# Maths IA - Written 8

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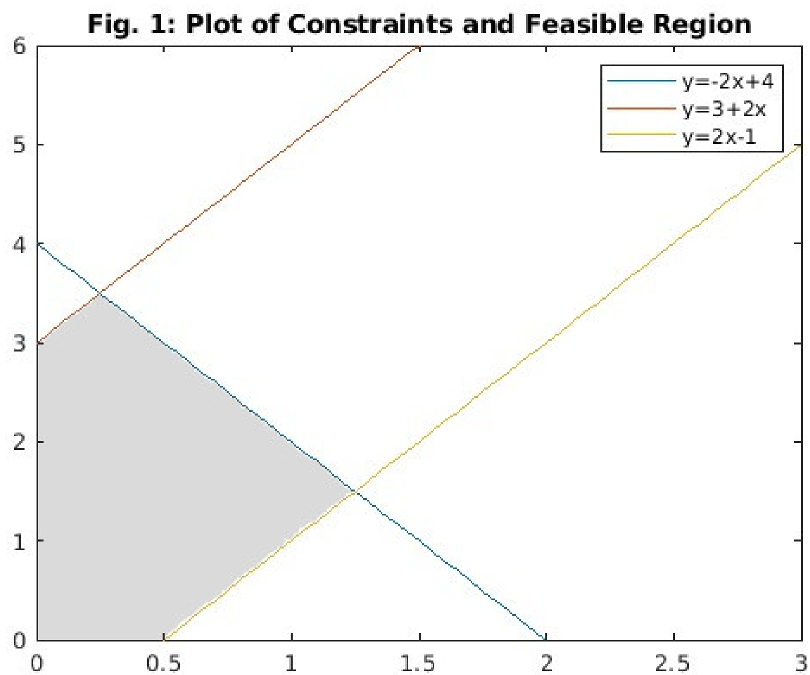
## Algebra

The plot of the given constraints, with feasible region shaded, is shown in Figure 1. It was generated using MATLAB.




### (b)

$$\begin{bmatrix} 2 & 1 \\ -2 & y \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x & y \end{bmatrix} \leq \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} x & y \end{bmatrix} \geq 0$$

### (c)

The vertices of this feasible region can be found by finding each point where constraint lines intersect, if such points exist. In this case given that $y = 3 + 2x$ is parallel to $y = 2x - 1$, only the points where the $x$ or $y$ axes or $y = -2x + 4$ intersects either of these lines need to be considered, leading us to the following five vertices:

1. $(0, 0)$
2. $(0, 3)$
3. $(0.5, 0)$
4. $(1.25, 1.5)$
5. $(0.25, 3.5)$

## Calculus

The first step in calculating the given integral is to split it up into an integral of $f(x)$ and an integral of $g(x)$, both with coefficients, rather than one large compound integral:

$$2\int_{-1}^{5} f(x)dx - 4\int_{-1}^{5} g(x)dx$$

Now this can be broken further still into the pieces which comprise the piecewise functions $f(x)$ and $g(x)$ as follows:

$$2(\int_{-1}^{2} f(x)dx + \int_{2}^{3} f(x)dx + \int_{3}^{5} f(x)dx) - 4(\int_{-1}^{2} g(x)dx + \int_{2}^{4} g(x)dx)$$

And from here we can do away with calculus in favour of simple geometry. Armed with the knowledge that the integral of a function is equal to the area bounded by it and the x axis, and is negative when it is below the x axis, it can be deduced by observation that:

$$\int_{-1}^{2} f(x)dx = \frac{3 \times 3}{2} = 4.5$$

$$\int_{2}^{3} f(x)dx = \frac{3 \times 1}{2} = 1.5$$

$$\int_{3}^{5} f(x)dx = \frac{-1 \times 2}{2} = -1$$

And similarly that:

$$\int_{-1}^{2} g(x)dx = \pi \times 1.5^2 \times 0.5 = \frac{9\pi}{8}$$

$$\int_{-1}^{2} g(x)dx = \pi \times -1 \times 0.5 = \frac{-\pi}{2}$$

SinceSo the integral can be rewritten as:

$$2(4.5 + 1.5 - 1) - 4(\frac{9\pi}{8} - \frac{\pi}{2})$$

Which is much nicer on the eyes, and is equal to:

$$10 - \frac{5\pi}{2}$$