Maths IA - Written 1

Jack Larkin

Algebra

(a)

Since **X** is a solution to the linear system, the unkown values can be found via simple substitution and algebra. After substituting **x**, the system becomes:

$$3(1) + 3(0) - 2 + w = 0$$

$$-4(0) + 2(2) + t(w) = 0$$

And then:

$$3 - 2 + w = 0$$

$$4 + t(w) = 0$$

From which it can be shown that $w = -1$, and consequently that $t = 4$.

Answer: $w = -1, t = 4$.

(b)

As any solution to a linear system will also be a solution to an equivalent linear system, this question too can be answered via substituting **X** into the linear system as follows:

$$1 + 0 - 2 - 1 = 0$$

$$3(1) - 0 + 2 + 5(-1) = 0$$

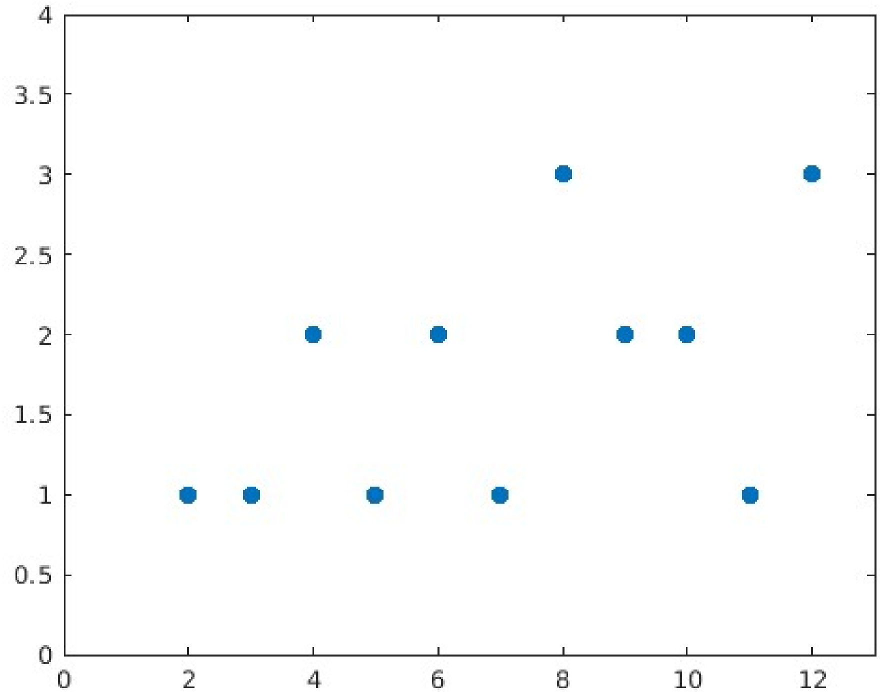
If and only if **both** of these equations can be shown to be true, then the linear system could equivalent, however, looking at the first equation in the system, it's clear that $1+0-2-1 = -2 \neq 0$. Even though the second equation can be said to be sound, this is enough to claim that the two systems are not equivalent.

Answer: Not Equivalent.

Calculus

(a)

The graph of $f(n) = p_n$, where p_n is the number of primes in the prime factorisation of of n, is shown below over the resticted domain $\{1, 2, ..., 12\}$.



The values for $f(n)$ were found by manually computing the prime factorisation of each integer in the domain and are as follows:

$$f(2) = 1, \; f(3) = 1, \; f(4) = 2, \; f(5) = 1, \; f(6) = 2, \; f(7) = 1$$

$$f(8) = 3, \; f(9) = 2, \; f(10 = 2, \; f(11) = 1, \; f(12) = 3$$

(b)

f is not one to one, as it can be easily demonstrated that for two values such that $n_1 \neq n_2$, $f(n_1) = f(n_2)$. Indeed, for the graph in question (a), it requires nothing more than observation to notice that while $2 \neq 3 \neq 5 \neq 7 \neq 11$, it is the case that $f(2) = f(3) = f(5) = f(7) = f(11)$. Therefore, the function is clearly not one-to-one, as it fails the horizontal line test (more than once!).

Answer: Not one-to-one

(c)

To show why the range of f is equal to $\mathbf{N}$, it needs to be demonstrated that $f(n) = m,\ m \in \mathbf{N}$.

First, express n differently such that $n = 2^m,\ n, m \in \mathbf{N}$, and notice that the prime factorisation of any number 2^m will look like $2 \times 2 \times ... \times 2$, m times. Ergo, $f(2^m) = m$, where m can be any natural number. Therefore $f(n)$ can take any value in the set of natural numbers, and the range of f is equal to $\mathbf{N}$.