Q1.1

For two camera projection matrix M1 and M2, we have

$$p = M_1 * P$$
$$q = M_2 * P$$

where P = [X Y Z 1]’, p = [x y 1]’, q = [x y 1]’ and M is a 3 by 4 matrix.

We can find a plane that Z in [X Y Z 1] is zero,

After that, consider M_1 and M_2 as two 3 by 3 matrix and P as [X Y 1], which will give us a invertible M matrix.

So that we have

$$p = M_1 * {M_2}^{-1} * q$$

In that equation, H exists and can be calculated by $M_1 * {M_2}^{-1}$

Q 1.2

For $x_1 = K_1[I\ 0]X$ and $X = [x\ y\ z\ 1]'$ we can derive that

$$x_1 = K_1[x\ y\ z]^T$$

So that $[x\ y\ z]^T = {K_1}^{-1}x_1$, then

$$X = [{K_1}^{-1}x_1\ \ 1]^T$$

For $x_2 = K_2[R\ 0]X$, we can derive that

$$x_2 = K_2[R\ 0]\left[{K_1}^{-1}x_1\ \ 1\right]^T$$

$$x_2 = K_2R{K_1}^{-1}x_1$$

which gives us the H:

$$H = K_2R{K_1}^{-1}$$

Q1.3

1) h has 8 degrees of freedom since the 9th one is for normalization and has the same effect if it is only scaled by a constant factor.

2) One correspondence will give us two equations so that 4 point pairs will help us solve h.

3) for each point pair, we have

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & h_9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

we multiply it out:

$x'(h7x + h8y + h9) = (h1x + h2y + h3)$

$x'(h7x + h8y + h9) = (h4x + h5y + h6)$

so that

$h7xx' + h8yx' + h9x' - h1x - h2y - h3 = 0$

$\mathrm{h7}xy' + h8yy' + h9y' - h4x - h5y - h6 = 0$

we can reform it into

$$A_i = \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & yx' & x' \\ 0 & 0 & 0 & -x & -y & 1 & xy' & yy' & y' \end{bmatrix}$$

and h is $[h1\ h2\ h3\ h4\ h5\ h6\ h7\ h8\ h9]^T$

Q1.4

Since $H = KRK^{-1}$

$$H^2 = KRK^{-1} * KRK^{-1}$$
$$H^2 = KRRK^{-1}$$

and when it rotates around y axis,

$$R = \begin{bmatrix} \cos\theta & -sin\theta & 0 \\ sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R * R = \begin{bmatrix} \cos\theta\cos\theta - \sin\theta\sin\theta & -2sin\theta\cos\theta & 0 \\ 2sin\theta\cos\theta & \cos\theta\cos\theta - \sin\theta\sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which is exactly

$$R = \begin{bmatrix} \cos 2\theta & -sin2\theta & 0 \\ sin2\theta & \cos 2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

other situations is similar to this circumstance.

So H^2 is the homography corresponding to a rotation of 2θ.

Q1.5

Planar homography is not completely sufficient to map any arbitrary scene image to another viewpoint since it use matched points to calculate the homography matrix. However, if there is so many wrong matched points in a pair of pictures, like a highly repeated texture or some pure colors, planar homography won’t work at all.

Q1.6

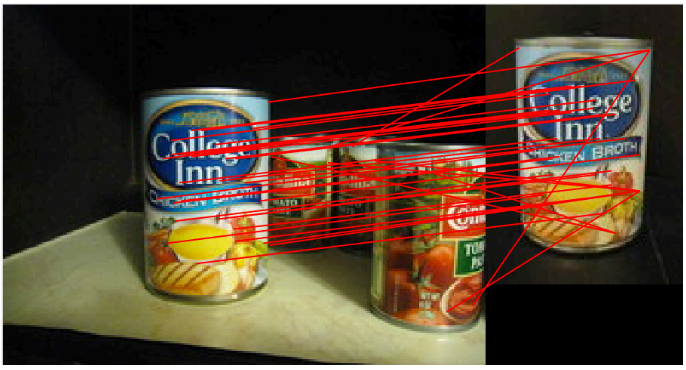
Assume that there are 3 points on a line in 3D space X1, X2 and X3, which follows the ax+by+cz+d=0.

$$[a\ b\ c\ d]\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = 0$$

if we apply a transformation P: x = PX and since it’s linear,

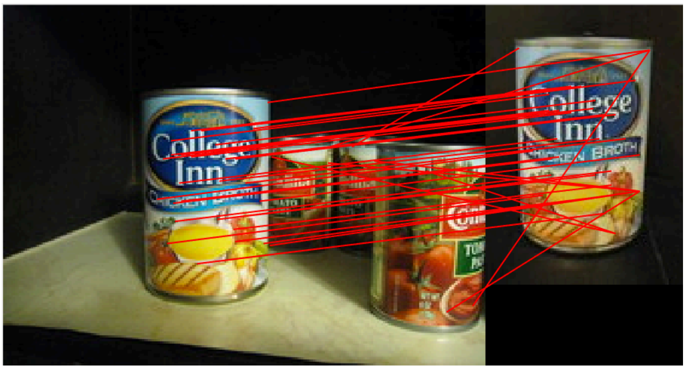
the new x1 x2 x3 will still follow a line equation px+qy+r=0

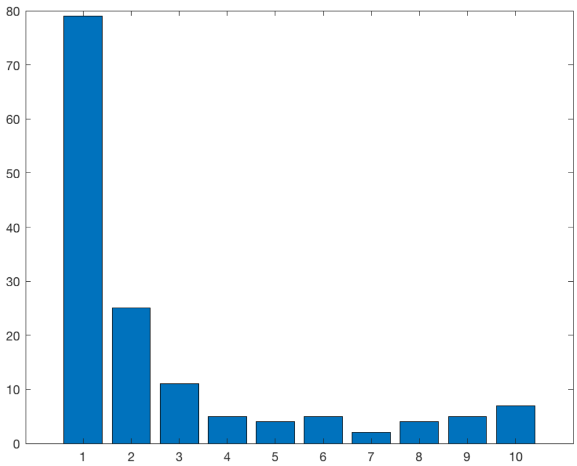
Q2.4.1



If I use other test patterns instead of the best way mentioned in the paper, the number of matched points decreases slightly. When I use symmetric pattern, I got the fewest matched points.

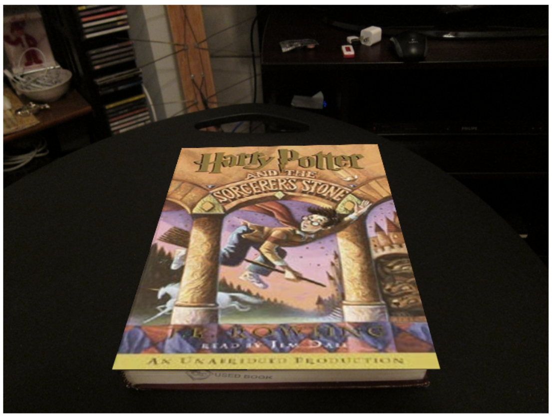
Q2.5





The BRIEF descriptor only has the same pattern for features with the same orientation. This is because once you rotate the picture, the generated encoder's patterns are not going to rotate as well. The same point on two pics are going to have different BRIEF descriptors just after a slight rotation.

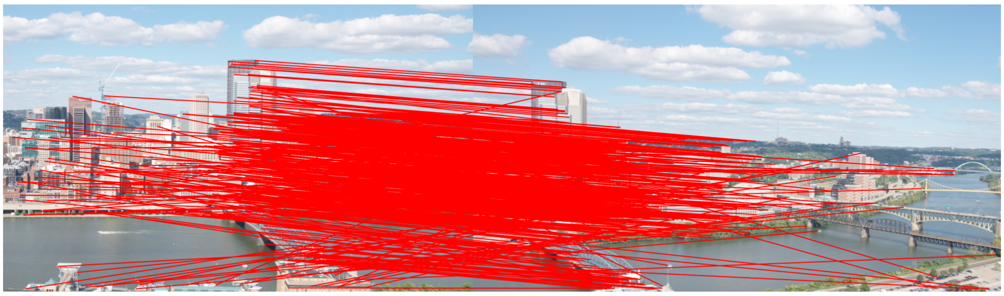
Q3.4



H =

2.5620	0.8081	-769.0239
0.0979	4.6679	-921.6505
0.0002	0.0043	1.0000

Q4.1

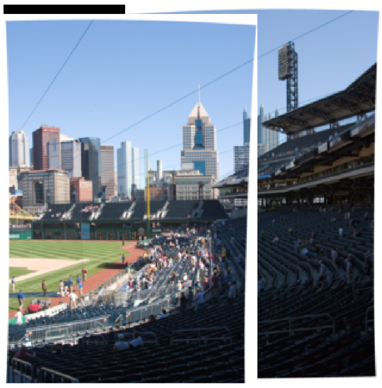


H =

$$\begin{matrix} 0.6770 & -0.0476 & 363.3622 \\ -0.0729 & 0.8696 & -16.0606 \\ -0.0003 & -0.0000 & 1.0000 \end{matrix}$$

Q4.2

Q4.3



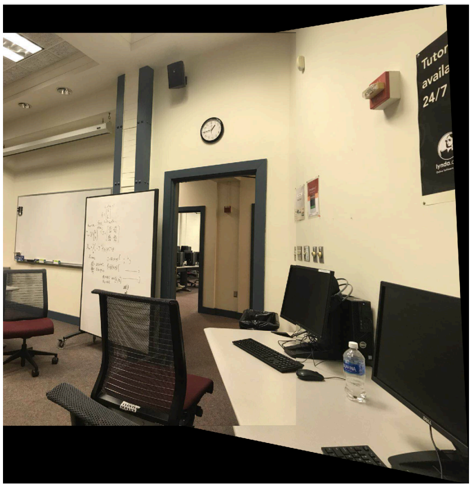
H =

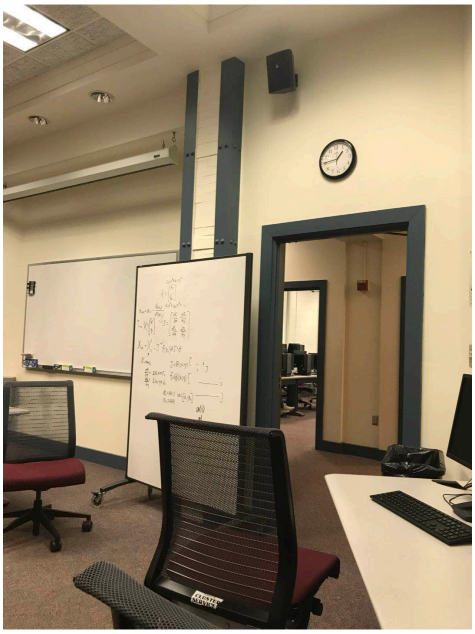
1.0193	0.0050	100.7683
0.0011	0.9998	-9.0055
0.0000	-0.0000	1.0000

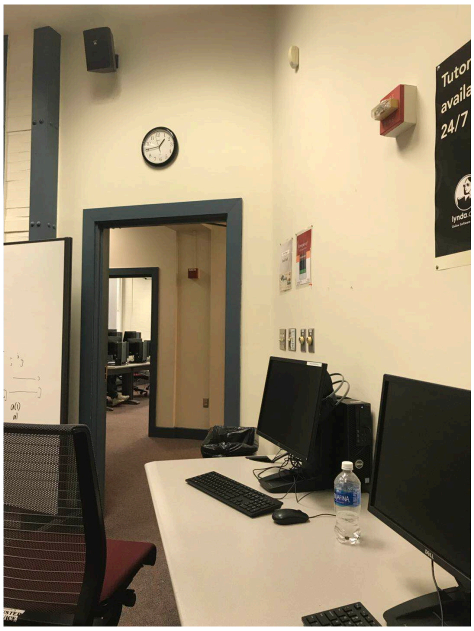
H =

0.6754	-0.0410	362.4359
-0.0751	0.8780	-17.7889
-0.0003	-0.0000	1.0000

Q4.4







H =

0.9387	0.2003	254.8148
-0.1242	1.1544	-40.7063
-0.0002	0.0001	1.0000