

Q1.1

Two points $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (0, 0)$

$$\text{So that } \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0.$$

After we multiply them out, we can get $[f_7 \ f_8 \ f_9]' [0 \ 0 \ 1] = 0$.

So that we have $f_9 = 0$.

Q1.2

If two cameras only differ from each other by a pure translation that is parallel to the x-axis, the translation matrix will be $[t_1 \ 0 \ 0]$ and their rotation matrix is an identical matrix.

And if they are calibrated cameras, $E = R[tx] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$

Also, we have epipolar lines $l_2 = E * x_1$

Which will give us $l_2 = [0, -t_1, y_1]$ for the line equation.

Substitute into $ax + by + c = 0$, we will have an equation without x , which means that the lines are parallel to x-axis. Vice Versa.

Q1.3

For a 3D point $[X \ Y \ Z \ 1]'$, we have $x = PX$, where $P = K[R|t]$ and $x = [x \ y \ 1]'$.

We want to find out R_{rel} and t_{rel} that give us $x_2 = R^*x_1 + t$

So we currently have $\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = K(R_1 * \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + t_1)$ and $\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = K(R_2 * \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + t_2)$

We substitute $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$ in the above equations, we will have

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = KR_1R_2^{-1}K^{-1}\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - KR_1R_2^{-1}t_2 + Kt_1$$

So that $R_{rel} = KR_1R_2^{-1}K^{-1}$ and $t_{rel} = -KR_1R_2^{-1}t_2 + Kt_1$.

$E = [tx]R$ and $F = K^{-T}[tx]RK^{-1}$

Q1.4

The rotation matrix between these two objects is an 3 by 3 identical matrix and we assume the $t = [t_1, t_2, t_3]$.

$$\text{So that } [t]_x = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}$$

Since the essential matrix is $E = [t]_x R$,

$$\text{We have } E = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}.$$

We notice that $E^T = -E$ and $F = K^{-T} E K^{-1}$

$$\text{So that } F^T = (K^{-T} E K^{-1})^T = K^{-T} E K^{-1} = -K^{-T} E K^{-1} = -F$$

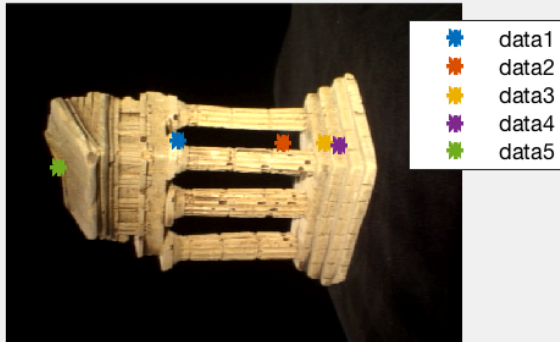
So that F is skew-symmetric.

2

F =

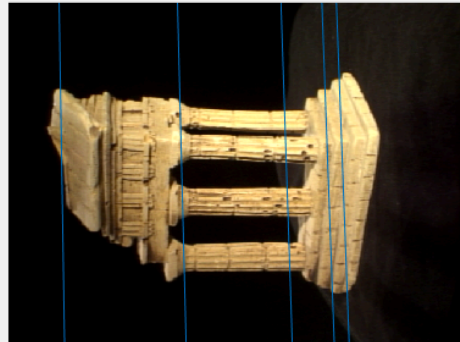
-0.0000	-0.0000	0.0011
-0.0000	0.0000	-0.0000
-0.0011	0.0000	-0.0042

Epipole is outside image boundary



Select a point in this image
(Right-click when finished)

Epipole is outside image boundary



Verify that the corresponding point
is on the epipolar line in this image

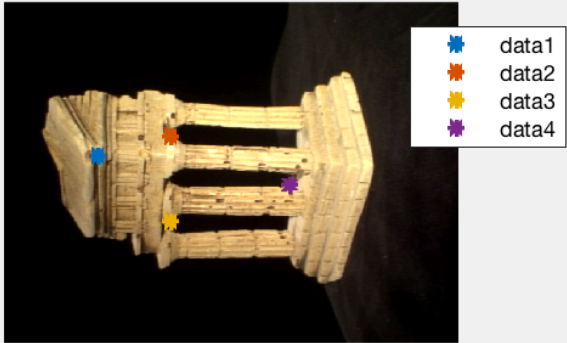
Q2.2

F{1}

ans =

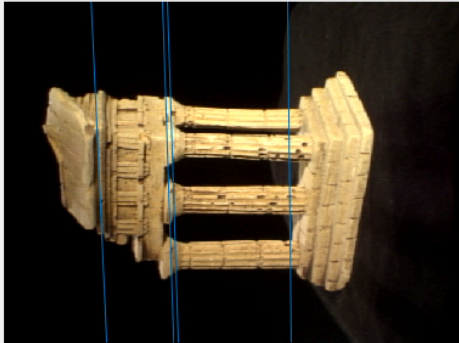
-0.0000	0.0000	0.0014
0.0000	-0.0000	-0.0001
-0.0014	0.0001	0.0040

Epipole is outside image boundary



Select a point in this image
(Right-click when finished)

Epipole is outside image boundary



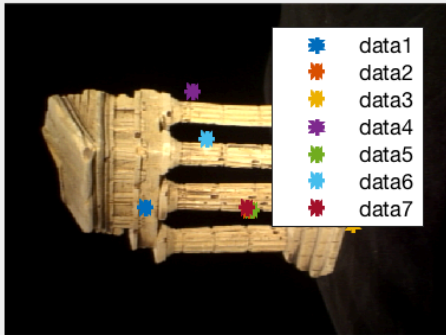
Verify that the corresponding point
is on the epipolar line in this image

>> F{2}

ans =

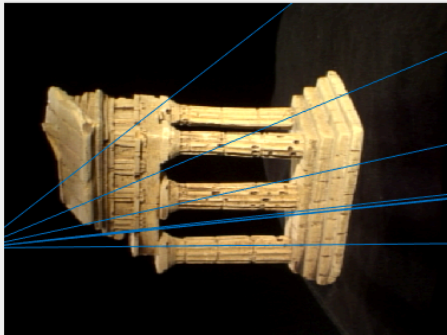
-0.0000	-0.0000	0.0004
0.0000	0.0000	-0.0001
-0.0002	-0.0004	0.0351

Epipole is outside image boundary



Select a point in this image
(Right-click when finished)

Epipole is outside image boundary



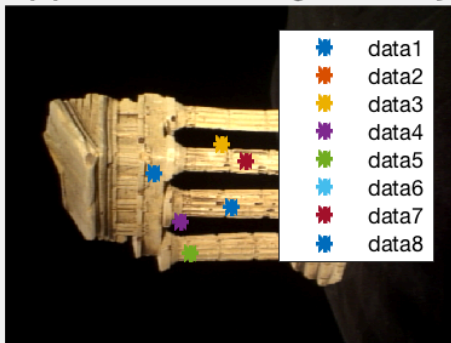
Verify that the corresponding point
is on the epipolar line in this image

```
>> F{3}
```

```
ans =
```

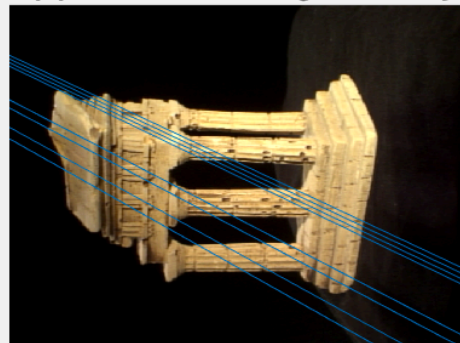
```
-0.0000    -0.0000    0.0001  
 0.0000     0.0000   -0.0001  
 0.0001   -0.0005    0.0439
```

Epipole is outside image boundary



Select a point in this image
(Right-click when finished)

Epipole is outside image boundary



Verify that the corresponding point
is on the epipolar line in this image

Q3.1

E =

-0.0030	-0.3032	1.6604
-0.1368	0.0300	-0.0468
-1.6650	-0.0092	-0.0007

M2 =

-0.9997	0.0259	0.0028	-0.0306
0.0260	0.9948	0.0989	-1.0000
-0.0003	0.0989	-0.9951	0.0821

P2 =

1.0e+03 *

-1.5200	0.0693	-0.2965	-0.0216
0.0397	1.5423	-0.0948	-1.5056
-0.0000	0.0001	-0.0010	0.0001

Q3.2

For

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \alpha \begin{bmatrix} p1 & p2 & p3 & p4 \\ p5 & p6 & p7 & p8 \\ p9 & p10 & p11 & p12 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

and $P1 = [p1 \ p2 \ p3 \ p4]$,

we can express A_i with

$$A_i = \begin{bmatrix} y_i P_3 - P_2 \\ P_1 - x_i P_3 \\ y_i' P_3 - P_2 \\ P_1 - x_i' P_3 \end{bmatrix}$$

Q3.3

For points1 reconstruction error, err1 =

57.9226

For points2 reconstruction error, err2 =

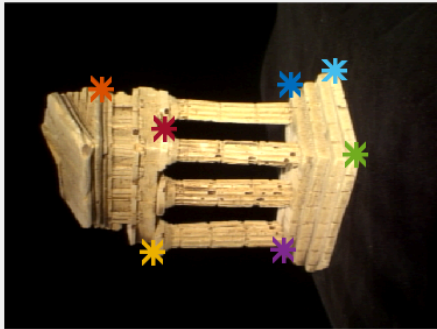
58.8861

Total Reprojection error err = 116.8087

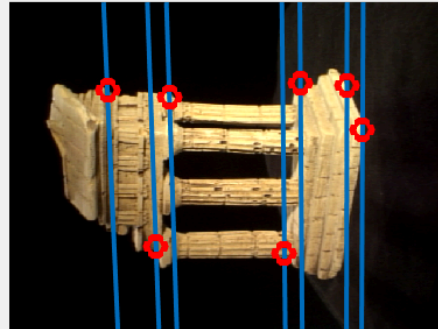
M2 =

-0.9997	0.0259	0.0028	-0.0306
0.0260	0.9948	0.0989	-1.0000
-0.0003	0.0989	-0.9951	0.0821

Q4.1

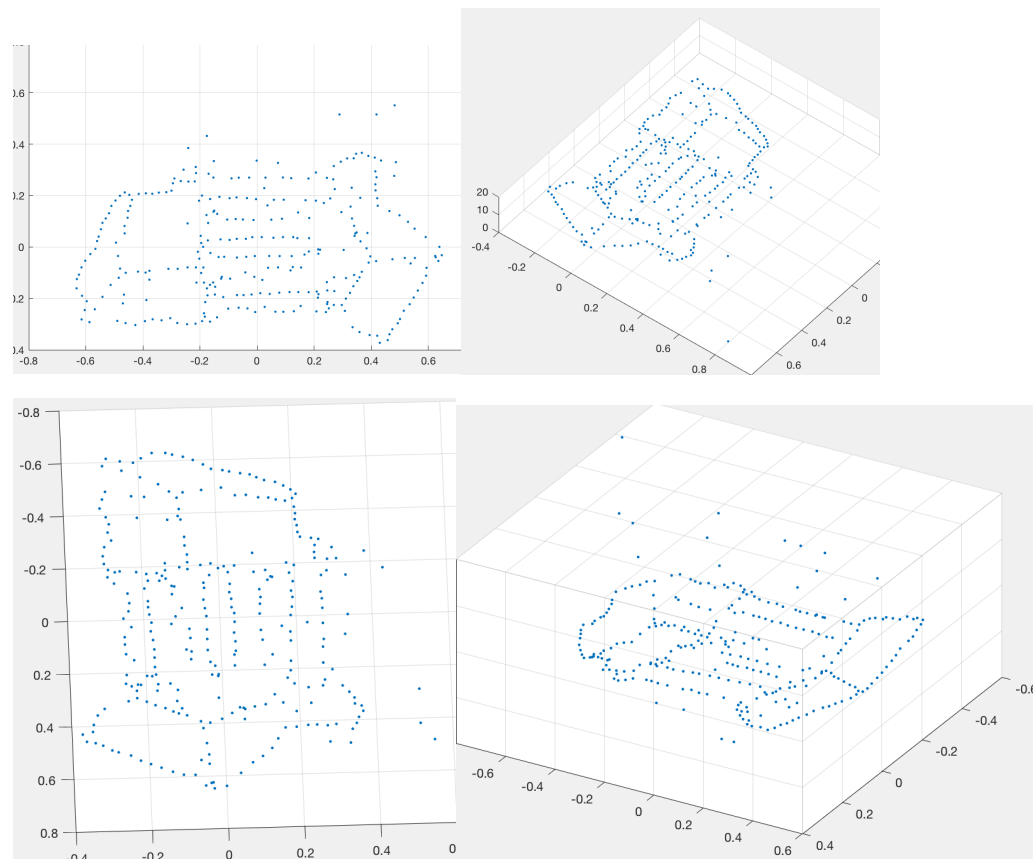


Select a point in this image
(Right-click when finished)



Verify that the corresponding point
is on the epipolar line in this image

Q4.2



Q5.1

Eight Point:

F =

0.0000	0.0000	-0.0004
-0.0000	-0.0000	0.0006
0.0000	0.0000	-0.0256

RANSAC:

73.57% inliers with RANSAC

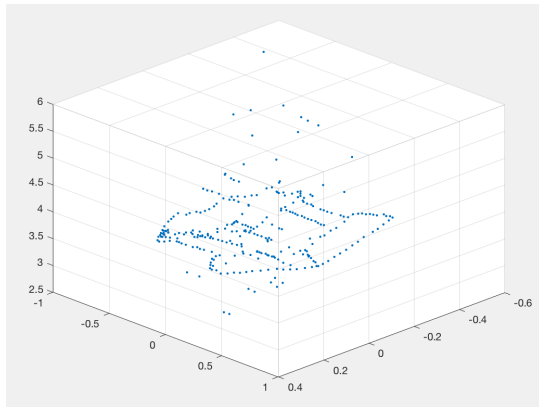
F =

-0.0000	-0.0000	0.0011
-0.0000	0.0000	-0.0000
-0.0011	0.0000	-0.0038

For the error metrics, I use $l = F \cdot x$ to compute the estimated corresponding epipolar line on image 2 for each point and then use ground truth points compute $ax+by+c$ to see if the value is zero. If the result is less than 10^{-3} , it will be considered as zero, which means the ground truth point is exactly on the estimated epipolar line. I also count the correct inliers so that the F with most inliers is the best F.

Using Eight Point, we cannot deal with the noise point pairs in the dataset while RANSAC actually solve this problem by counting inliers and choose the best F with most inliers.

Q5.3



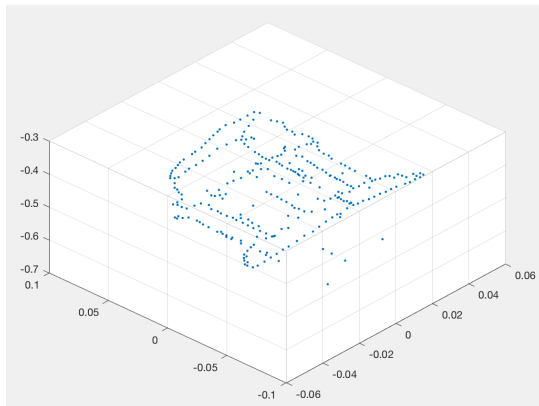
err_before = 116.8087

M2 =

0.9994	0.0340	0.0081	-0.0306
-0.0349	0.9650	0.2598	-1.0000
0.0010	-0.2599	0.9656	0.0821

P2 =

1.0e+03 *			
1.5198	-0.0269	0.3043	-0.0216
-0.0531	1.4084	0.6348	-1.5056
0.0000	-0.0003	0.0010	0.0001



err_after = 44.7298

M2_new =

0.9994	0.0332	0.0082	0.0035
-0.0342	0.9652	0.2594	0.1190
0.0007	-0.2595	0.9657	-0.0113

P2_new =

1.0e+03 *			
1.5197	-0.0279	0.3044	0.0019
-0.0521	1.4087	0.6342	0.1787
0.0000	-0.0003	0.0010	-0.0000