Maths IA - Written 4

Jack Larkin

Algebra

(a)

To determine the value(s) of a for which the given vectors are linearly dependant, we need to show that the system of linear equations which they represent has a non-trivial solution, the system of equations being:

$$x1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x2 \begin{bmatrix} 1 \\ 1 \\ a \end{bmatrix} + x3 \begin{bmatrix} a \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Expanding this equation gives:

$$x_1 + x_2 + ax_3 = 0$$

$$x_2 + x_3 = 0$$

$$2x_1 + ax_2 - x_3 = 0$$

Which can be written as follows as an augmented matrix:

$$\begin{bmatrix} 1 & 1 & a & 0 \\ 0 & 1 & 1 & 0 \\ 2 & a & -1 & 0 \end{bmatrix}$$

Which, via Gauss-Jordan elimination, can be written in row echelon form as follows:

$$\begin{bmatrix} 1 & 1 & a & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -3a+1 & 0 \end{bmatrix}$$

From which it can be observed that $x_3(-3a+1) = 0$, and therefore either $x_3 = 0$ or $a = \frac{1}{3}$.

If we let $x_2 = 0$, we will arrive at a trivial solution ($x_1 = x_2 = x_3 = 0$), so the only value of a for which the given vectors are linearly dependant is $\frac{1}{3}$.

Answer: $a = \frac{1}{3}$.

(b)

The answer in the previous section gives us the ability to rewrite the vectors in row-reduced matrix as follows:

$$\begin{bmatrix} 1 & 0 & \frac{-2}{3} & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Which then lets us write v_3 as a linear combination of v_1 and v_2 by simple observation:

$$v_3 = \frac{-2}{3} v_1 + v_2$$

Calculus

(a)

Given $x^3 = x^2 - 3y^2$, $\frac{dy}{dx}$ can be found with implicit differentiation. The first step is to differentiate both sides of the equation:

$$3x^2 = 2x - 6y\frac{dy}{dx}$$

and then simply isolate $\frac{dy}{dx}$ as the subject of the equation:

$$\frac{dy}{dx} = -\frac{3x^2 - 2x}{6y}$$

(b)

At the point on the Tschirnhausen cubic where $x = -2$ and $y > 0$, $y = 2$. Therefore the slope of the tangent to the curv is given by $\frac{dy}{dx}$ at $(x, y) = (-2, 2)$.

$$\frac{dy}{dx} = -\frac{3(-2^2) - 2(-2)}{6(2)}$$

$$\therefore \frac{dy}{dx} = -\frac{4}{3}$$