Maths IA - Written 9

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Algebra

(a)

By adding non-negative slack variables to the given inequalities, the following linear system is obtained:

$$2x_1 + x_2 + x_3 = 4$$

$$2x_1 + 3x_2 + x_4 = 6$$

(b)(i)

The basic solution corresponding to the basic variables x_3 and x_4 is a somewhat trivial case. Upon converting the linear system to an augmented matrix, it becomes clear that pivots already exist in the corresponding collumns.

$$\begin{bmatrix} 2 & 1 & 1 & 0 & 4 \\ 2 & 3 & 0 & 1 & 6 \end{bmatrix}$$

So setting all other variables equal to zero gives the solution $(0, 0, 4, 6)$.
Answer: $(0, 0, 4, 6)$

(ii)

The basic solution corresponding to the basic variables x_2 and x_4 can be found with the same logic as the previous part, however with the added step of employing Gauss-Jordan elimination to convert the corresponding columns to pivot columns, if they aren't already, as follows. Beginning with the same augmented matrix:

$$\begin{bmatrix} 2 & 1 & 1 & 0 & 4 \\ 2 & 3 & 0 & 1 & 6 \end{bmatrix}$$

And applying the row operation $R_2 = R_2 - 3R_1$:

$$\begin{bmatrix} 2 & 1 & 1 & 0 & 4 \\ -4 & 0 & -3 & 1 & -6 \end{bmatrix}$$

Gives the solution: $(0, 4, 0, -6)$.
Answer: $(0, 4, 0, -6)$

(iii)

For the solution corresponding to x_2 and x_3, we can pick up where we left of with the matrix in the previous section.

$$\begin{bmatrix} 2 & 1 & 1 & 0 & 4 \\ -4 & 0 & -3 & 1 & -6 \end{bmatrix}$$

$R_2 = \frac{-1}{3}R_2$:

$$\begin{bmatrix} 2 & 1 & 1 & 0 & 4 \\ \frac{4}{3} & 0 & 1 & \frac{-1}{3} & 2 \end{bmatrix}$$

$R_1 = R_1 - R_2$:

$$\begin{bmatrix} \frac{2}{3} & 1 & 0 & \frac{1}{3} & 2 \\ \frac{4}{3} & 0 & 1 & \frac{-1}{3} & 2 \end{bmatrix}$$

Answer: $(0, 2, 2, 0)$

(c)

The feasible region is defined by the additional constraints $x_1, x_2 \geq 0$, therefore the vertices of the region are given by the x_1 and x_2 coordinates of the basic solutions where each component is ≥ 0, i.e vertices exist at:

$$(0,0), (0,2), (2,0) \text{ and } (\frac{3}{2}, 1)$$

Calculus

$$\int sin(\sqrt{x})dx$$

Letting $u = \sqrt{x}$:

$$\therefore x = u^2$$

$$\therefore dx = 2udu$$

Which gives:

$$2 \times \int u \times sin(u)du$$

Applying integration by parts, letting $u = u$ and $sin(u) = v'$

$$\therefore u' = 1 \text{ and } v = -cos(u)$$

$$\int u \times sin(u)du = \therefore -u \times cos(u) + \int cos(u)du$$

$$= sin(u) - u \times cos(u)$$

$$= sin(\sqrt{x}) - \sqrt{x}cos(\sqrt{x})$$

Which gives the solution:

$$\int sin(\sqrt{x})dx = 2(sin(\sqrt{x}) - \sqrt{x}cos(\sqrt{x})) + c$$