Fast Matrix Multiplication Attempts

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Brent‘s Equations *)

Substituting (5.01) in (5.02), equating coefficients, and using the definition of matrix multiplication, gives the set of equations

$$\sum_{p=1}^{T} \alpha_{ijp}\beta_{kLp}\gamma_{mnp} = \delta_{ni}\delta_{jk}\delta_{Lm} , \qquad (5.03)$$

where δ is Kronecker's delta. (The subscripts on the c_{nm} were reversed to increase the symmetry of (5.03).) For the multiplication of $M \times N$ matrices by $N \times P$ matrices, (5.03) gives $(MNP)^2$ equations as i, j, k, L, m, and n range over the integers $1 \le i,n < M$, $1 \le j,k < N$, $1 \le L,m < P$.

Set of non-linear equations.
Each equation is a sum of „triples“, products of three coefficients.
Approach is to solve the equations modulo 2 in a first phase

*) R.P. Brent, Stanford University, 1970 http://maths-people.anu.edu.au/~brent/pub/pub002.html

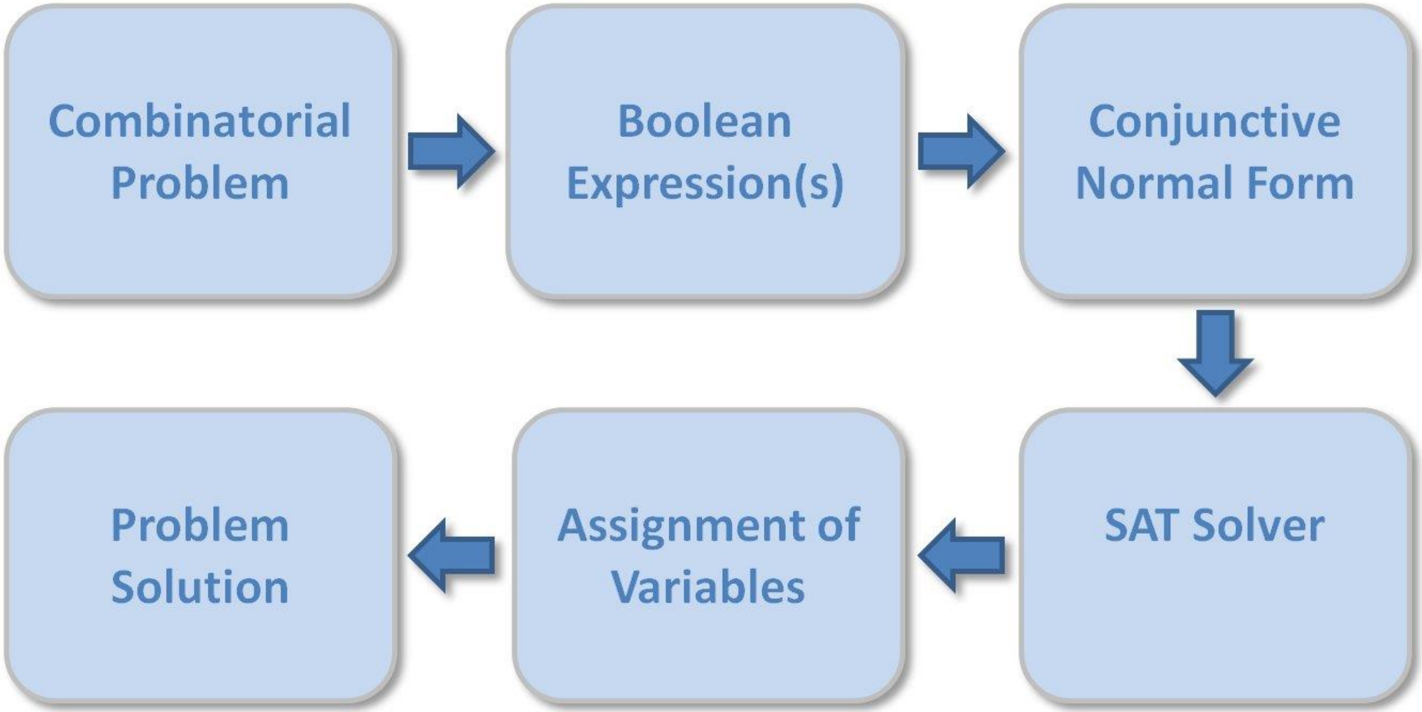
Attempts to solve Brent‘s Equations

- Encode in solver-specific format:

CNF	Boolean Satisfiability
SMT	Satisfiability Modulo Theories
ASP	Answer Set Programming
PBS	Pseudo Boolean Satisfaction
MiniZinc	Constraint Programming
Linear Program	Linear Integer Programming

- Add constraints to break symmetries
- Fix certain variables („odd triples“)
- Assume properties of products and triples
- Use 0 as initial value for variables
- Partition set of equations
- Use common subexpressions
- Shuffle constraints and variables

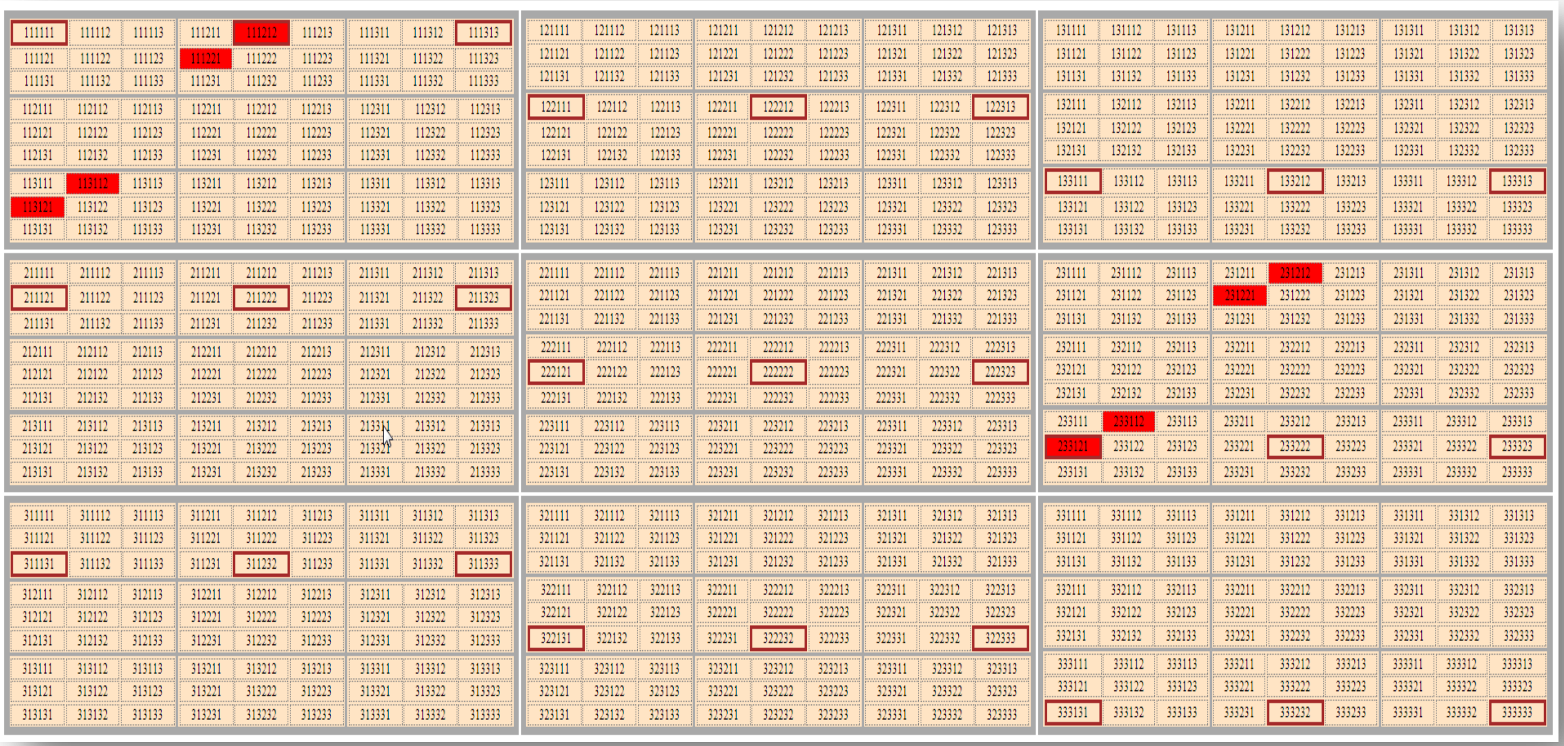
Solution Flow



Observations with Brent‘s Equations

- Small cases can be solved easily:
  2x2x2 with 7 products in 50ms
  2x3x3 with 11 products in 1-2s
  3x3x3 with 27 products in 15s
- Solutions for 3x3x2 and 3x3x3 take excessive runtime
- Quite a few SAT solvers have problems even with small cases
- SAT solvers tend to beat all other solvers tried so far
- All known solutions for 3x3x3 with 23 products
  have the same 376 „all-zero“ equations
- Product columns of odd triples can be used as symmetry breaking sort criteria
- Composition of EXORn from EXOR3 yields good results
- Solutions of equations are unambiguous
- Overlap of odd triples is essential

Brent Equation Explorer



Encoding of Brent‘s Equations

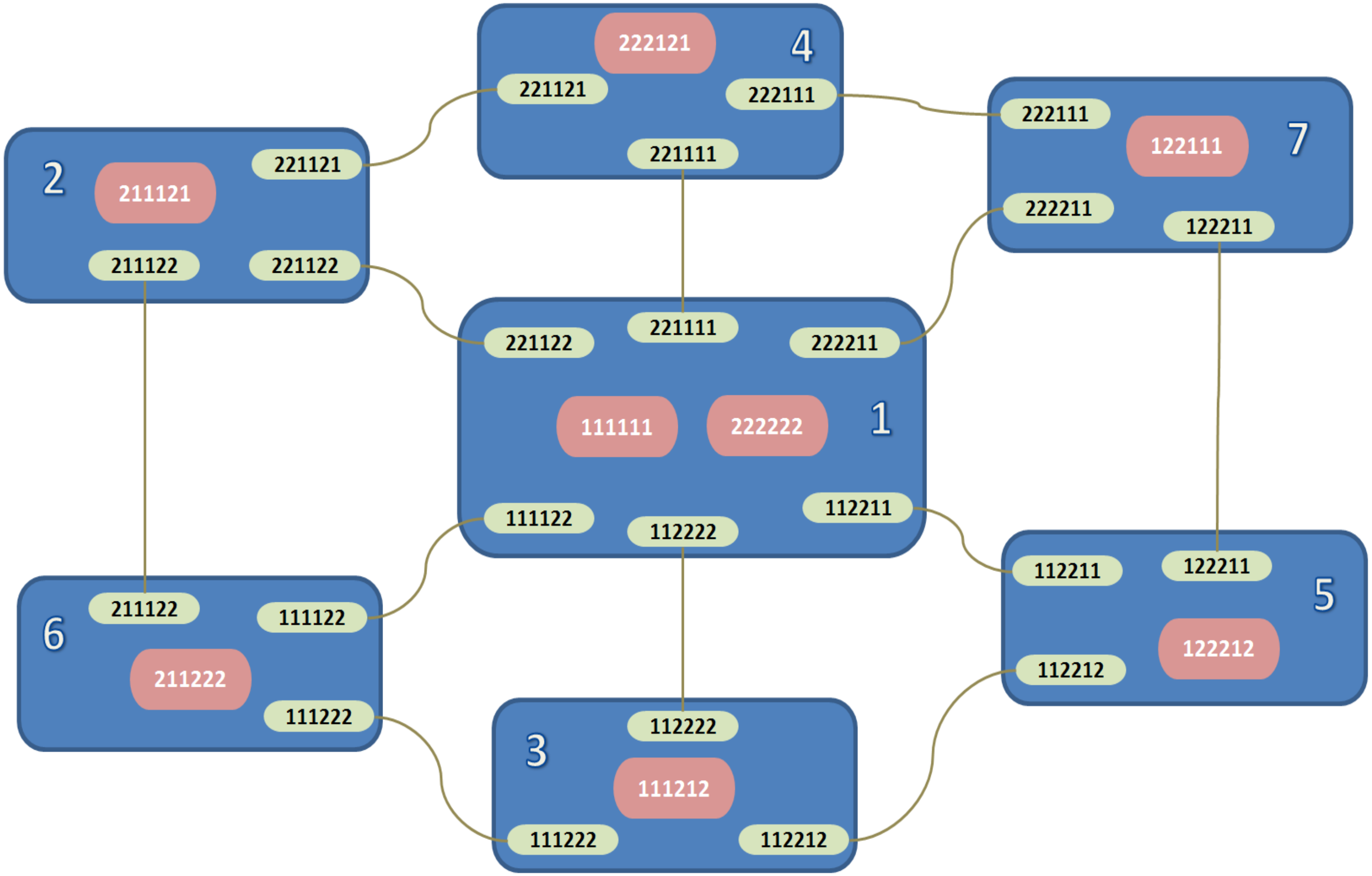
- Mod 2: every equation is an XOR or XNOR constraint
- Summands of the equations are AND3 “triples”
- Common AND2 (“dyads”) subexpressions are used
- Tseitin encoding to prevent exponential blow-up
- Big XOR/XNOR expressions are broken down using XOR cut proposed by Gregory V. Bard in his dissertation
- Random selection of triples in “odd” equations set to 1 (one per equation, one or two per product)
- Fruitless experiments with symmetry breaking constraints

Analysis of Strassen's Algorithm 2x2x2

Highly regular odd equations:

122111	1	=	1	+	0	+	0	+	0	+	0	+	0	+	0	+	0
111111	1	=	0	+	1	+	0	+	0	+	0	+	0	+	0	+	0
222222	1	=	0	+	1	+	0	+	0	+	0	+	0	+	0	+	0
211222	1	=	0	+	0	+	1	+	0	+	0	+	0	+	0	+	0
122212	1	=	0	+	0	+	0	+	1	+	0	+	0	+	0	+	0
111212	1	=	0	+	0	+	0	+	0	+	1	+	0	+	0	+	0
222121	1	=	0	+	0	+	0	+	0	+	0	+	1	+	0	+	0
211121	1	=	0	+	0	+	0	+	0	+	0	+	0	+	1	+	0

Analysis of Strassen's Algorithm 2x2x2



Other Results

- Detailed algorithm for 5x5x5 with 100 products (Makarov showed the feasibility but omitted the algorithm)
- Solution for 3x3x2 for $a_{33}=0$ with 14 products
- An improved solution for 3x3x3 would improve 5x5x5
- Implementation of Sýkora's method (yields Laderman's solution for 3x3x3)
- Tools to analyze, visualize and convert solutions
- Implemented search algorithm of Jinsoo Oh, et al. *)
- Tools to convert Boolean circuits to CNF
- Collection of small-factor solutions
- Tried to derive 3x3x3 solutions from larger solutions

*) http://www.sciencedirect.com/science/article/pii/S0020019013001531

Questions

- Is there any hope to find a solution for 3x3x3 with 22 or fewer products?
- How to exploit symmetries?
- How did J. Laderman find his solution?

Contact

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