

```

find_log := proc(n, a, m)
description "Find log of a";
for i from 1 to n
do
if a · i mod n = m
then
return i;
fi;
enddo;
endproc;

```

>

```
ifactor(3267)
```

$$(3)^3 (11)^2 \quad (1)$$

```
n := 3^3 · 11^2;
```

$$n := 3267 \quad (2)$$

g = x<sup>13</sup>, h = x<sup>157</sup>, n = 3267; [g, h] generatees the group of order 3267;

Steps 1:

```
g1 := x13·112 mod 27;
```

$$g1 := x^7 \quad (3)$$

```
h1 := x157·112 mod 27;
```

$$h1 := x^{16} \quad (4)$$

We need to find the log of h1 = x<sup>16</sup> in the cyclic group of order 27 generated by g1 = x<sup>7</sup>. By trial and error, we get log(h1) = 10

```
find_log(27, 7, 16);
```

>

$$10 \quad (5)$$

So our first congruence is x1 = 10 mod 27 — (1)

Step 2:

```
g2 := x13·33 mod 121;
```

$$g2 := x^{109} \quad (6)$$

```
h2 := x157·33 mod 121;
```

$$h2 := x^4 \quad (7)$$

We need to find the log of  $h2 = x^4$  in the cyclic group of order 121 generated by  $g2 = x^{109}$ ; using the proc `find_log` above, it is = 118  
`find_log(121, 4, 109);`

$$118 \quad (8)$$

We get our second congruence as:  $x2 = 118 \bmod 121$  — (2)

Hence we need to find the unique solution to  $x = 10 \bmod 27$ , and  $x = 118 \bmod 121$  using Chinese Remainder Theorem.

`with(NumberTheory);`

$$\begin{aligned} &[AreCoprime, CalkinWilfSequence, CarmichaelLambda, \\ &ChineseRemainder, ContinuedFraction, \\ &ContinuedFractionPolynomial, CyclotomicPolynomial, \\ &Divisors, FactorNormEuclidean, HomogeneousDiophantine, \\ &ImaginaryUnit, InhomogeneousDiophantine, IntegralBasis, \\ &InverseTotient, IsCyclotomicPolynomial, IsMersenne, \\ &IsSquareFree, IthFermat, IthMersenne, JacobiSymbol, \\ &JordanTotient, KroneckerSymbol, Landau, LargestNthPower, \\ &LegendreSymbol, Möbius, ModExtendedGCD, ModularLog, \\ &ModularRoot, ModularSquareRoot, Moebius, \\ &MultiplicativeOrder, Möbius, NearestLatticePoint, \\ &NextSafePrime, NumberOfIrreduciblePolynomials, \\ &NumberOfPrimeFactors, \Omega, \Phi, PrimeCounting, PrimeFactors, \\ &PrimitiveRoot, PseudoPrimitiveRoot, QuadraticResidue, \\ &Radical, RepeatingDecimal, RootsOfUnity, \\ &SimplestRational, SumOfDivisors, SumOfSquares, ThueSolve, \\ &Totient, \lambda, \mu, \phi, \pi, \sigma, \tau, \varphi] \end{aligned} \quad (9)$$

`ChineseRemainder([10, 118], [27, 121]);`

$$118 \quad (10)$$