-IIP
GDP	PPP-adjusted Brazil's GDP (current \$)	OECD Stats
GDP Deflator	U.S. GDP deflator	Fred

Following Caballero and Simsek (2020), we categorize gross inflows into three main components: Foreign Direct Investment (FDI), portfolio investment, and other investment inflows. Similarly, gross outflows consist of four components: FDI, portfolio investment, other investment, and official reserve outflows. External liabilities and assets are documented as their end-of-period gross external liabilities market values. All these metrics are expressed in current dollars. Therefore, we transform them into real terms using the U.S. GDP deflator. Regarding the GDP of Brazil, we first obtain the PPP-adjusted Brazil's GDP in current dollars and again convert it into its real counterpart using the U.S. GDP deflator.

To align the data with our model, we normalize both the real capital flows and real gross positions using the concurrent real GDP trend, derived from an HP filter. This process effectively strips away trends present in the capital flows and domestic income, rendering the data stationary and more apt for our analysis.

Beyond capital flows, it's essential to compute the excess returns on Brazil's external balance sheet. We do this by separately assessing the returns on external liabilities and assets. Adopting the conventional register method, the gross return for the liability side is characterized as the aggregate of investment payments and valuation changes, divided by the previous period's liabilities. The equation below helps us deduce the unobserved valuation change:

$$\text{valuation change}_t = \text{liabilities}_t - \text{liabilities}_{t-1} - \text{gross capital inflows}_t$$

After accounting for inflation, we can derive the real returns on Brazil's external liabilities using the valuation change. A parallel method is employed to compute the gross returns on Brazil's external assets. The excess return is then ascertained by finding the difference between the two gross returns, which signifies the excess returns attributed to Brazil's external balance sheet.

8.2 Derivation of The Social Planner's Decision Rule

Given the Bellman equation of the social planner in Section 3.4, we can derive the first-oder conditions regarding A' and B' as follows:

$$-u'(C_{SP}(s))\Big(\frac{1}{R_f}+\frac{\partial Q(W_H,A',B')}{\partial A'}(B'-(1-\delta)B)\Big)+\beta E_{W'|W}\frac{\partial V_{SP}(W',A',B')}{\partial A'}+\mu_{SP}^A=0 \tag{J}$$

$$-u'(C_{SP}(s))\Big(Q(W_H,A',B')+\frac{\partial Q(W_H,A',B')}{\partial B'}(B'-(1-\delta)B)\Big)+\beta E_{W'|W}\frac{\partial V_{SP}(W',A',B')}{\partial B'}+\mu_{SP}^B-\gamma_{SP}=0 \tag{K}$$

where μ_{SP}^A is the Lagrangian multiplier concerning the non-nongetive constraint of A', μ_{SP}^B is the Lagrangian multiplier concerning the constraint $B' \geq \underline{B}$ and γ_{SP} is the Lagrangian multiplier concerning the constraint $B' \leq 0$.

Next, we need to solve $\frac{\partial V_{SP}(W',A',B')}{\partial A'}$ and $\frac{\partial V_{SP}(W',A',B')}{\partial B'}$. Here, we need to note that the form of these two partial derivatives can vary with W, the financiers' wealth. This is due to our assumption that the social planner does not exist during financial tightening episodes. During financial easing periods, according to the envolope theorem, we have

$$\frac{\partial V_{SP}(W_H,A,B)}{\partial A}=u'(C)$$
$$\frac{\partial V_{SP}(W_H,A,B)}{\partial B}=u'(C)(\delta+(1-\delta)Q(W_H,A,B))$$

In contrast, During financial tightening periods, the social planner's problem is formulated as

$$V_{SP}(W_L,A,B)\equiv \max_{\substack{A'\geq 0\\ \underline{B}\leq B'\leq 0}} u(\bar{y}+A+\delta B-q^{SP}(W_L,A,B)(B'-(1-\delta)B)-\frac{A'}{R_f})$$
$$+\beta E_{W'|W_L}V(W',A',B')$$

the envolope theorem then implies

$$\frac{\partial V_{SP}(W_L,A,B)}{\partial A}=u'(C)\Big(1-\frac{\partial q^{SP}(W_L,A,B)}{\partial A}\big(B'-(1-\delta)B\big)\Big)$$
$$\frac{\partial V_{SP}(W_L,A,B)}{\partial B}=u'(C)\Big(\delta+(1-\delta)q(W_L,A,B)-\frac{\partial q^{SP}(W_L,A,B)}{\partial B}\big(B'-(1-\delta)B\big)\Big)$$

Therefore, we have the following equations regarding $\frac{\partial V_{SP}(W',A',B')}{\partial A'}$ and $\frac{\partial V_{SP}(W',A',B')}{\partial B'}$:

$$\frac{\partial V_{SP}(W_H', A', B')}{\partial A'} = u'(C(W_H, A', B')) \tag{L}$$
$$\frac{\partial V_{SP}(W_H, A', B')}{\partial B'} = u'(C(W_H, A', B'))(\delta + (1-\delta)Q(W_H, A^{''}, B^{''})) \tag{M}$$
$$\frac{\partial V_{SP}(W_L, A', B')}{\partial A'} = u'(C(W_L, A', B'))\Big(1 - \frac{\partial q^{SP}(W_L, A', B')}{\partial A'}(B^{''} - (1-\delta)B')\Big) \tag{N}$$
$$\frac{\partial V_{SP}(W_L, A', B')}{\partial B'} = u'(C(W_L, A', B'))\Big(\delta + (1-\delta)q(W_L, A', B') - \frac{\partial q^{SP}(W_L, A', B')}{\partial B'}(B^{''} - (1-\delta)B')\Big) \tag{O}$$

Plugging L to O back into J and K and utilizing the relationship that $Q(W_H, A'', B'') = q^{SP}(W_H, A', B')$, we can get the Euler equations for the social planners shown in Section 3.4.

8.3 The Construction of $MV(W_H, A, B)$

First, we apply the implicit function theorem to the budget constraint:

$$\bar{y} + \delta B + A - \frac{\tilde{A}'}{R_f} - Q(W_H, \tilde{A}', B')(B' - (1-\delta)B) - \bar{C} = 0$$

From this, we deduce

$$\frac{\partial \tilde{A}'}{\partial B'} = \frac{-\frac{\partial Q}{\partial B'}(B' - (1-\delta)B) - Q}{\frac{1}{R_f} + \frac{\partial Q}{\partial A'}(B' - (1-\delta)B)}$$

where $Q(W_H, \tilde{A}', B) \equiv= \beta\delta + (1-\delta)E_{W'|W_H}[SDF_{W_H,W'}q^{LF}(W', \tilde{A}', B')]$. If the social planner intervenes in the market only once, the welfare regarding the choice of B' is

$$V_{SP}(W_H, B') \equiv \max_{B'} u(\bar{C}) + E_{W'|W_H}[V_{LF}(W', \tilde{A}', B')]$$
$$\text{s.t. } \frac{\partial \tilde{A}'}{\partial B'} = \frac{-\frac{\partial Q}{\partial B'}(B' - (1-\delta)B) - Q}{\frac{1}{R_f} + \frac{\partial Q}{\partial A'}(B' - (1-\delta)B)}$$

Differentiating $V_{SP}(W_H, B')$ with respect to B' yields:

$$\frac{V_{SP}(W_H, B')}{\partial B'} = E_{W'|W}\Big[\frac{\partial V_{LF}(W', \tilde{A}', B')}{\partial B'} + \frac{\partial V_{LF}(W', \tilde{A}', B')}{\partial A'} \times \frac{\partial \tilde{A}'}{\partial B'}\Big] \tag{P}$$

Note that we have

$$\frac{\partial V_{LF}(W', A', B')}{\partial A'} = u'(C_{LF}(W', A', B'))\Big(1 - \frac{\partial q^{LF}(W', A', B')}{\partial A'}(B_{LF}^{''} - (1-\delta)B')\Big) \tag{Q}$$
$$\frac{\partial V_{LF}(W', A', B')}{\partial B'} = u'(C_{LF}(W', A', B'))\Big(\delta + (1-\delta)q^{LF}(W', A', B') - \frac{\partial q^{LF}(W_L, A', B')}{\partial B'}(B_{LF}^{''} - (1-\delta)B')\Big) \tag{R}$$

Utilizing Equations (Q) and (R), and plugging them into Equation (P), we get:

$$\frac{V_{SP}(W_H,B')}{\partial B'} = E_{W'|W}\Big[u'(C_{LF}(W',\tilde{A}',B'))\Big(1-\frac{\partial q^{LF}(W',\tilde{A}',B')}{\partial A'}\big(B''_{LF}-(1-\delta)B'\big)\Big)*\frac{\partial \tilde{A}'}{\partial B'}$$
$$+u'(C_{LF}(W',\tilde{A}',B'))\Big(\delta+(1-\delta)q^{LF}(W',\tilde{A}',B')-\frac{\partial q^{LF}(W',\tilde{A}',B')}{\partial B'}\big(B''_{LF}-(1-\delta)B'\big)\Big)\Big] \tag{S}$$

Note that if we evaluate $\frac{V_{SP}(W_H,B')}{\partial B'}$ in S at $B'_{LF}(W_H,A,B)$, then we have

$$\frac{\partial V_{SP}(W_H,B'_{LF}(W_H,A,B))}{\partial B'}$$
$$=E_{W'|W}\Big[u'(C_{LF}(W',A'_{LF},B'_{LF}))\Big(1-\frac{\partial q^{LF}(W',A'_{LF},B'_{LF})}{\partial A'}\big(B''_{LF}-(1-\delta)B'_{LF}\big)\Big)*\frac{\partial \tilde{A}'}{\partial B'}$$
$$+u'(C_{LF}(W',A'_{LF},B'_{LF}))\Big(\delta+(1-\delta)q^{LF}(W',A'_{LF},B'_{LF})-\frac{\partial q^{LF}(W',A'_{LF},B'_{LF})}{\partial B'}\big(B''_{LF}-(1-\delta)B'_{LF}\big)\Big)\Big]$$

By evaluating $\frac{\partial V_{SP}(W_H,B'_{LF}(W_H,A,B))}{\partial B'}$ from Equation (S) at $B'_{LF}(W_H,A,B)$, we get the value of balance sheet contraction at the laissez-faire's portfolio choice. Since in our model B' indicates long-term bond holding and not issuance, the marginal value assessed by the social planner for balance sheet expansion around $(A'_{LF}(W_H,A,B), B'_{LF}(W_H,A,B))$ is the opposite value. This appears as $MV(W_H,A,B)$ in Section 6.2.2.

8.4 A Discussion of Calibration and Estimation

In our model, we calibrate three parameters: the international risk-free rate R_f, the coupon decay rate δ, and the relative risk-aversion σ.

For Brazil's international risk-free rate, we adopt the average return on Brazil's external assets. This is a departure from many other studies that typically default to a quarterly rate of 1.01 for R_f. Instead of using this conventional value, we employ the time-average of Brazil's external asset returns as a proxy. Given a significant portion of our data post-dates the 2008 Global Financial Crisis, marked by extraordinary interest rates, the typical value of 1.01 may not serve as an accurate reflection. Our method, rooted in Brazil's long-term foreign asset returns, ensures our risk-free rate aligns more closely with real-world data. Consequently, our deduced value for R_f stands at 1.0017.

For the coupon decay rate δ, our estimation hinges on the average ratio of capital inflows to the prior period's liabilities, represented as $\frac{\text{Capital inflow}_t}{\text{Liabilities}_{t-1}}$. This is premised on the long-term equilibrium notion, where stationary capital inflows and liabilities would mean that capital inflows should offset the diminishing portion of liabilities. This relationship can be articulated as mean$(-q_t(B_{t+1}-(1-\delta)B_t) = -\delta * mean(q*B) = \delta * mean(\text{Liabilities})$. This methodology yields a δ value of 0.0174. Though our model doesn't settle into a clear steady-state, the simulations suggest a model-implied ratio of $\frac{\text{Capital inflow}_t}{\text{Liabilities}_{t-1}}$ at 0.0180, quite close to 0.0174, thus further validating our hypothesis.

At first glance, a value of 0.0174 might appear unusually low for emerging markets, particularly for Brazil. However, this calibrated value aligns well with the observed low ratio of investment payment to past-period's liabilities. Specifically, the data indicates that the average of $\frac{\text{Investment payment}_t}{\text{Liabilities}_{t-1}}$ is 0.0136. Within our model's framework, the average of this ratio is given by $\frac{\delta B_t}{q_{t-1} B_t} = \frac{\delta}{q_{t-1}}$. Given that the long-term bond price in our model is consistently below 1, the 0.0174 value seems reasonable, especially since it exceeds the 0.0136 figure, effectively setting a lower limit for δ.

For the relative risk-aversion parameter σ, we reference established literature and set its value to 4. While the oft-preferred value is 2, our own estimations find that 4 is the most fitting integer choice for σ within the range of 2 to 10. This value of 4 is also consistent with estimates from a range of life-cycle models that rely on granular, micro-level data.

We calibrate our model using eight empirical moments to determine a parameter set. This set incorporates two financier inefficiency parameters: risk aversion during financial easing, R^{κ}, and wealth during financial tightness, W_L. It also includes transition matrix parameters for the financial state, π_{UU} and π_{CC}, as well as the households' subjective discount factor, β. The empirical moments span two means, three standard deviations, and three correlations, encapsulating metrics like excess returns, gross capital flows, and gross external positions of Brazil.

For parameter estimation, we adopt the Continuously Updated Simulated Method of Moments (CU SMM). This approach, akin to the two-step SMM, offers efficiency by updating the weighting matrix iteratively and requiring only a single optimization. Notably, the CU SMM is scale-invariant and doesn't necessitate an initial weight matrix. Empirical findings, such as in Hansen et al. (1996), highlight its superior finite sample performance compared to other GMM methods, even though it's asymptotically analogous to the two-step SMM.

8.5 A Full Description of The Numerical Algorithm

I use the FiPit method by Mendoza and Villalvazo (2020) to solve the laissez-faire and constrained efficient allocation. Here I will use the laissez-faire case as an example to illustrate the algorithm. Before the iteration, I create grids for (A, B), namely $\mathbf{A}$ and $\mathbf{B}$. I select 41 grids for $\mathbf{A}$ between $\underline{A} = 0$ and $\bar{A} = 2$ and 41 grids for $\mathbf{B}$ between $\underline{B} = -6$ and $\bar{B} = -2$. Then, I initialize the decision rules and the bond price function with $A'_1(s)$, $B'_1(s)$, and $q'_1(s)$, respectively, where the subscripts index the iteration. After the initialization, I start the iteration. Particularly, for iteration $i = 1, 2, ..., n$, I implement the following steps

- Step 1: Construct the consumption $c_i(s)$ for grid $s \equiv (W, A, B)$ by setting

$$c_i(s) = y + \delta B + A - \frac{A'_i(s)}{R_f} - q_i(s)(B'_i(s) - (1-\delta)B)$$

  and then interpolate $c_i(s)$ over grids of s linearly to get the consumption function $c_i(\cdot)$.

- Step 2: Solve for c_i^b by applying the fixed-point iteration method to the Euler equation for the long-term bond with

$$c_i^b(s) = \left(\beta E_{W'|W} \frac{c_i\big(W', A'_i(s), B'_i(s)\big)^{-\sigma}\big(\delta + (1-\delta) q_i(W', A'_i(s), B'_i(s))\big)}{q_i(s)}\right)^{-\frac{1}{\sigma}}$$

  As $A'_i(s), B'_i(s)$ generally do not match node grids in **A** and **B**, respectively, $c_i(\cdot)$ and $q_i(\cdot)$ are evaluated at $\big(W', A'_i(s), B'_i(s)\big)$ to determine $c_i(W', A'_i(s), B'_i(s))$ and $q_i(W', A'_i(s), B'_i(s))$ using cubic spline interpolations.

- Step 3: Solve for $B'_{i+1}(s)$ using the budget constraint

$$B'_{i+1}(s) = \frac{\bar{y} + \delta B + A - c_i^b(s) - \frac{A'_i(s)}{R_f}}{q_i(s)} + (1-\delta)B$$

  If $B'_{i+1}(s)$ is between $\underline{B}$ and $\bar{B}$, keep it; If$B'_{i+1}(s)$ is lower than $\underline{B}$, change it to $\underline{B}$; If $B'_{i+1}(s)$ is higher than $\bar{B}$, change it to $\bar{B}$.

- Step 4: Solve for c_i^a by applying the fixed-point iteration method to the Euler equation for the one-period bond with

$$c_i^a(s) = \left[\beta R_f E_{W'|W} c_i(W', A'_i(s), B'_i(s))^{-\sigma}\right]^{-\frac{1}{\sigma}}$$

- Step 5: Solve for $A'_{i+1}(s)$ using the budget constraint

$$A'_{i+1}(s) = R_f\big(\bar{y} + \delta B + A - q_i(s)(B'_{i+1}(s) - (1-\delta)B) - c_i^a(s)\big)$$

  If $A'_{i+1}(s)$ is between $\underline{A}$ and $\bar{A}$, keep it; If $A'_{i+1}(s)$ is lower than $\underline{A}$, change it to $\underline{A}$; If $A'_{i+1}(s)$ is higher than $\bar{A}$, change it to $\bar{A}$.

- Step 6: Update the long-term bond price.

$$q_{i+1}(W, A, B) = \min \left\{ E_t\left[SDF_{W'|W}\left(\delta + (1-\delta) q_i(W', A'_{i+1}(W, A, B), B'_{i+1}(W, A, B))\right)\right], -\frac{W}{B'_{i+1}(W, A, B)} \right\}$$

  Like in Step 2, $q_i(\cdot)$ needs to be evaluated at $(W', A'_{i+1}(W, A, B), B'_{i+1}(W, A, B))$ using the cubic spline interpolation.

- Step 7: Check the convergence. Compute $||A_{i+1}(s) - A_i(s)||$, $||B_{i+1}(s) - B_i(s)||$, and $||q_{i+1}(s) - q_i(s)||$ to see whether they are smaller than the numerical tolerance, which is 10^{-6}. If they are, stop the algorithm and collect $A_i(s)$, $B_i(s)$, and $q_i(s)$ as the decision rules and the price function. If not, return to Step 1 with the updated decision rules and price function $A_{i+1}(s)$, $B_{i+1}(s)$, and $q_{i+1}(s)$.

To determine the social planner's allocation, we can employ the same method used for the laissez-faire scenario. The sole distinction is the need to compute the partial derivatives related to bond price functions. For these derivatives, we utilize Julia's 'ForwardDiff' capacity.

References

Akinci, O., Kalemli-Ozcan, S., and Queralto, A. (2022). Uncertainty shocks, capital flows, and international risk spillovers. Technical report, National Bureau of Economic Research.

Alfaro, L. and Kanczuk, F. (2009). Optimal reserve management and sovereign debt. *Journal of International Economics*, 77(1):23–36.

Avdjiev, S., Hardy, B., Kalemli-Özcan, Ş., and Servén, L. (2022). Gross capital flows by banks, corporates, and sovereigns. *Journal of the European Economic Association*, 20(5):2098–2135.

Bertaut, C. C., Bruno, V., and Shin, H. S. (2021). Original sin redux. *Available at SSRN 3820755*.

Bianchi, J. (2011). Overborrowing and systemic externalities in the business cycle. *American Economic Review*, 101(7):3400–3426.

Bianchi, J., Hatchondo, J. C., and Martinez, L. (2018). International reserves and rollover risk. *American Economic Review*, 108(9):2629–70.

Bianchi, J. and Sosa-Padilla, C. (2020). Reserve accumulation, macroeconomic stabilization, and sovereign risk. Technical report, National Bureau of Economic Research.

Bräuning, F. and Ivashina, V. (2020). Monetary policy and global banking. *The Journal of Finance*, 75(6):3055–3095.

Bruno, V. and Shin, H. S. (2015). Cross-border banking and global liquidity. *The Review of Economic Studies*, 82(2):535–564.

Caballero, R. J. and Krishnamurthy, A. (2003). Excessive dollar debt: Financial development and underinsurance. *The Journal of Finance*, 58(2):867–893.

Caballero, R. J. and Simsek, A. (2020). A model of fickle capital flows and retrenchment. *Journal of Political Economy*, 128(6):2288–2328.

Cerutti, E., Claessens, S., and Rose, A. K. (2019). How important is the global financial cycle? evidence from capital flows. *IMF Economic Review*, 67(1):24–60.

Chari, A., Stedman, K. D., and Lundblad, C. (2020). Capital flows in risky times: Risk-on/risk-off and emerging market tail risk. Technical report, National Bureau of Economic Research.

Chari, A., Stedman, K. D., and Lundblad, C. (2022). Global fund flows and emerging market tail risk. Technical report, National Bureau of Economic Research.

Davis, J. S., Valente, G., and Van Wincoop, E. (2021). Global drivers of gross and net capital flows. *Journal of International Economics*, 128:103397.

Devereux, M. B. and Sutherland, A. (2011). Country portfolios in open economy macro-models. *Journal of the european economic Association*, 9(2):337–369.

Erten, B., Korinek, A., and Ocampo, J. A. (2021). Capital controls: Theory and evidence. *Journal of Economic Literature*, 59(1):45–89.

Farhi, E. and Werning, I. (2016). A theory of macroprudential policies in the presence of nominal rigidities. *Econometrica*, 84(5):1645–1704.

Forbes, K. J. and Warnock, F. E. (2012). Capital flow waves: Surges, stops, flight, and retrenchment - sciencedirect. *Journal of International Economics*, 88(2):235–251.

Gelos, M. R. (2011). *International mutual funds, capital flow volatility, and contagion–a survey*. International Monetary Fund.

Gourinchas, P.-O. and Rey, H. (2022). Exorbitant privilege and exorbitant duty.

Hansen, L. P., Heaton, J., and Yaron, A. (1996). Finite-sample properties of some alternative gmm estimators. *Journal of Business & Economic Statistics*, 14(3):262–280.

Ivashina, V., Scharfstein, D. S., and Stein, J. C. (2015). Dollar funding and the lending behavior of global banks. *The Quarterly Journal of Economics*, 130(3):1241–1281.

Jeanne, O. and Ranciere, R. (2011). The optimal level of international reserves for emerging market countries: A new formula and some applications. *The Economic Journal*, 121(555):905–930.

Jeanne, O. and Sandri, D. (2023). Global financial cycle and liquidity management. *Journal of International Economics*, page 103736.

Jiang, Z., Krishnamurthy, A., and Lustig, H. (2020). Dollar safety and the global financial cycle.

Kekre, R. and Lenel, M. (2021). The flight to safety and international risk sharing. Technical report, National Bureau of Economic Research.

Korinek, A. and Simsek, A. (2016). Liquidity trap and excessive leverage. *American Economic Review*, 106(3):699–738.

Lorenzoni, G. (2008). Inefficient credit booms. *The Review of Economic Studies*, 75(3):809–833.

Mendoza, E. G. and Villalvazo, S. (2020). Fipit: A simple, fast global method for solving models with two endogenous states & occasionally binding constraints. *Review of Economic Dynamics*, 37:81–102.

Miranda-Agrippino, S., Nenova, T., and Rey, H. (2020). *Global footprints of monetary policies*. CFM, Centre for Macroeconomics.

Miranda-Agrippino, S. and Rey, H. (2022). The global financial cycle. In *Handbook of international economics*, volume 6, pages 1–43. Elsevier.

Obstfeld, M. (2021). Trilemmas and tradeoffs: living with financial globalization. In *The Asian Monetary Policy Forum: Insights for Central Banking*, pages 16–84. World Scientific.

Obstfeld, M., Ostry, J. D., and Qureshi, M. S. (2019). A tie that binds: Revisiting the trilemma in emerging market economies. *Review of Economics and Statistics*, 101(2):279–293.

Raddatz, C. and Schmukler, S. L. (2012). On the international transmission of shocks: Micro-evidence from mutual fund portfolios. *Journal of International Economics*, 88(2):357–374.

Tille, C. and Van Wincoop, E. (2010). International capital flows. *Journal of international Economics*, 80(2):157–175.