Motilal Nehru National Institute of Technology Allahabad, Prayagraj
End Semester Examination(session: 2019-20)
B.Tech. Semester I
Mathematics I (Course Code MA 11101)

Student Reg. No.:
Note : Solve all **Six** questions. All questions carry equal marks.
Time: 3 Hrs

Max Marks: 60

---

1. (a) Find the volume of the solid of revolution of $r^2 = a^2 cos\ 2\theta$ about the line $\theta = \pi/2$. **3 Marks**

   (b) Evaluate $\int_0^a \int_{y^2/4a}^{y} \frac{x^2-y^2}{x^2+y^2}\, dxdy$. **3 Marks**

   (c) (i) State and prove Euler's theorem for homogeneous functions of two variables.
   (ii) If $u = x\ log\ xy$, where $x^3 + y^3 + 3xy = 1$. Find $\frac{du}{dx}$. **2 + 2 = 4 Marks**

2. (a) If $\lim_{n\to\infty} a_n = l$, then show that $\lim_{n\to\infty} \frac{a_1 + a_2 + \ldots + a_n}{n} = l$. **3 Marks**

   (b) Let $\sum a_n$ be infinite series of positive terms and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = l$. Then show that series converges if $l < 1$ and diverges if $l > 1$. **3 Marks**

   (c) Test the convergence of the series
   $1 + \frac{\alpha\beta}{1.\gamma}x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{1.2\gamma(\gamma+1)}x^2 + \ldots$ **4 Marks**

3. (a) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8x^2e^{2x}sin\ 2x$ **3 Marks**

   (b) Solve $\frac{dy}{dx} + y = z + e^x$, $\frac{dz}{dx} + z = y + e^x$ **3 Marks**

   (c) State the existence and uniqueness theorem for the initial value problems. Test the uniqueness of solutions of the initial value problem $(x - 2y + 1)\ dy - (3x - 6y + 2)\ dx = 0$, $y(0) = 0$ over the rectangle $R : |x| \leq 4$, $|y| \leq 4$. Determine the interval for x, over which the solution is gauranteed.

   4 Marks

4. (a) Find the work done by the force $(x^2 + y^2)\hat{i} + (y + 2x)\hat{j}$ in moving a particle along the closed curve C in the first quadrant bounded by the curves $y^2 = x$ and $x^2 = y$. **3 Marks**

   (b) Find the flux of the vector field $x\hat{i} + y\hat{j} - z\hat{k}$ through the portion of the plane $\frac{x}{4} + \frac{y}{8} + \frac{z}{10} = 1$ in the first octant, where positive flow is defined to be in the positive z-direction. **3 Marks**

   (c) State and prove the Gauss Divergence Theorem.

or

PTO

State Stoke's Theorem and verify it for the field $\bar{F} = x^2\hat{i} + 2x\hat{j} + z^2\hat{k}$ on the surface $S = \{(x, y, z) : 4x^2 + y^2 \leq 4,\ z = 0\}$.

4 Marks

5. (a) Consider the map $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} 1 & if\ y \geq 0 \\ -1 & if\ y < 0, \end{cases}$$

Test the continuity $f(x, y)$ at $(0, 0)$? Justify. 3 Marks

(b) Test the differentiability of the function $f(x, y) = \sqrt{|xy|}$ at the point $(0, 0)$. 3 Marks

(c) (i) Examine the convergence of $\int_0^{\pi/2} \frac{\sin x\ dx}{x^n}$

(ii) Show that $\int_0^{\infty} \frac{x^c}{c^x}\, dx = \frac{\Gamma(c+1)}{(\log c)^{c+1}}$. $2 + 2 = 4$ Marks

6. (a) Show that $grad(\bar{a}.\bar{b}) = \bar{a} \times curl\ \bar{b} + \bar{b} \times curl\ \bar{a} + (\bar{a}.\nabla)\ \bar{b} + (\bar{b}.\nabla)\ \bar{a}$. 3 Marks

(b) By the method of variation of parameters find the general solution of non-homogeneous linear differential equation

3 Marks

$$y'' - 2y' + y = e^x \log x$$

(c) Obtain series solution of the equation 4 Marks

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - xy = 0.$$

B.Tech.I Sem End Semester Examination 2009-10
Mathematics-I (MA-101)

Time: $2\frac{1}{2}$ **Hours** MM: **40**

Note: Attempt any **FIVE** questions. Each question carries **equal** marks.

1(a) Show that the following function is continuous at (0 , 0)

$f(x, y) = (x+y)\sin\left(\frac{1}{x+y}\right)$, *if* $(x+y) \neq 0$ and $f(x, y) = 0$ if $(x + y) = 0$.

(b) State Euler's theorem and use it to evaluate

$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ if $u(x, y) = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$.

(c) If $f(x, y) = 0$, $g(y, z) = 0$, show that $\frac{\partial f}{\partial y}\frac{\partial g}{\partial z}\frac{dz}{dx} = \frac{\partial f}{\partial x}\frac{\partial g}{\partial y}$.

(d) If function u, v, w of the three independent variables x, y, z are not independent, then show that the Jacobian of u, v, w with respect to x, y, z vanishes.

2(a) Show that $\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$

(b) By changing the order of integration, evaluate $\int_0^1\int_{2y}^2 e^{x^2}\,dx\,dy$.

(c) Find by double integration , the area lying inside the circle $r = a\sin\theta$ and outside the cardioid $r = a(1-\cos\theta)$.

(d) Show that the sequence $\left\{\frac{n!}{2^n}\right\}$ is not convergent.

3(a) Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at the point (1 , -2 , 1) and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$.

(b) Test for convergence of the series $1 + \frac{2x}{2!} + \frac{3^2x^2}{3!} + \frac{4^3x^3}{4!} + \ldots\ldots,\ x > 0$.

4(a) If $V = f(r)$, where $r^2 = x^2 + y^2 + z^2$, show that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{d^2V}{dr^2} + \frac{2}{r}\frac{dV}{dr}$

(b) Find the total mass and the centre of gravity of the region bounded by $x^{2/3} + y^{2/3} = a^{2/3}$., $x \geq 0,\ y \geq 0$, when the density is constant k.

5(a) Show that the vector field $\vec{F} = 2x(y^2 + z^3)\,i + 2x^2y\,j + 3x^2z^2\,k$ is conservative field. Find its scalar field potential and the work done in moving a particle from(-1 , 2 , 1) to (2 , 3 , 4).

(b) Prove the identity

$grad(\vec{A}\bullet\vec{B}) = (\vec{A}\bullet\nabla)\vec{B} + (\vec{B}\bullet\nabla)\vec{A} + \vec{A}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\vec{A})$

6(a) Verify divergence theorem for vector function $\vec{V} = 4x\,i - 2y^2\,j + z^2\,k$ taken over the region bounded by the cylinder $x^2 + y^2 = 4$, $z = 0$, $z = 3$.

(b) State Stoke's theorem and use it to evaluate $\iint_S curl\vec{V}\bullet\hat{n}\,dS$ where $\vec{V} = y\,i + z\,j + x\,k$ and S is the part of surface of the paraboloid $z = 1 - x^2 - y^2$ for which $z \geq 0$.

F. d

Motilal Nehru National Institute of Technology Allahabad, Prayagraj

B. Tech. I Semester

Supplementary Examination, 2019

Subject: Mathematics I **Subject Code: MA11101/1101**

Time: 3 hours **Maximum Marks: 60**

Note: Attempt all six questions. Each question carries equal marks. Solve all parts of a question on continuation.

Q. 1 (a) Discuss the convergence of the series $\sum \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n, (x > 0)$.

(b) State the Lagrange's mean value theorem and hence find c given

$$f(x) = x(x-1)(x-2);\ a = 0,\ b = \frac{1}{2}.$$

Q.2 (a) If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$ and hence show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}.$$

(b) Compute $f_{xy}(0,0)$ and $f_{yx}(0,0)$ for the function

$$f(x,y) = \begin{cases} xy^3/(x+y^2), & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Also discussed the continuity of f_{xy} and f_{yx} at (0, 0).

Q.3 (a) Evaluate the triple integral $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ if the region of integration is bounded by coordinate planes and the plane $x+y+z=1$.

(b) Evaluate the integral $\int_0^\infty \int_0^x x \exp\left(-\frac{x^2}{y}\right) dx\, dy$ by changing the order of integration.

Q.4 (a) Prove that the relation between beta and gamma function is given by

$$B(m,n) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(m+n)}.$$

(b) Show that the functions $u = x+y+z$, $v = xy+yz+zx$, $w = x^3+y^3+z^3-3xyz$ are not independent. Find the relation between them.

P.T.O.

Q. 5. (a) State Gauss divergence theorem. Let D be the region bounded by the closed cylinder $x^2+y^2=16$, $z=0$ and $z=4$. Verify the divergence theorem for $\vec{F}=3x^2\vec{i}+6y^2\vec{j}+z\vec{k}$.

(b) If $\phi(x, y, z)=3x^2y-y^3z^2$, then find $grad\phi$ at $(1,-2,-1)$.

Q. 6. (a) Apply the method of variation of parameter to solve $(D^2+1)y=\tan x$.

(b) Solve the differential equation $(2x\log(x)-xy)dy+2ydx=0$;

Motilal Nehru National Institute of Technology Allahabad
B.Tech.. I Semester
Mathematics-I (Code MA 11101)
End-Semester Examination: 2018-19

Time: 3 Hrs | Max Marks: 60

Note :
- *Each question carries equal marks.*
- *Solve each part of a question in continuation.*
- *Questions 1-4 are compulsory. Attempt any one question from questions 5 and 6.* .

1. (a) State and prove the Lagrange's Mean Value Theorem.
   (b) Test the convergence of the series (i) $\Sigma(1 - n \sin\frac{1}{n})$ and (ii) $\Sigma\frac{e^{n^2}}{(n!)^2}$.
   (c) Define Gamma function. Using Gamma function evaluate the integral $\int_0^1 x^m(\log x)^n dx$.

2. (a) Evaluate $\iint_R (x+y)dxdy$, where R is the trapezoidal region with vertices given by $(0,0)$, $(5,0)$, $(\frac{5}{2},\frac{5}{2})$ and $(\frac{5}{2},-\frac{5}{2})$ using the transformation $x = 2u + 3v$ and $y = 2u - 3v$.
   (b) The Cardioid $r = a(1 + \cos\theta)$ revolves about the initial line. Find the surface and volume of revolution.
   (c) (i) Show that the functions $u = 3x+2y-z, v = x-2y+z$ and $w = x(x+2y-z)$ are functionally related.
   (ii) If $u = \sin^{-1}\left(\frac{x+2y+3z}{x^8+y^8+z^8}\right)$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

3. (a) Find the values of a, b and c such that the maximum value of the directional derivative of $f(x,y,z) = axy^2 + byz + cx^2z^2$ at $(1,-1,1)$ is in the direction parallel to y-axis and has magnitude 6.
   (b) State and prove the Gauss Divergence theorem.
   (c) If $\vec{F} = g\hat{i} - f\hat{j}$, $\vec{T}$ is the unit tangent vector to C , then using Green's theorem show that $\oint_C \vec{F}.\vec{T}ds = \iint_R (\nabla\times\vec{F}).\hat{k}dxdy$.

4. (a) Solve the simultaneous linear differential equations $y'' - y + 5x' = t, 2y' - x'' + 4x = 2$, where x and y are functions of t.
   (b) Find the general solution of the equation $y'' - 2y' + y = xe^x \log x$, using the method of the variation of parameters.
   (c) State the Uniqueness Theorem for the solution of IVP $\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0$. Test the uniqueness of solution of the IVP $y' = 1 + y^2$, $y(0) = 0$ over the rectangle $R = ((x,y) : |x| \leq 10, |y| \leq 1)$. Determine the interval for x, over which solution is guaranteed.

5. (a) Show that the function $f(x,y) = 2x^4 - 3x^2y + y^2$ has niether a maximum nor a minimum at $(0,0)$.
   (b) Evaluate the integral $\iint_S y\,dA$, where S is the portion of the cylinder $x = 6 - y^2$ in the first octant bounded by the planes $x = 0, y = 0, z = 0$ and $z = 8$.

PTO

B.Tech.I Sem End Semester Examination 2009-10
Mathematics-I (MA-101)

Time: $2\frac{1}{2}$ **Hours** MM: 40

Note: Attempt any **FIVE** questions. Each question carries **equal** marks.

1(a) Show that the following function is continuous at (0 , 0)

$$f(x, y) = (x+y)\sin\left(\frac{1}{x+y}\right), \ if\ (x+y) \neq 0 \ \text{and}\ f(x, y) = 0 \ \text{if}\ (x+y) = 0.$$

(b) State Euler's theorem and use it to evaluate

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} \quad \text{if} \quad u(x , y) = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right).$$

(c) If $f(x , y) = 0$, $g(y , z) = 0$, show that $\frac{\partial f}{\partial y}\frac{\partial g}{\partial z}\frac{dz}{dx} = \left(\frac{\partial f}{\partial x}\right)\frac{\partial g}{\partial y}$.

(d) If function u, v, w of the three independent variables x, y, z are not independent, then show that the Jacobian of u, v, w with respect to x, y, z vanishes.

2(a) Show that $\overline{|}\tfrac{1}{2} = \sqrt{\pi}$

(b) By changing the order of integration, evaluate $\int_0^1\int_{2y}^2 e^{x^2}\,dx\,dy$.

(c) Find by double integration , the area lying inside the circle $r = a\sin\theta$ and outside the cardioid $r = a(1-\cos\theta)$.

(d) Show that the sequence $\left\{\frac{n!}{2^n}\right\}$ is not convergent.

3(a) Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at the point (1 , -2 , 1) and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$.

(b) Test for convergence of the series $1 + \frac{2x}{2!} + \frac{3^2x^2}{3!} + \frac{4^3x^3}{4!} + \ldots\ldots, \quad x > 0.$

4(a) If $V = f(r)$, where $r^2 = x^2 + y^2 + z^2$, show that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{d^2V}{dr^2} + \frac{2}{r}\frac{dV}{dr}$

(b) Find the total mass and the centre of gravity of the region bounded by $x^{2/3} + y^{2/3} = a^{2/3}$., $x \geq 0$, $y \geq 0$, when the density is constant k.

5(a) Show that the vector field $\vec{F} = 2x(y^2 + z^3)\,i + 2x^2y\,j + 3x^2z^2\,k$ is conservative field. Find its scalar field potential and the work done in moving a particle from(-1 , 2 , 1) to (2 , 3 , 4).

(b) Prove the identity

$$grad(\vec{A}\bullet\vec{B}) = (\vec{A}\bullet\nabla)\vec{B} + (\vec{B}\bullet\nabla)\vec{A} + \vec{A}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\vec{A})$$

6(a) Verify divergence theorem for vector function $\vec{V} = 4x\,i - 2y^2\,j + z^2\,k$ taken over the region bounded by the cylinder $x^2 + y^2 = 4$, $z = 0$, $z = 3$.

(b) State Stoke's theorem and use it to evaluate $\iint curl\,\vec{V}\bullet\hat{n}\,dS$ where $\vec{V} = y\,i + z\,j + x\,k$ and S is the part of surface of the paraboloid $z = 1 - x^2 - y^2$ for which $z \geq 0$.

B.Tech-I Semester End Semester Examination
(Session: 2011-12).

Subject: Mathematics-I
Time: 2.30 Hrs
Subject Code: MA-101
Maximum Marks: 40

Note: Attempt all questions. Each question carries equal marks.

1. (a) Show that the plane $z=0$ cuts the enveloping cone of the sphere $x^2+y^2+z^2=11$ which has its vertex at (2, 4, 1) in a rectangular hyperbola.
   (b) Define absolute and conditional convergence of a series. Give an example of a series which is conditionally convergent. Prove that the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\cos nx}{n\sqrt{n}}$ converges absolutely.

2. (a) Define Beta and Gamma functions. Using Beta and Gamma functions evaluate the following improper integrals:

   (i) $\int_0^{\infty}\sqrt{x}e^{-x^3}dx$ (ii) $\int_{-1}^{1}(1-x^2)^n dx$, where n is a positive integer

   (b) If $u=\ln(x^3+y^3-x^2y-xy^2)$, then show that $u_{xx}+2u_{xy}+u_{yy}=-\frac{4}{(x+y)^2}$.

3. (a) Expand $f(x,y)=21+x-20y+4x^2+xy+6y^2$ in Taylor's series of maximum order about the point (-1, 2).
   (b) The temperature T at any point (x,y,z) in space is $T(x,y,z)=k\,x\,y\,z^2$, where k is a constant. Find the highest temperature on the surface of the sphere $x^2+y^2+z^2=a^2$.

4. (a) Evaluate $\int_{-2}^{1}\int_{x^2+4x}^{3x+2}dy\,dx$ by changing the order of integration.
   (b) Define directional derivative of a function in a given direction. Find the directional derivative of the function $f(x,y,z)=x^2-y^2+2z^2$ at the point P(1, 2, 3) in the direction of the line PQ, where Q is the point (5, 0, 4). Also, find the magnitude of the maximum value of the directional derivative.

5. (a) Prove the following identities:

   (i) $curl(f\,\overline{v})=(grad\,f)\times\overline{v}+f(curl\,\overline{v})$ (ii) $curl\,(curl\,\overline{v})=\nabla(\nabla.\overline{v})-\nabla^2\,\overline{v}$

   (b) State Green's theorem in a plane. Using Green's theorem, evaluate $\oint_c(x^2+y^2)dx+(y+2x)dy$, where c is the boundary of the region in the first quadrant that is bounded by the curves $y^2=x$ and $x^2=y$.

End Term Examination, Odd Semester, Academic Session 2012-13

Programme: B.Tech I year Subject: Mathematics-I (MA-1101)

Duration: 2.30 Hrs. Date: 06-12-12 M.M. 40

Note: Attempt **All** questions. Each question carries equal marks.

1(a): Prove that every convergent sequence has unique limit.

1(b): Explain Rolle's theorem. Also state the conditions when Rolle's theorem does not hold good.

1(c): Define Cauchy sequence and

test the convergence of $1+\frac{3}{7}x+\frac{3\cdot6}{7\cdot10}x^2+\frac{3\cdot6\cdot9}{7\cdot10\cdot13}x^3+\frac{3\cdot6\cdot9\cdot12}{7\cdot10\cdot13\cdot16}x^4+\cdots$

2(a): Let $f(x,y)=\begin{cases}\frac{x^4y-3x^2y^3+y^5}{(x^2+y^2)^2}; & \text{if } (x,y)\neq(0,0)\\ 0 \quad ; & \text{if } (x,y)=(0,0)\end{cases}$

Find a $\delta>0$ such that $|f(x,y)-f(0,0)|<0\cdot01$, whenever $\sqrt{x^2+y^2}<\delta$.

2(b): Find the dimensions of a rectangular parallelepiped of maximum volume with edges parallel to the coordinate axes that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

3(a): The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes in A, B and C. Apply Dirichlet's integrals to find the volume of the tetrahedron OABC. Also find its mass if the density at any point is $kxyz$.

3(b): Evaluate the integral $\iint_R (x-y)^2\cos^2(x+y)\,dx\,dy$ using suitable transformation of variables, where **R** is the rhombus with successive vertices at $(\pi,\ 0)$, $(2\pi,\ \pi)$, $(\pi, 2\pi)$ and $(0,\ \pi)$.

4: State Gauss Divergence theorem. Verify Divergence theorem for $\iint_S \vec{F}\cdot\hat{n}\ ds$, where $\vec{F}=2xy\,\hat{i}+yz^2\,\hat{j}+xz\,\hat{k}$ and S is the surface of the region bounded by $x=0$, $y=0$, $y=3$, $z=0$ and $x+2z=6$.

5(a): State Existence and Uniqueness theorem for a first order ordinary differential equations. Also solve $3x^2y^2+\cos(xy)-xy\sin(xy)+\{2x^3y-x^2\sin(xy)\}\frac{dy}{dx}=0$.

5(b): Solve $\frac{d^2y}{dx^2}+a^2y=cosec\,(ax)$ using variation of parameters.

Motilal Nehru National Institute of Technology Allahabad
I B.Tech I Semester End Examinations, November 2017.

MATHEMATICS-I

(Common to all Branches)

Time: 3 hours

Max Marks: 60

Answer all questions
All questions carry equal marks

*****

1 a) Find the interval of convergence of the series $\frac{x}{1}-\frac{x^2}{\sqrt{2}}+\frac{x^3}{\sqrt{3}}-\frac{x^4}{\sqrt{4}}+\ldots\ldots$

b) State and prove the Rolle's Theorem and verify the theorem for $f(x)=x(x+3)e^{-\frac{1}{2}x}$ in $(-3,0)$.

2 a) If $u=f(r), where\ r^2=x^2+y^2$, prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$

b) Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

3 a) Define the Dirichlet Integrals. Hence evaluate the Dirichlet integral $I=\iiint_T x^{1/2}y^{1/2}z^{1/2}\,dxdydz$ where T is the region in the first octant bounded by the plane $x+y+z=1$ and the coordinate planes.

b) Change the order of integration in $I=\int_0^1\int_{x^2}^{2-x} xy\,dx\,dy$ and hence evaluate the same.

4 a) Define Beta and Gamma functions and hence show that $\Gamma\left(n+\frac{1}{2}\right)=\frac{\sqrt{\pi}\,\Gamma(2n+1)}{2^{2n}\Gamma(n+1)}$.

b) Evaluate $\int_0^\infty \frac{e^{-\alpha x}\sin x}{x}dx,\ \alpha>0$ and hence deduce that

i) $\int_0^\infty \frac{\sin x}{x}dx=\frac{\pi}{2}$ and ii) $\int_0^\infty \frac{\sin ax}{x}dx=\frac{\pi}{2},\ a>0$.

5 State Gauss divergence theorem. Verify the divergence theorem for $\vec{F}=2x^2y\,\vec{i}-y^2\,\vec{j}+4xz^2\,\vec{k}$ taken over the region in the first octant bounded by a cylinder $y^2+z^2=9$ and the plane $x=2$.

6 a) Solve $\left(D^3+2D^2+D\right)y=x^2e^{2x}+\sin^2 x$

b) Solve $\frac{d^2y}{dx^2}+4y=4\sec^2 2x$ using the method of variation of parameters.

function

Motilal Nehru National Institute of Technology, Allahabad

Department of Mathematics

B. Tech 1st Semester Tutorial Questions

Subject: MA1101: Mathematics-I   Unit 1 : Infinite Series & Mean Value Theorems

1. Test the convergence of the series: $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+.....$ Ans: Convergent(cgt)
2. Test the convergence of the series $1+\frac{2}{5}x+\frac{6}{9}x^2+........+\frac{2^n-2}{2^n+1}x^{n-1}+..........(x>0)$. Ans:: Cgt: $cgt: x<1, dgt: x\geq 1$
3. Test the convergence of the following series: i). $\sum \frac{n!2^n}{n^n}$ ii) $\sum \frac{1}{n}\log\frac{1}{n}$

   iii) $\sum\left(\frac{n}{n+1}\right)^{n^2}$
4. Prove that a positive term series $\sum u_n$ convergences or divergence according to whether $\underset{n\to\infty}{Lt}\, n\log\left(\frac{u_n}{u_{n+1}}\right)>1 or <1$
5. Find whether the series $\sum_{n=1}^{\infty}(-1)^n\frac{\log n}{n^2}$ convergence absolutely or conditional convergence.
6. Discuss the convergence of the series: $1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^2}x^3+...............(x>0)$
7. Discuss the convergence of the series: $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p}, (p>0)$
8. Verify Rolle's theorem for the function $f(x)=\frac{\sin x}{e^x}$ in$[0,\pi]$
9. Verify Rolle's theorem for the function $\log\left[\frac{x^2+ab}{x(a+b)}\right]$ in $[a,b]$, $a>0, b>0$
10. Verify Rolle's theorem for $f(x)=\frac{x^2-x-6}{x-1}$ in the interval (-2,3)
11. Verify Rolle's theorem for $f(x)=x(x+3)e^{-x/2}$ in the interval [-3,0]
12. Use us Rolle's theorem ,show that $g(x)=8x^3-6x^2-2x+1$ has a zero between 0 and 1.

Motilal Nehru National Institute of Technology, Allahabad
End Semester Examination- Session: 2014-15
B.Tech. I Semester
Mathematics I (Code- MA 1101)

2014
D-3

Time: 3 Hrs Maximum Marks: 60

Note : Solve all questions. All the questions carry equal marks.

1. (a) Find the linear Taylor series polynomial approximation to the function $f(x,y) = 2x^3 + 3y^3 - 4x^2y$ about the point $(1,2)$. Obtain the maximum absolute error in the region $|x-1| \leq 0.01$ and $|y-2| \leq 0.1$. 3 Marks

(b) Show that the function $f(x,y) = \begin{cases} (x+y)sin(\frac{1}{x+y}), & x+y \neq 0 \\ 0, & x+y = 0 \end{cases}$ 4 Marks

is continuous at $(0,0)$ but $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ do not exist.

(c) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{log^2 n}{n^{3/4}}$ 3 Marks

2. (a) Find the values of constants a, b, c such that the maximum value of the directional deivative of $f(x,y,z) = axy^2 + byz + cz^2x^2$ at $(1,-1,1)$ is in the direction parallel to the axis of y and has magnitude 6. 4 Marks

(b) State Stoke's theorem and verify this theorem for the vector field $\overrightarrow{v} = (3x-y)i - 2yz^2j - 2y^2zk$, where S is the surface of the sphere $x^2+y^2+z^2 = 16$, $z > 0$. 6 Marks

OR

(b) State and prove Gauss Divergence theorem.

3. (a) Evaluate $\int\int\int sin(x+y+z)\ dx\ dy\ dz$ over portion cut off by the plane $x+y+z = \pi$ 4 Marks

(b) If $z = f(x,y)$, $x = rcos\theta, y = rsin\theta$, then show that 3 Marks

$$(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2 = (\frac{\partial f}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial f}{\partial \theta})^2$$

(c) Solve the system of differential equation 3 Marks

$$(2D-4)y_1 + (3D+5)y_2 = 3t+2$$
$$(D-2)y_1 + (D+1)y_2 = t$$

4. (a) Show that $\beta(m+\frac{1}{2}, m+\frac{1}{2}) = \frac{\pi}{m2^{4m-1}\beta(m,m)}$. 3 Marks

(b) State Leibniz formula. Use it to evaluate the integral $\int_0^\infty \frac{tan^{-1}(ax)}{x(1+x^2)}dx$, $a > 0, a \neq 1$. 3 Marks

(c) Evaluate $\int\int_R \ (x-y)^2 cos^2(x+y)\ dx\ dy$, where R is the rhombus with succesive vertices given by $(1,0), (2,1), (1,2)$ and $(0,1)$. 4 Marks

5. (a) Define Gamma function. Show that $\Gamma(1/2) = \sqrt{\pi}$. 3 Marks

(b) Show that the vector field $\overrightarrow{A} = 3x^2y\mathrm{i} + (x^3 - 2yz^2)\mathrm{j} + (3z^2 - 2y^2z)\mathrm{k}$ is irrotational but not solenoidal. Also find its scalar potential. 4 Marks

(c) Solve the differential equation $y'' + 4y' + 29y = e^{-2x} sin5x$. 3 Marks

6. (a) State uniqueness theorem for solution of an initial value problem $y' = f(x,y), \quad y(x_0) = y_0$. Test the existence and uniqueness of IVP $y' = \sqrt{y}, \ y(1) = 0$ in a suitable rectangle R. If more than one solution exists, find all of them. 3 Marks

(b) By the method of variation of parameters only solve the differential equation $y''' - 6y'' + 11y' - 6y = e^{-x}$. 4 Marks

(c) Solve the differential equation $(1 + e^{x/y})dx + e^{x/y}(1 - (x/y))dy = 0$. 3 Marks

मोतीलाल नेहरू राष्ट्रीय प्रौद्योगिकी संस्थान
इलाहाबाद 211004

Department of Mathematics
Motilal Nehru National Institute of Technology, Allahabad

End Semester Examination-(Odd Sem 2015)
Mathematics I (B.Tech. I Sem): MA1101

Time: 3 hours M.M.: 60.

Note: Attempt all questions. Each question carries equal marks.

Q. 1. Find the interval of convergence of the series $x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \cdots \cdots$

Q. 2. If $u = f(x, y, z)$ and $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, then show that

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2 = \left(\frac{\partial f}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2 + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial f}{\partial \phi}\right)^2$$

Q. 3. State and prove Taylor's theorem for the expansion of functions of several variables. Find the quadratic Taylor's series polynomial approximations to the function $f(x, y) = 2x^3 + 3y^3 - 4x^2y$ about the point$(1, 2)$. Obtain the maximum absolute error in the region $|x - 1| < 0.01$ and $|y - 2| < 0.1$.

Q. 4. Examine for maximum and minimum values of $\sin x + \sin y + \sin(x + y)$ and find the minimum value of$x^2 + y^2 + z^2$, subject to $ax + by + cz = p$.

Q. 5. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

Q. 6. State and prove Green's theorem and verify it for the $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region defined by $y = \sqrt{x}$, $y = x^2$.

Q. 7. State Gauss divergence theorem and apply to evaluate $\int_S (a^2x^2 + b^2y^2 + c^2z^2)^{-1/2}\, dS$, where, S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$.

Q. 8. Show that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ and express $\int_0^1 x^m(1 - x^p)^n\, dx$ in terms of Beta function and hence evaluate the integral $\int_0^1 x^{3/2}(1 - \sqrt{x})^{1/2}$.

Q. 9. (a) Solve $(1 + y^2)dx = (\tan^{-1} y - x)dy$ (b) Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \sin(\log x)$.
(c) Find the solution of $y'' + 16y = 32\sec 2x$, using the method variation of parameters

Q. 10. (a) Find the solution of $y'' - 4y' + 13y = 12e^{2x}\sin 3x$ using method of undetermined coefficients
(b). Find the power series solution of $(1 - x^2)y'' - 2xy' + 2y = 0$, about $x = 0$.

Subject code: MA1101

Motilal Nehru National Institute of Technology Allahabad

I B.Tech I Semester End Examinations, December 2013.

MATHEMATICS-I

(Common to all Branches)

Time: 3 hours Max Marks: 60

Answer any SIX questions

All questions carry equal marks

*****

1 a) Discuss the convergence of the series $\frac{x}{1}+\frac{1}{2}\frac{x^3}{3}+\frac{1.3}{2.4}\frac{x^5}{5}+\frac{1.3.5}{2.4.6}\frac{x^7}{7}+......(x>0)$.

b) Solve the following simultaneous equations: $\frac{d^2x}{dt^2}-\frac{dy}{dt}=2x+2t,\ \frac{dx}{dt}+4\frac{dy}{dt}=3y$.

2 a) Evaluate the triple integral $\iiint_V \frac{dxdydz}{(x+y+z)^3}$ taken over the volume bounded by the planes $x=0, y=0, z=0$ and the plane $x+y+z=1$.

b) Find the centre of gravity of a plate whose density $\rho(x,y)$ is constant and is bounded by the curves $y=x^2$ and $y=x+2$. Also, find the moments of inertia about the axes.

3 a) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

b) Prove that $\nabla^2 f(r)=\frac{d^2f}{dr^2}+\frac{2}{r}\frac{df}{dr}$. Also, find $f(r)$ such that $\nabla^2 f(r)=0$.

4 State Gauss divergence theorem. Verify the divergence theorem for $\bar{F}=2x^2y\,\bar{i}-y^2\,\bar{j}+4xz^2\,\bar{k}$ taken over the region in the first octant bounded by a cylinder $y^2+z^2=9$ and the plane $x=2$.

5 a) Express $\int_0^1 x^m(1-x^p)^n dx$ in terms of Beta function and hence evaluate the integral $\int_0^1 x^{3/2}(1-\sqrt{x})^{1/2}dx$.

b) Calculate the length of the arc of the cycloid $x=a(\theta-\sin\theta),\ y=a(1-\cos\theta)$.

6 a) Solve the differential equation $(3x^2y^3e^y+y^3+y^2)dx+(x^3y^3e^y-xy)dy=0$.

b) Solve the differential equation $y'''+2y''+y'=e^{2x}+x^2+x+\sin 2x$.

7 a) Using the result $\int_0^\infty e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$, evaluate the integral $\int_0^\infty e^{-x^2}\cos(2\alpha x)dx$. (3M)

b) If $u+v=e^x\cos y$ and $u-v=e^x\sin y$, then find the Jacobian of the functions u and v with respect to x and y. (4M)

c) State the Leibnitz's theorem. Also, test the convergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\sqrt{n+1}-\sqrt{n}\right)$. (3M)

8 a) If $u=\log(x^3+y^3+z^3-3xyz)$, then prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$ and hence show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=-\frac{9}{(x+y+z)^2}$.

b) State the Lagranges mean value theorem and hence prove that $0<[\log(1+x)]^{-1}-x^{-1}<1$.