Random Search proof

The search space of F1 contains $2^n$ solutions.

Therefore, the probability of finding the optimal search point is $\frac{1}{2^n}$.

Let $m = 2^{n/2}$; the probability that an optimal search point is found within the budget 'm' fitness evaluations is:

$$P(success\ in\ m) = 1 - \left(1 - \frac{1}{2^n}\right)^m$$

Using the general inequality $1 - x \leq e^{-x}$ for all $x$:

$$1 - \left(1 - \frac{1}{2^n}\right)^m \leq \exp\left(-\frac{m}{2^n}\right)$$

Plug in $m = 2^{n/2}$:

$$1 - \left(1 - \frac{1}{2^n}\right)^m \leq \exp\left(-\frac{2^{\frac{n}{2}}}{2^n}\right)$$
$$= \exp\left(-2^{\frac{n}{2}-n}\right)$$
$$= \exp\left(-2^{-\frac{n}{2}}\right)$$
$$\therefore 1 - \left(1 - \frac{1}{2^n}\right)^{2^{n/2}} \approx \exp\left(-2^{-n/2}\right)$$

For large n, $2^{-n/2}$ is tiny, so:

$$1 - \left(1 - \frac{1}{2^n}\right)^{2^{n/2}} \leq \exp\left(-2^{-n/2}\right) \approx 1 - 2^{-n/2} \approx 1$$

Thus:

$$P\left(success\ in\ 2^{n/2}\right) = 1 - P(miss\ in\ 2^{n/2}) = 1 - 2^{-\frac{n}{2}}$$

Since $1 - 2^{-n} = 1 - e^{-\Omega(n)}$, with probability at least $1 - e^{-\Omega(n)}$, the probability that an optimal search point cannot be located within the first $2^{n/2}$ fitness evaluations is very close to 1. Therefore, random search requires at least $2^{n/2}$ fitness evaluations to have a non-negligible chance to locate an optimal search point on F1.