8.2.24

Theory of Relativity

Frame of Reference :- It is a 3D coordinate system which is used to analyse the motion of an object.

Types :-

1. Inertial frame of reference :- All such frame where newton's law of motion is valid or frame which is moving with constant velocity or at rest

2. Non Inertial frame of reference :- All such frame from where newton's law of motion is not valid or frame which is moving with accelerated velocity

Michaelson Morley Experiment :-

Obj :- To determine the velocity of earth w.r.t a medium which is considered to be at rest always

Our Earth is surrounded by a gaseous medium called Ether.

Property of ether :-
1. massless
2. rigid
3. transparent
4. Invisible
5. insensitive (means doesn't affect the motion of earth)

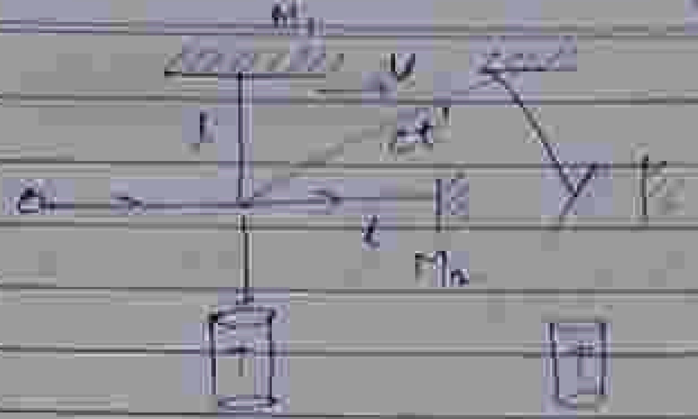


Fig: Michaelson Morley setup

$$t_1 = \frac{l}{c-v} + \frac{l}{c+v} = \frac{lc+lv+lc-lv}{c^2-v^2}$$

$$t_1 = \frac{2lc}{c^2-v^2} = \frac{2lc}{c^2\left(1-\frac{v^2}{c^2}\right)} = \frac{2l}{c\left(1-\frac{v^2}{c^2}\right)}$$

Applying Pythagoras theorem

$$\Rightarrow (l)^2 + (vt')^2 = (ct')^2$$

$$\Rightarrow (t')^2(c^2-v^2) = l^2$$

$$\Rightarrow (t') = \frac{l}{(c^2-v^2)^{1/2}}$$

$$t_2 = 2t' = \frac{2l}{(c^2-v^2)^{1/2}} = \frac{2l}{c\left(1-\frac{v^2}{c^2}\right)^{1/2}}$$

$$\Delta t = t_1 - t_2 = \frac{2l}{c}\left(1-\frac{v^2}{c^2}\right)^{-1} - \frac{2l}{c}\left(1-\frac{v^2}{c^2}\right)^{-1/2}$$

$$\Delta t = \frac{2l}{c}\left(\cancel{1}+\frac{v^2}{c^2} - \cancel{1} - \frac{v^2}{2c^2}\right) = \frac{2lv^2}{2c^3} = \frac{lv^2}{c^3}$$

Path difference, $\Delta x = c\,\Delta t$

$$\Rightarrow \Delta x = \frac{l v^2}{c^2}$$

For constructive interference, Path difference $= n\lambda$

$$\Rightarrow \quad n\lambda = \frac{l v^2}{c^2} \quad - (1)$$

If the setup is rotated by 90° the reflected & transmitted beams get interchanged & the path difference of $\frac{lv^2}{c^2}$ will be produced in the opposite direction.

Net path difference $= \frac{2 l v^2}{c^2}$

Now, $n\lambda = \frac{2 l v^2}{c^2}$

$$\Rightarrow n = \frac{2 l v^2}{c^2 \cdot \lambda}$$

Putting, $c = 3\times10^8$ m/s, $v = 3\times10^4$ m/s, $l = 11$ m, $\lambda = 5.5\times10^{-7}$ m

$$\Rightarrow n = \frac{2\times11\times9\times10^{8}}{9\times10^{16}\times5.5\times10^{-7}}$$

$$\Rightarrow n = \frac{2\times11\times9\times10}{9\times5.5\times10}$$

$$\Rightarrow n = 0.4$$

Theoretically there was a shift in fringes but there was no shift of fringe is observed.

Explanation of negative result of michaelson morley expt :-

(1) Constancy of Speed of light

⇒ If speed of light would remain same in every direction i.e. c. Hence there will no any time difference. Hence there would be no any shift of fringe.

(2) Ether Drag Hypothesis

⇒ As per this hypothesis, Ether is also carried by Earth with its own velocity. Hence there would be no any relative motion b/w the Earth. Therefore no shift of fringe is obtained.

(3) Lorentz Fitzgerald contraction hypothesis :-

$L = L_0\sqrt{1 - v^2/c^2}$

$$t_1 = \frac{2l}{c}\left(1 + \frac{v^2}{c^2}\right) = \frac{2L_0\sqrt{1 - v^2/c^2}}{c}\left(1 + \frac{v^2}{c^2}\right)$$

$$t_1 = \frac{2L_0}{c}\left(1 + \frac{v^2}{2c^2}\right) = t_2$$ {ignoring higher powers of v/c}

Hence there is no time difference
Hence there shouldn't be any shift of fringes

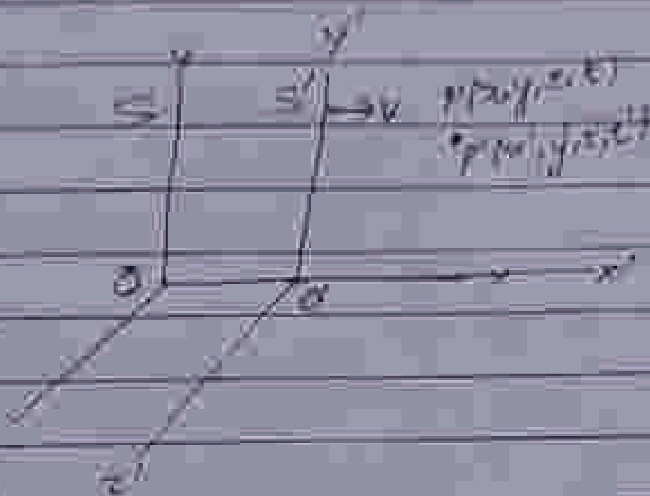
(4) Postulates of special theory of relativity

(i) All the fundamental laws of physics remain same in every inertial frame

(ii) The velocity of light remain constant in every direction.

(5) Conclusion :- (i) The velocity of light is constant in all direction
(ii) The concept of making universal frame of reference is meaningless
(iii) A new theory with different concept of space, mass & time is required

(4) Galilean Transformation



$x' = x - vt$

$y' = y$

$z' = z$

$t' = t$

Galilean reverse transformation

$x = x' + vt$

$y = y'$

$z = z'$

$t' = t$

on differentiating

$x' = x - vt \quad \text{--- (i)}$

$\Rightarrow \frac{dx'}{dt} = \frac{dx}{dt} - \frac{d(vt)}{dt}$

$\Rightarrow \boxed{u_x' = u_x - v}$

$y' = y \quad \text{--- (ii)} \qquad z' = z$

$\Rightarrow \frac{dy'}{dt} = \frac{dy}{dt} \qquad \Rightarrow \frac{dz'}{dt} = \frac{dz}{dt}$

$\Rightarrow \boxed{u_y' = u_y} \qquad \Rightarrow \boxed{u_z' = u_z}$

again differentiating

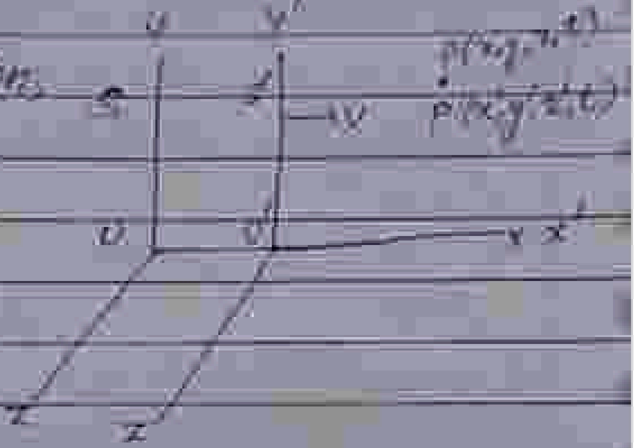
$\Rightarrow a_x' = a_x$

$a_y' = a_y$

$a_z' = a_z$.

④ **Lorentz transformation:-**

Using galilean transformation



$x' = K(x - vt)$ — (i)

$x = K(x' + vt')$ — (ii)

$\Rightarrow x = K[K(x - vt) + vt']$

$\Rightarrow \frac{x}{K} = Kx - Kvt + vt'$

$\Rightarrow vt' = \frac{x}{K} + Kvt - Kx$

$\Rightarrow t' = \frac{x}{Kv} + Kt - \frac{Kx}{v}$

$\Rightarrow t' = Kt - \frac{Kx}{v}\left(1 - \frac{1}{K^2}\right)$

we know $(x = ct)$ — (iii)

$x' = ct'$ — (iv)

$\Rightarrow ct = K(x' + vt')$

$\Rightarrow ct = K(ct' + vt')$

$\Rightarrow ct = Kt'(c + v)$ —— (v)

$\Rightarrow ct' = K(x - vt)$

$\Rightarrow ct' = K(ct - vt)$

$\Rightarrow ct' = Kt(c - v)$ — (vi)

Multiplying (v) & (vi)

$\Rightarrow c^2 tt' = K^2 t t'(c^2 - v^2)$

$\Rightarrow K^2 = \frac{c^2}{c^2 - v^2}$

$$\Rightarrow 1-\frac{v^2}{c^2}=\frac{1}{K^2}$$

$$\Rightarrow\left(1-\frac{1}{K^2}\right)=\frac{v^2}{c^2} \quad \text{--- (vii)} \qquad K=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Using (vii)

$$\Rightarrow t'=Kt-\frac{Kx}{v}\left(\frac{v^2}{c^2}\right)$$

$$\Rightarrow t'=Kt-\frac{Kxv}{c^2}$$

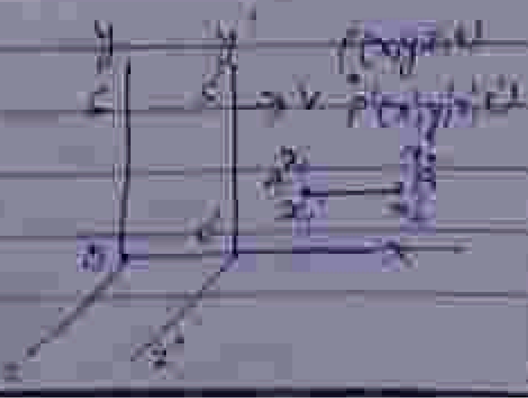
$$\Rightarrow \boxed{t'=\frac{t-\frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$\boxed{x'=\frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}}, \quad y'=y, \quad z'=z$$

Inverse Lorentz transformation

$$\boxed{x=\frac{x'+vt'}{\sqrt{1-\frac{v^2}{c^2}}}}, \quad y=y', \quad z=z', \quad \boxed{t=\frac{t'+\frac{x'v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}}$$

(4) Length Contraction



$$l_0=x_2'-x_1'$$

$$l=x_2-x_1$$

Using Lorentz transformation

$$x_1' = \frac{x_1 - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$x_2' = \frac{x_2 - vt}{\sqrt{1-v^2/c^2}}$$

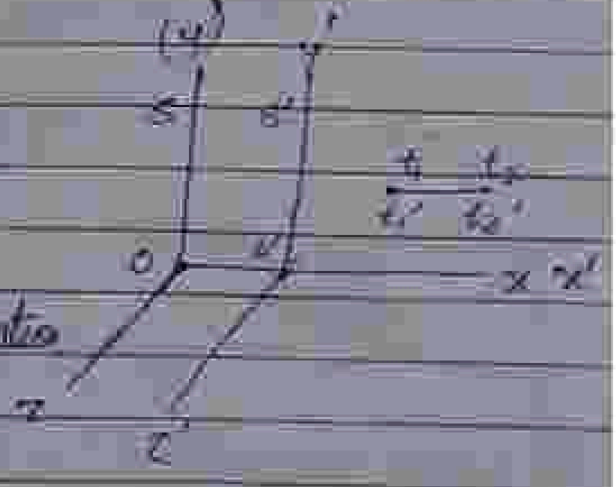
On substracting

$$x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1-v^2/c^2}}$$

$$\Rightarrow \quad \boxed{L = L_0 \sqrt{1-v^2/c^2}}$$

Length get contracted by a factor of $\sqrt{1-v^2/c^2}$ in the direction of motion.

(#) Time Dilation



$$t_2' - t_1' = t_0$$
$$t_2 - t_1 = t$$

Applying Lorentz transformation

$$t_1 = \frac{t_1' + \frac{vx'}{c^2}}{\sqrt{1-v^2/c^2}}$$

$$t_2 = \frac{t_2' + \frac{vx'}{c^2}}{\sqrt{1-v^2/c^2}}$$

On subtracting

$$t_2 - t_1 = \frac{t_2' - t_1'}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow \boxed{t = \frac{t_0}{\sqrt{1 - v^2/c^2}}}$$

Hence time get dilated i.e clock at rest moves slower than the clock moving with velocity v

(iii) Simultanity in relativity:

Applying Lorentz transformation.

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$t_1' = \frac{t_0 - \frac{vx_1}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$t_2' = \frac{t_0 - \frac{vx_2}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$t_1' - t_2' = \frac{(x_2 - x_1)\frac{v}{c^2}}{\sqrt{1 - v^2/c^2}} \Rightarrow \boxed{\Delta t' = \frac{(x_2 - x_1)\, v/c^2}{\sqrt{1 - v^2/c^2}}}$$

→ Observer O observes the glow of bulb simultaneously, observer O' observes the glow of bulb at different time.

When $x_2 = x_1 = x$

$\Delta t' = 0$ — When the bulb is placed at one x

$t_1' - t_2' = 0$ — then that event is simultaneous

$\Rightarrow t_1' = t_2'$ — event if it is moving frame

A event which is simultaneous in one frame may be simultaneous or non-simultaneous w.r.t other frame

④ Addition of Velocities :-

From Inverse Lorentz transformation, we have

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}, \quad y = y', \quad z = z', \quad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - v^2/c^2}} \quad \text{— (i)}$$

On differentiating these equations we get

$$dx = \frac{dx' + v\,dt'}{\sqrt{1 - v^2/c^2}}, \quad dy = dy', \quad dz = dz', \quad dt = \frac{dt' + \frac{v\,dx'}{c^2}}{\sqrt{1 - v^2/c^2}} \quad \text{— (ii)}$$

From (i) & (ii) we have

$$u_x = \frac{dx}{dt} = \frac{dx' + v\,dt'}{dt' + \frac{v\,dx'}{c^2}} = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}}$$

$$\boxed{u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}}}$$

Similarly,

$$u_y = \frac{dy}{dt} = \frac{dy'\sqrt{1-v^2/c^2}}{dt' + \frac{v\,dx'}{c^2}} = \frac{\frac{dy'}{dt'}\sqrt{1-v^2/c^2}}{1+\frac{v\,dx'}{c^2 dt'}}$$

$$\boxed{u_y = \frac{u_y'\sqrt{1-v^2/c^2}}{1+\frac{v u_x'}{c^2}}}$$

Similarly, $$\boxed{u_z = \frac{u_z'\sqrt{1-v^2/c^2}}{1+\frac{v u_x'}{c^2}}}$$

(1) If $u_x' = c$

$$u_x = \frac{c+v}{1+\frac{vc}{c^2}} = \frac{(c+v)c}{c+v} = c$$

(2) If $v = c$

$$u_x = \frac{u_x'+c}{1+\frac{c u_x'}{c^2}} = \frac{(u_x'+c)c}{(u_x'+c)} = c$$

(3) If $v = c$, $u_x' = c$

$$u_x = \frac{c+c}{1+\frac{c^2}{c^2}} = \frac{2c}{2} = c$$

(4) Prove:- $$\sqrt{1-\frac{(u')^2}{c^2}} = \frac{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-u^2/c^2}}{1-\frac{vu_x}{c^2}}$$

⇒ A/c to addition of velocities.

$$u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}, \quad u_y' = \frac{u_y \sqrt{1 - v^2/c^2}}{1 - \frac{v u_x}{c^2}}, \quad u_z' = \frac{u_z \sqrt{1 - v^2/c^2}}{1 - \frac{v u_x}{c^2}}$$

$$(u')^2 = (u_x')^2 + (u_y')^2 + (u_z')^2$$

$$= \frac{(u_x - v)^2 + (u_y^2 + u_z^2)(1 - v^2/c^2)}{\left(1 - \frac{v u_x}{c^2}\right)^2}$$

$$\Rightarrow \frac{u_x^2 + v^2 - 2u_x v + (u^2 - u_x^2)(1 - v^2/c^2)}{\left(1 - \frac{v u_x}{c^2}\right)^2}$$

$$\Rightarrow \frac{\cancel{u_x^2} + v^2 - 2u_x v + u^2 - \frac{u^2 v^2}{c^2} - \cancel{u_x^2} + \frac{u_x^2 v^2}{c^2}}{\left(1 - \frac{v u_x}{c^2}\right)^2}$$

$$1 - \frac{(u')^2}{c^2} \Rightarrow 1 - \frac{\left(\frac{v^2}{c^2} - \frac{2u_x v}{c^2} + \frac{u^2}{c^2} - \frac{u^2 v^2}{c^4} + \frac{u_x^2 v^2}{c^4}\right)}{\left(1 - \frac{v u_x}{c^2}\right)^2}$$

$$\Rightarrow \frac{1 + \cancel{\frac{v^2 u_x^2}{c^4}} - \cancel{\frac{2 v u_x}{c^2}} - \frac{v^2}{c^2} + \cancel{\frac{2u_x v}{c^2}} - \frac{u^2}{c^2} + \frac{u^2 v^2}{c^4} - \cancel{\frac{u_x^2 v^2}{c^4}}}{\left(1 - \frac{v u_x}{c^2}\right)^2}$$

$$\Rightarrow \frac{\left(1 - \frac{v^2}{c^2}\right) - \frac{u^2}{c^2}\left(1 - \frac{v^2}{c^2}\right)}{\left(1 - \frac{v u_x}{c^2}\right)^2}$$

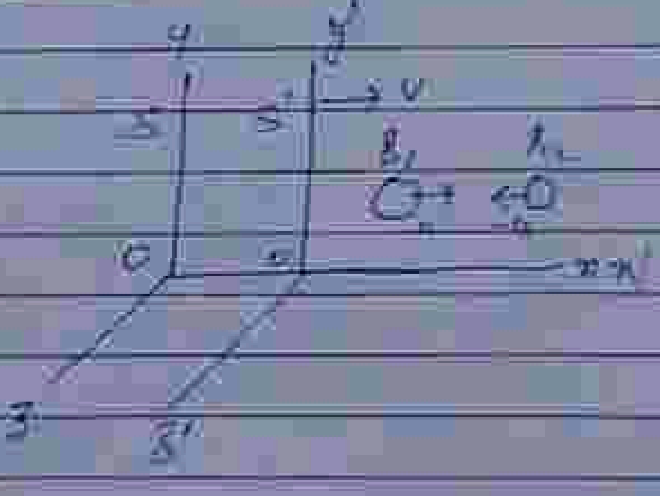
Square root both side.

$$\sqrt{1-\left(\frac{u'}{c}\right)^2} = \frac{\sqrt{1-\frac{u^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}}}{\left(1-\frac{vu_x}{c^2}\right)}$$

Its inverse form

$$\sqrt{1-\frac{u^2}{c^2}} = \frac{\sqrt{1-\frac{(u')^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}}}{\left(1+\frac{vu_x}{c^2}\right)}$$

Variation of mass with velocity



By C.L.M

$\Rightarrow m(u) - mu) = 0$

$$u_1 = \frac{u+v}{1+uv/c^2} \quad -(i)$$

$$u_2 = \frac{-u+v}{1-uv/c^2} \quad -(ii)$$

Again applying C.L.M

$\Rightarrow \quad m_1 u_1 + m_2 u_2 = (m_1 + m_2)V$

on substituting

$$m_1\left(\frac{u+v}{1+uv/c^2}\right) + m_2\left(\frac{-u+v}{1-\frac{uv}{c^2}}\right) = (m_1+m_2)V$$

$$\Rightarrow m_1\left(\frac{u+v}{1+uv/c^2}\right) - m_1 v = m_2 v - m_2\left(\frac{-u+v}{1-uv/c^2}\right)$$

$$\Rightarrow m_1\left(\frac{u+v}{1+\frac{uv}{c^2}} - v\right) = m_2\left(v - \left(\frac{-u+v}{1-\frac{uv}{c^2}}\right)\right)$$

$$\Rightarrow m_1\frac{\left(u+\cancel{v}-\cancel{v}-\frac{uv^2}{c^2}\right)}{1+\frac{uv}{c^2}} = m_2\frac{\left(\cancel{v}-\frac{uv^2}{c^2}+u-\cancel{v}\right)}{\left(1-\frac{uv}{c^2}\right)}$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{1+uv/c^2}{1-uv/c^2}$$

From Eq^n (1)

$$1-\frac{u_1^2}{c^2} = 1-\frac{\left(\frac{u+v}{c}\right)^2}{\left(1+\frac{uv}{c^2}\right)^2} = \frac{1+\frac{u^2v^2}{c^4}+\cancel{\frac{2uv}{c^2}}-\frac{u^2}{c^2}-\frac{v^2}{c^2}-\cancel{\frac{2uv}{c^2}}}{\left(1+\frac{uv}{c^2}\right)^2}$$

$$\Rightarrow 1-\frac{u_1^2}{c^2} = \frac{\left(1-\frac{u^2}{c^2}\right)-\frac{v^2}{c^2}\left(1-\frac{u^2}{c^2}\right)}{\left(1+\frac{uv}{c^2}\right)^2}$$

$$\Rightarrow 1-\frac{u_1^2}{c^2} = \frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)}{\left(1+\frac{uv}{c^2}\right)^2}$$

$$\Rightarrow \left(1+\frac{uv}{c^2}\right) = \frac{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}}{\sqrt{1-\frac{u_1^2}{c^2}}} \quad \text{--- (iii)}$$

Similarly

$$\left(1 - \frac{uv}{c^2}\right) = \frac{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}}{\sqrt{1-\frac{u_2^2}{c^2}}} \quad \text{--- (iv)}$$

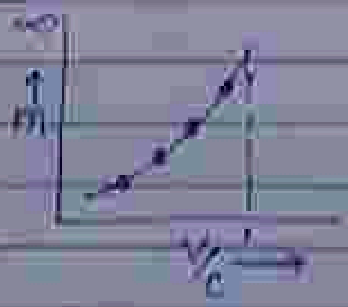
Using (iii) & (iv)

$$\frac{m_1}{m_2} = \frac{\sqrt{1-u_2^2/c^2}}{\sqrt{1-u_1^2/c^2}}$$

$$\Rightarrow \quad m_1\sqrt{1-u_1^2/c^2} = m_2\sqrt{1-u_2^2/c^2} = m_0$$

$$\Rightarrow \quad \boxed{m = \frac{m_0}{\sqrt{1-v^2/c^2}}}$$

As $\frac{v}{c} \to 1$, $m \to \infty$



Einstein's Mass Energy Relation

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt}$$

$$dW = dE_K = F\,dx$$

$$dE_K = \frac{d(mv)}{dt}\,dx$$

$$\Rightarrow \quad dE_K = v\,d(mv) = v[v\,dm + m\,dv] \quad \text{--- (i)}$$

$$\Rightarrow \quad dE_K = v^2 dm + mv\,dv$$

We have,

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

$$\Rightarrow m^2 = \frac{m_0^2}{1-v^2/c^2}$$

$$\Rightarrow m^2\left(1-v^2/c^2\right) = m_0^2$$

$$\Rightarrow m^2(c^2-v^2) = m_0^2 c^2$$

$$\Rightarrow 2mc^2 dm - 2mv^2 dm - 2m^2 v\,dv = 0$$

$$\Rightarrow c^2 dm = v^2 dm + mv\,dv \quad \text{—— (ii)}$$

Using (i) & (ii)

$$\int_0^{E_k} dE_K = c^2 \int_{m_0}^{m} dm$$

$$\Rightarrow E_k = (m-m_0)c^2$$

$$\Rightarrow E_K = mc^2 - m_0c^2$$

on Comparing

$$KE = E - PE$$

$\underline{E = mc^2}$. $PE = m_0c^2$ (rest mass Energy)

(iii) Deduce, $E^2 - P^2c^2 = m_0^2c^4$

Sol^n:- $E = mc^2 = \frac{m_0}{\sqrt{1-v^2/c^2}}\, c^2$

$$\Rightarrow E = \frac{m_0}{\sqrt{1-\frac{m^2c^2v^2}{m^2c^4}}}\, c^2 = \frac{m_0}{\sqrt{1-\frac{P^2c^2}{E^2}}}\, c^2$$

(22)

On squaring,

$$\Rightarrow E^2\left(1 - \frac{P^2C^2}{E^2}\right) = m_0^2C^4$$

$$\Rightarrow E^2 - P^2C^2 = m_0^2C^4$$

↳ Since this is a constant quantity

Hence $E^2 - P^2C^2$ is invariant.

For a particle of mass $m_0 = 0$ (Photon)

$$E^2 = P^2C^2 \Rightarrow E = PC \Rightarrow P = \frac{E}{C} = \frac{hc}{C\lambda} \Rightarrow \boxed{P = \frac{h}{\lambda}}$$

(4) Transformation of momentum & Energy:-

(i) For Momentum

$$P = mu$$

$$\Rightarrow P = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}\left(\frac{u'+v}{1+\frac{u'v}{c^2}}\right)$$

We know,

$$\sqrt{1-\frac{u^2}{c^2}} = \frac{\sqrt{1-\frac{v^2}{c^2}}\sqrt{1-\frac{u'^2}{c^2}}}{\left(1+\frac{vu'}{c^2}\right)}, \text{ on substituting}$$

$$\Rightarrow P = \frac{m_0}{\sqrt{1-\frac{u'^2}{c^2}}\sqrt{1-v^2/c^2}}(u'+v)$$

$$\Rightarrow P = \frac{m'(u'+v)}{\sqrt{1-v^2/c^2}} = \frac{m'u' + m'c^2\,v/c^2}{\sqrt{1-v^2/c^2}} = \frac{P' + E'v/c^2}{\sqrt{1-v^2/c^2}}$$

$$\Rightarrow \boxed{P = \frac{P' + \frac{E'V}{c^2}}{\sqrt{1 - V^2/c^2}}}$$

$, P_y' = P_y , P_z' = P_z$

Now for Energy.

$$E = mc^2$$

$$\Rightarrow E = \frac{m_0}{\sqrt{1 - \frac{u^2}{c^2}}} c^2$$

$$\Rightarrow E = \frac{m_0}{\frac{\sqrt{1 - \frac{u'^2}{c^2}}\sqrt{1 - \frac{V^2}{c^2}}}{\left(1 + \frac{Vu'}{c^2}\right)}} c^2$$

$$\Rightarrow E = \frac{m'c^2\left(1 + \frac{Vu'}{c^2}\right)}{\sqrt{1 - V^2/c^2}}$$

$$\Rightarrow E = \frac{m'c^2 + m'u'V}{\sqrt{1 - V^2/c^2}}$$

$$\Rightarrow \boxed{E = \frac{E' + P'V}{\sqrt{1 - V^2/c^2}}}$$

(#) Twin Paradox :- It is a thought experiment in special relativity involving identical twins one of whom makes a journey into space in a high speed rocket and returns home to find that the twin who remained on earth has aged more. In both views there is no symmetry between the space time paths of the twins.

Thermal Physics

Intro to the Kinetic theory of gases:

(i) A gas consist of molecules that perform random motion

(ii) Gas molecules influence each other only collision i.e. they donot exert force on each other

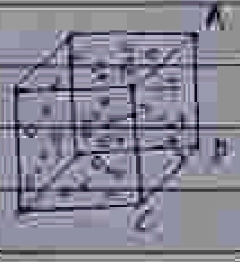
(iii) All collision between gas molecules are perfectly elastic all KE is conserved.

(iv) The volume actually occupied by the molecules of a gas is negligibly small.

Pressure on the surface area ABCD.

$$F = \frac{\Delta P}{\Delta t}$$

$$\Rightarrow P = \frac{F}{A}$$



Macroscopic properties of gas (P, V, T)

Microscopic properties of gas (x, v)

$$\Rightarrow F = \frac{\Delta p}{\Delta t} = \frac{m \Delta v}{\Delta t} = \frac{m(2v_x)}{\Delta t} = \frac{2 m v_x}{\Delta t}$$

$$\Delta t = \frac{2L}{v_x}$$

$$\Rightarrow F = \frac{2m\, v_x \times v_x}{2L} \Rightarrow \frac{m v_x^2}{L}$$

Total force on the wall, $\frac{F}{N} = \frac{m}{L} \frac{(v_{x_1}^2 + v_{x_2}^2 + \ldots + v_{x_n}^2)}{N}$

$$\Rightarrow \frac{f}{N} = \frac{m(\bar{v}_x)^2}{l}$$

$$(\bar{v})^2 = (\bar{v}_x)^2 + (\bar{v}_y)^2 + (\bar{v}_z)^2$$

$$(\bar{v}_x) = (\bar{v}_y) = (\bar{v}_z)$$

$$\Rightarrow 3(\bar{v}_x)^2 = (\bar{v})^2$$

$$\Rightarrow (\bar{v}_x)^2 = \frac{(\bar{v})^2}{3}$$

$$\Rightarrow \frac{f}{A} = \frac{Nm}{Al}\frac{(\bar{v})^2}{3}$$

$$\Rightarrow PV = \frac{1}{3} Nm(\bar{v})^2$$

$$\Rightarrow \frac{3}{2} PV = N\underbrace{\left[\frac{1}{2} m(\bar{v})^2\right]}_{\text{Avg. kinetic energy of a gas molecule}}$$

$$\Rightarrow \frac{3}{2} PV = N (KE)_{avg}$$

$$\Rightarrow \boxed{(KE)_{avg} = \frac{3}{2} K_B T} \rightarrow \text{for monoatomic gas molecule}$$

$$\Rightarrow \boxed{(KE)_{avg} = \frac{5}{2} K_B T} \rightarrow \text{for diatomic gas molecule}$$

(#) Maxwell Boltzmann distribution of molecular velocities :-

The gas molecules perform random motion with wide range of velocities. Maxwell Boltzmann proposed a law for distribution of velocity of the gas molecule at any temp T, called law of Maxwell Boltzmann distribution of velocity:

The gas molecules follow this function for distribution of the velocity:

$$f(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} \quad \text{— (1)}$$

m = mass of one molecule

v = velocity, k = Boltzmann constant

$f(v)$ = Distribution function of probability

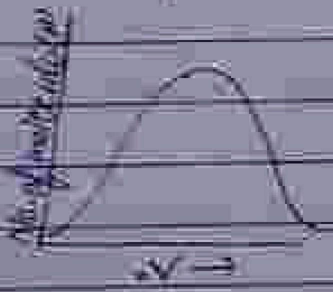
T = Temperature

Here, $f(v)$ is a normalised function

$$\Rightarrow \int_0^\infty f(v)\,dv = 1$$

If the $f(v)$ is not normalised

$$\boxed{\int_0^\infty f(v)\,dv = N} \rightarrow \text{total no. of molecules}$$



If we increase T, V ↑ but dN/N will ↓

If we increase m, V ↓ but dN/N will ↑

There are three ways to calculate the velocity of the gas molecules:-

(i) Average velocity

(ii) Most probable velocity

(iii) RMS velocity

(i) Average velocity

$$V_{av} = \int_0^\infty v \frac{dN}{N}$$

$$\frac{dN}{N} = 4\pi\left(\frac{m}{2\pi KT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2KT}} dv$$

$$\Rightarrow V_{av} = \int_0^\infty \underbrace{4\pi\left(\frac{m}{2\pi KT}\right)^{3/2}}_{C_1} v^3 e^{-\underbrace{\frac{mv^2}{2KT}}_{C_2}} dv$$

$$\Rightarrow V_{av} = \int_0^\infty C_1 v^3 e^{-C_2 v^2} dv \qquad \left[\int_0^\infty x^{2n+1} e^{-ax^2} dx = \frac{\lfloor n}{2a^{n+1}}\right]$$

$$2n+1 = 3$$
$$n = 1$$

$$\Rightarrow V_{av} = \frac{C_1 \lfloor 1}{2 \cdot C_2^2} = \frac{C_1}{2(C_2)^2}$$

$$\Rightarrow V_{av} = \frac{4\pi\left(\frac{m}{2\pi KT}\right)^{3/2}}{2\left(\frac{m}{2KT}\right)^2} \Rightarrow \frac{\frac{4}{2^{3/2}} \cdot \frac{\pi}{\pi^{3/2}}}{\frac{1}{2} \cdot \frac{m^{1/2}}{(KT)^{1/2}}}$$

$$\boxed{V_{avg} \Rightarrow \sqrt{\frac{8KT}{\pi m}}}$$

(ii) Most Probable Velocity

Maximum no of molecule have this velocity called most probable velocity.

$$f(v) = C_1 e^{-C_2 v^2} v^2$$

$$f'(v) = \frac{df(v)}{dv} = \frac{d}{dv}\left(C_1 v^2 e^{-C_2 v^2}\right)$$

$$f'(v) = 2C_1 v e^{-C_2 v^2} + C_1 v^2 e^{-C_2 v^2}(-2C_2 v)$$

For the molecules having max'm v

$$f'(v) = 0$$

$$\cancel{2C_1 v e^{-C_2 v^2}} = 2C_1 C_2 v^3 e^{-C_2 v^2}$$

$$v = \frac{1}{\sqrt{C_2}} \neq 0$$

Hence max'm

$$\Rightarrow v = \frac{1}{\sqrt{\frac{m}{2KT}}} = \sqrt{\frac{2KT}{m}}$$

$$\Rightarrow \boxed{v = \sqrt{\frac{2KT}{m}}}$$

(iii) RMS Velocity :- It is defined as the square root of mean of the square of the velocity of all the molecules

$$v_{av}^2 = \int_0^\infty f(v)\, v^2\, dv$$

$$\Rightarrow \int_0^\infty C_1 v^2 e^{-C_2 v^2} v^2\, dv$$

$$\Rightarrow C_1 \int_0^\infty v^4 e^{-C_2 v^2}\, dv$$

$$\left\{ \begin{aligned} 2n+1 &= 4 \\ n &= 3/2 \end{aligned} \right.$$

$$\Rightarrow \frac{C_1 \Gamma 5/2}{2 \cdot C_2^{5/2}} \Rightarrow \frac{4\pi\left(\frac{m}{2\pi KT}\right)^{3/2} \frac{\sqrt{\pi}}{2}}{2 \cdot \left(\frac{m}{2KT}\right)^{5/2}} \Rightarrow$$ [illegible]

$$(V_{avg})^2 = \frac{3KT}{m} \Rightarrow V_{avg} = \sqrt{\frac{3KT}{m}}$$

$$\boxed{V_{RMS} = \sqrt{V_{avg}^2} = \sqrt{\frac{3KT}{M}}}$$

$$V_{avg} = \sqrt{\frac{8KT}{\pi m}}, \quad V_{rms} = \sqrt{\frac{3KT}{m}}, \quad V_{mp} = \sqrt{\frac{2KT}{m}}$$

$V_{rms} > V_{avg} > V_{mp}$

$\Rightarrow V_{rms} : V_{avg} : V_{mp} = \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2}$

$= 1.732 : 1.595 : 1.414$

$\Rightarrow V_{rms} : V_{avg} : V_{mp} = 1.224 : 1.128 : 1$

(#) Transport phenomena in gases

Mean free Path :- The average distance travelled by a gas molecule between two successive collisions is called mean free path and it is determined by λ.

If a molecule travelled total distance S after N number of collisions, λ is given by $\boxed{\lambda = \frac{S}{N}}$

The average time taken by a gas molecule between two successive collisions is called mean free time (τ)

$\boxed{\tau = \frac{\lambda}{C}}$, where C is the average velocity of a gas molecule

Expression for mean free path :

⇒ Molecules are perfectly elastic sphere

⇒ Diameter of each molecule is $= \sigma$

⇒ The molecule under consideration is moving and others are supposed to be at rest

⇒ Average velocity of the molecule is C

⇒ No. of molecules per unit volume $= \eta$

molecule will collide with all molecule whose centre lie within a cylinder of radius σ and length C

No. of molecule in the cylinder $= \pi\sigma^2 C\eta$

No of collision made by the molecule in 1 s $= \pi\sigma^2 C\eta$

⇒ Time taken in 1 collision $= \dfrac{1}{\pi\sigma^2 C\eta}$

⇒ Time between two successive collision $= \dfrac{1}{\pi\sigma^2 C\eta}$

Distance travelled b/w two successive collision $= C \times \dfrac{1}{\pi\sigma^2 C\eta}$

$$\boxed{\lambda = \frac{1}{\pi\sigma^2\eta}}$$

Maxwell provided a correct expression for λ by considering the motion of all the molecules into account and it is equal to $\boxed{\lambda = \dfrac{1}{\sqrt{2}\,\pi\sigma^2\eta}}$

Collision frequency, $\boxed{f = \dfrac{C}{\lambda}}$

Variation of λ with temperature and pressure

For 1 mol of an ideal gas

$$\frac{PV}{N} = \frac{RT}{N}$$

$$\Rightarrow P = \frac{N}{V} kT$$

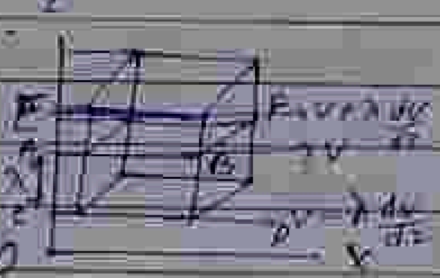
$$\Rightarrow n = \frac{P}{kT}$$

$$\lambda = \frac{1}{\sqrt{2}\pi\sigma^2 n}$$

$$\Rightarrow \boxed{\lambda \propto T} \ \& \ \boxed{\lambda \propto \frac{1}{P}}$$

Transport Phenomena:— The gas attains state of equilibrium by transporting momentum, heat & mass from one layer to another layer giving rise to viscosity, conductivity and diffusion respectively and the phenomena is called transport phenomena. It occurs only at non equilibrium state of a gas.

⟹ Viscosity



Velocity of layer AB = V

Gradient of velocity along z-axis $= \frac{dv}{dz}$

The layers are separated by a distance λ (mean free path)

Velocity of layer EF = $v + \lambda \frac{dv}{dz}$

Velocity of layer CD = $v - \lambda \frac{dv}{dz}$

n = no of molecules per unit volume.
m = mass of each gas molecules
c = average velocity of gas molecules

$\frac{n}{6}$ = average no. of gas molecules along (+ve or -ve) axis

The no. of molecules moving downwards from EF to CD per unit area of the AB in one second = $\frac{nc}{6}$

Momentum lost per unit area per second by the layer EF =
$$\frac{m \times nc}{6} \times \left(v + \lambda \frac{dv}{dz}\right)$$

Similarly the no. of molecules passing upwards from CD to EF per unit area of layer in one second = $\frac{nc}{6}$

The momentum gained per unit area/second = $\frac{mnc}{6}\left(v - \lambda \frac{dv}{dz}\right)$

Net momentum lost by the layer EF per unit area/second =
$$= \frac{mnc}{6}\left(v + \lambda \frac{dv}{dz} - v + \lambda \frac{dv}{dz}\right)$$
$$\Rightarrow \frac{2mnc}{6} \lambda \frac{dv}{dz} \Rightarrow \frac{mnc}{3} \lambda \frac{dv}{dz} \quad \text{--- (i)}$$

The backward dragging force per unit area = gain or loss of momentum per unit area/sec

$$F = \eta \frac{dv}{dz} \quad \text{--- (ii)}$$

On comparing

$\Rightarrow \boxed{\eta = \frac{1}{3} mnc\lambda}$

$\boxed{mn = \rho}$

$\Rightarrow \boxed{\eta = \frac{1}{3}\rho c\lambda}$, As $c \propto \sqrt{T}$

$\Rightarrow \eta \propto \sqrt{T}$

Viscosity increases with increase of temperature

$\Rightarrow \rho$ increases with temp. but λ decreases with increase of pressure in same proportion.

$\Rightarrow \lambda\rho$ = constant

Hence η is independent of pressure

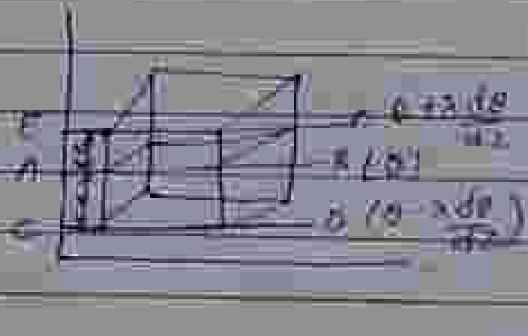
→ Thermal Conductivity:-

θ = Temp of layer AB

$\frac{d\theta}{dz}$ = Temp. gradient

$\theta + \lambda\frac{d\theta}{dz}$ = Temp of layer EF

$\theta - \lambda\frac{d\theta}{dz}$ = Temp of layer CD



The no. of molecules crossing unit area of the layer upward or downward/second = $\frac{nc}{6}$

Mass of the gas crossing unit area of layer AB upward or downward/sec = $\frac{mnc}{6}$

Heat energy carried by the molecules in crossing per unit area of the layer AB in the downward direction/second

$$= \text{mass} \times \text{specific heat} \times \text{temp}$$

$$= \frac{mnc}{6} \times C_v \times \left(\theta + \lambda \frac{d\theta}{dz}\right)$$

C_v - Specific heat of the gas

The heat energy carried by the gas molecules in crossing per unit area of the layer AB in the upward direction/sec

$$= \frac{mnc}{6} \times C_v \times \left(\theta - \lambda \frac{d\theta}{dz}\right)$$

Net heat transfer per unit area of the layer AB

$$Q = \frac{mnc}{6} \times C_v \left[\theta + \lambda \frac{d\theta}{dz} - \theta + \lambda \frac{d\theta}{dz}\right]$$

$$Q = \frac{mnc\, C_v}{3} \lambda \frac{d\theta}{dz} \quad \text{--- (1)}$$

In terms of coefficient of thermal conductivity :-

$$Q = K \frac{d\theta}{dz} \quad \text{--- (2)}$$

Comparing (1) & (2)

$$K = \frac{mnc\, C_v \lambda}{3}$$

$$(\rho = mn) \Rightarrow K = \frac{\rho c\, C_v \lambda}{3}$$

Relation between K & η

$$\eta = \frac{1}{3} \rho c \lambda$$

$$K = \frac{1}{3} \rho c\, C_v \lambda$$

$$\Rightarrow \boxed{K = \eta C_v} \quad \text{--- (3)}$$

The relation (8) doesnot hold good with the experiment value since it is true only when heat supplied to it is converted into translation K.E

A/c to chapman & Enskog, the corrected value is

$$\boxed{K = \epsilon \eta C_v}$$

Where, $\epsilon = \frac{(9\gamma - 5)}{4}$, γ = Ratio of specificheat $\boxed{\gamma = \frac{C_p}{C_v}}$

$$\boxed{K = \frac{1}{4}(9\gamma - 5)\eta C_v}$$

$\Rightarrow K = \frac{1}{3}\rho c \lambda C_v$

$= \frac{1}{3} mnc \lambda C_v$

$\Rightarrow \frac{1}{3} mnc\, C_v \frac{1}{\sqrt{2}\pi\sigma^2 n}$

$\Rightarrow K = \frac{mcC_v}{3\sqrt{2}\pi\sigma^2}$

$c \propto \sqrt{T}$

$K \propto \sqrt{T}$

K is independent of pressure.

(B) Show that the thermal conductivity of hydrogen molecules is largest

$\Rightarrow K = \frac{1}{3}\rho c \lambda C_v$

$\lambda = \frac{1}{\sqrt{2}\pi\sigma^2 n}$

$\Rightarrow K = \frac{mCC_v}{3\sqrt{2}\,\pi\sigma^2}$

$\Rightarrow K = \frac{M}{N}\,\frac{C}{\pi\sigma^2}\,\frac{1}{3\sqrt{2}}\,\frac{C_v}{M}$ → molar specific heat

$\Rightarrow K = \frac{1}{3\sqrt{2}}\,\frac{C_v}{\pi N\sigma^2}\,C$

$\Rightarrow K \propto C$

$\Rightarrow K \propto \sqrt{\frac{8KT}{\pi m}}$ $\Rightarrow K \propto \frac{1}{\sqrt{m}}$

As the mass of molecule of hydrogen is smallest. Hence K is largest.

$\Rightarrow$ <u>Diffusion</u> :- In diffusion mass transports from a region of higher concentration to a region of lower concentration to achieve equilibrium.

n = no of molecule/Volume on layer AB

λ = mean free path

$\frac{dn}{dz}$ = Conc^n gradient along Z axis

$n + \lambda\frac{dn}{dz}$ = Conc^n at the layer EF

$n - \lambda\frac{dn}{dz}$ = Conc^n at the layer CD

The no of molecules coming from layer EF and crossing AB downward per unit area and per unit second $= \frac{C}{6}\left(n+\lambda\frac{dn}{dz}\right)$

No. of molecules coming from layer CD and crossing AB upward per unit area per unit second $= \frac{C}{6}\left(n-\lambda\frac{dn}{dz}\right)$

Net no. of molecules crossing per unit area per second of layer AB in downward direction =

$$\Rightarrow \frac{C}{6}\left[n + \lambda\frac{dn}{dz} - n + \lambda\frac{dn}{dz}\right]$$

$$\Rightarrow \frac{C}{6} \times 2\lambda\frac{dn}{dz} = \frac{C\lambda}{3}\frac{dn}{dz}$$

The coefficient of diffusion is defined as the ratio of the number of molecule crossing 1 area in one second to the rate of change of conc^n

i.e. $D = \dfrac{\frac{C\lambda}{3}\cancel{\frac{dn}{dz}}}{\cancel{\frac{dn}{dz}}}$

$$\boxed{D = \frac{C\lambda}{3}}$$

(ii) Effect of Temperature & Pressure

$$\lambda = \frac{KT}{\sqrt{2}\,\pi\sigma^2 P} \qquad C = \sqrt{\frac{8KT}{\pi m}}$$

$$D = \frac{1}{3}\sqrt{\frac{8KT}{\pi m}} \times \frac{KT}{\sqrt{2}\,\pi\sigma^2 P}$$

Hence, $D \propto T^{3/2}$

$D \propto \frac{1}{P}$

Relation between η & D:-

$$\eta = \frac{1}{3}\rho c\lambda$$

$$D = \frac{c\lambda}{3}$$

$$\frac{M}{D} = \frac{\frac{ECA}{3}}{\frac{CA}{3}} = \rho \text{ (density)}$$

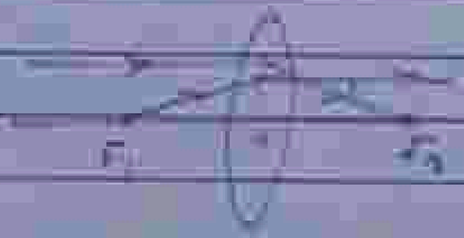
$$\Rightarrow \boxed{D = \frac{M}{\rho}}$$

Date 8.2.21

Geometrical Optics

Some properties of lens:

f_1 = First focal point
f_2 = Second focal point
O = optical centre
C_1, C_2 are the centres of the spheres.



Focal length of a lens

ΔABO & $\Delta A'B'O$

$\angle BAO = \angle B'A'O$

$\angle BOA = \angle B'OA'$

Hence similarity

$$\frac{AB}{A'B'} = \frac{OA}{OA'} = \frac{-u}{v} \quad \text{— (i)}$$

Similarly ΔEOF_2 and $\Delta B'A'F_2$ are similar.

$$\frac{EO}{B'A'} = \frac{OF_2}{A'F_2} = \frac{f}{v-f} = \frac{AB}{A'B'} \quad \text{— (ii)}$$

Equating (i) & (ii)

$$\Rightarrow \frac{f}{v-f} = \frac{-u}{v}$$

$$\Rightarrow vf = -uv + uf$$

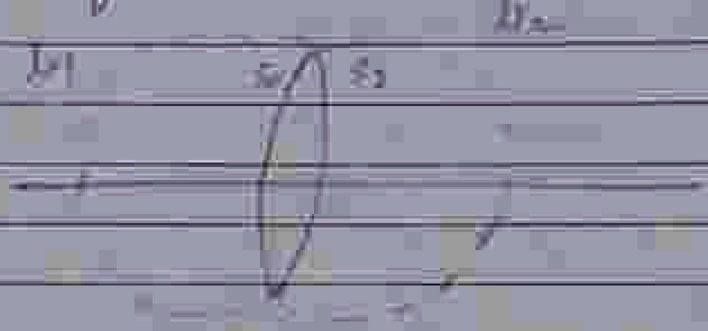
$$\Rightarrow vf + uv = uf$$

Divide by uvf

We get, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{A'B'}{AB} = -\frac{v}{u} = \frac{h_i}{h_o} = m$ (lateral magnification)

Lens Maker's formula



Refraction through spherical lenses.

For surface $S_1 \Rightarrow \frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}$

$S_2 \Rightarrow \frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2}$

On adding

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = ({}^{2}\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \boxed{\frac{1}{f} = ({}^{2}\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Combination of thin lenses in contact

for d_1,

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u} \quad \text{— (i)}$$

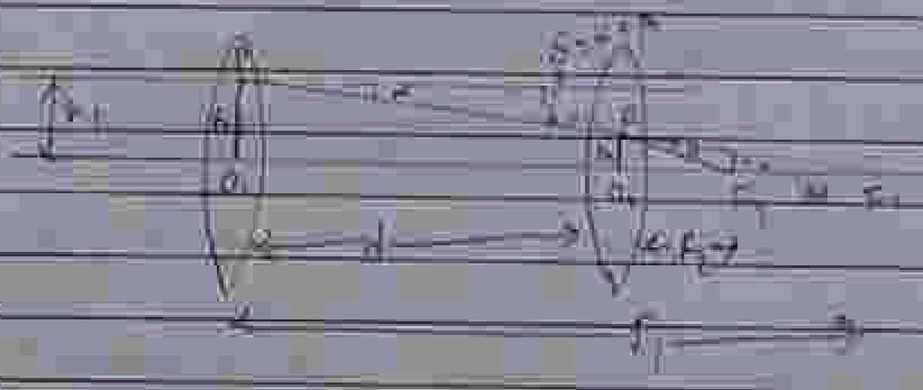
$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v_1} \quad \text{— (ii)}$$

(i) adding

$$\Rightarrow \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$$

④ <u>Combination of thin lenses separated by a distance 'd'</u>



ΔPO_1F_1

$$\alpha = \frac{PO_1}{O_1F_1} = \frac{h_1}{f_1}$$

$$\beta = \frac{h_2}{f_2}$$

$$\delta = \alpha + \beta \Rightarrow \frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2}$$

ΔPO_1F_1 and $\Delta RO_2F_1 = \angle PO_1F_1 = \angle RO_2F_1$

$\angle PF_1O_1 = \angle RF_2O_2$

$\Delta PO_1F_1 \cong \Delta RO_2F_1$

$$\frac{PO_1}{RO_2} = \frac{O_1F_1}{O_2F_1} \quad , O_2F_1 = f_1 - d$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{f_1}{f_1 - d}$$

$$\Rightarrow \frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 f_2}$$

$$\Rightarrow \boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$$

If μ is the refractive index of medium b/w b/w the lens.

$$\Rightarrow \boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{\mu f_1 f_2}}$$

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} \quad , f = \frac{f_1 f_2}{\Delta}$$

where $\Delta = f_1 + f_2 - d$, called optical interval b/w the two lenses.

(i) when $d > f_1 + f_2$ the system become divergent.

(ii)

Proof :-

$$\frac{2^{nd}\text{ focal length}}{1^{st}\text{ focal length}} = \frac{\text{refractive index of image space}}{\text{refractive index of object space}}$$

For 1^{st} lens, $\frac{f_1'}{f_1} = \frac{\mu}{1} \Rightarrow f_1' = \mu f_1$

For 2^{nd} lens, $\frac{f_2'}{f_2} = \frac{1}{\mu} \Rightarrow f_2' = \frac{f_2}{\mu} \Rightarrow f_2 = \mu f_2' = \mu f_2$

Now Eq. focal length $= \frac{f_1' \times f_2'}{f_1' + f_2' - d} = \frac{\mu f_1 f_2}{\mu f_1 + \mu f_2 - d}$

$$\boxed{f_{eq} = \frac{f_1 f_2}{f_1 + f_2 - \frac{d}{u}}}$$

<u>Power of a lens</u>:- The power of a lens is the measure of its ability to produce convergence of a parallel beam of light. The power of a converging lens is (+ve) and convex lens produce it. The concave lens produce divergence with (-ve) power.

$$\text{Power}(P) = \frac{1}{f(m)} \quad \text{diopter}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow P = P_1 + P_2$$

For the lens separated by a distance 'd'

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\boxed{P = P_1 + P_2 - d P_1 P_2}$$

(4) <u>Optical system & cardinal points</u>:- An optical system consists of a number of lenses placed apart, and having common principal axis.

<u>Cardinal Points</u>:- An optical system is characterised by six points called cardinal points

(1) Two principal points

(2) Two focal points

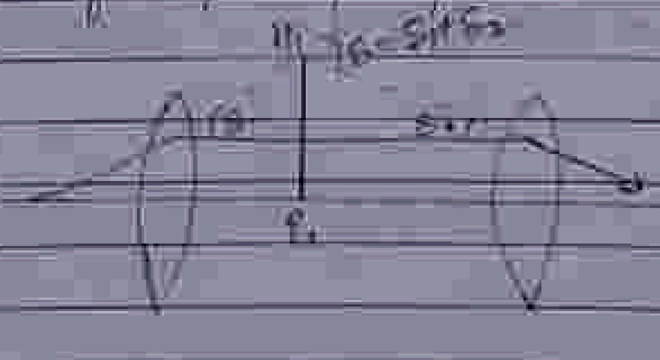
(3) Two nodal points

These six points are called cardinal points of an optical system. The planes passing through these points

And ⊥ to the principal axis are called cardinal points

The refraction of OA is presented in terms of single refraction at a plane passing through H_2. A ⊥ line drawn from a point H_2 meets the principal axis at a point P_2, called second principal points and the plane passing through the points H_2 & P_2 is called second principal plane

Here P_1 is called the first principal point and H_1P_1 is called the first principal plane



It seems that two incident rays directed towards H_1 & after refraction they seems to come from H_2. Thus H_2 is the image of H_1. $H_1P_1 = H_2P_2$, H_1 & H_2 are the conjugate points and H_1P_1 & H_2P_2 are called conjugate planes. Therefore lateral magnification is unity. Thus the first (P_1) and second (P_2) principal points are the conjugate point having unit lateral magnification.

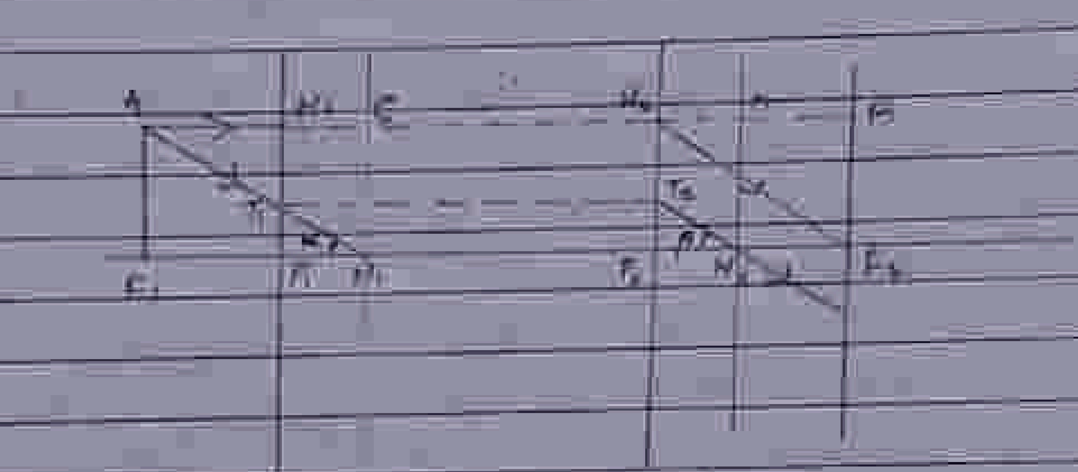
<u>Focal points & focal plane</u> :- The first focal point is a point on the principal axis of optical system such that a beam of light passing through it is rendered parallel to the principal axis after refraction.

The second focal point is a point on the principal axis of optical system such that a beam of light travelling parallel to the principal axis of the optical system

$C_1F_1 = f_1$ = first focal point
$C_2F_2 = f_2$ - second focal point
Plane AF_1 is first focal plane
& BF_2 is second focal plane.

(7) <u>Nodal points and nodal planes</u> :- Nodal points are the points on the principal axis of the optical system where light rays.

emerge without refraction or Nodal points are the pair of conjugate points on the principal axis having unit angular magnification and the planes passing through the nodal points & ⊥ to principal axis are called nodal planes.



N_1 & N_2 are called nodal points as

$$m = \frac{\tan\alpha}{\tan\beta} = 1 \text{ (unit angular magnification)}$$

CN_1 & DN_2 are called nodal planes.

(Q) Show that the distance b/w two nodal points is equal to the distance b/w two principal points.

Ans→ From above figure

$\Delta T_1P_1N_1$ and $\Delta T_2P_2N_2$ are similar

because, $\angle T_1N_1P_1 = \angle T_2N_2P_2$

$T_1P_1 = T_2P_2$

⇒ we can write $P_1N_1 = P_2N_2$

Add N_1P_2 to both side

$$\Rightarrow P_1N_1 + N_1P_2 = P_2N_2 + N_1P_2$$

$$\Rightarrow \boxed{P_1P_2 = N_1N_2}$$

(ii) The nodal points N_1 and N_2 coincide with the principal points P_1 and P_2 when refractive indices on either side of the lens are same.

Ans: From above figure $\Delta A F_1 N_1$ & $\Delta H_2 P_2 F_2$

$AF_1 = H_2 P_2$ & $\angle A N_1 F_1 = \angle H_2 F_2 P_2$

Hence, $\Delta A F_1 N_1 \sim \Delta H_2 P_2 F_2$

$\Rightarrow F_1 N_1 = P_2 F_2$

$\Rightarrow F_1 N_1 = F_1 P_1 + P_1 N_1$

$\Rightarrow F_1 P_1 + P_1 N_1 = P_2 F_2$

$\Rightarrow P_1 N_1 = P_2 F_2 - P_1 F_1$

$P_2 F_2 = f_2$ & $P_1 F_1 = -f_1$

$\Rightarrow P_1 N_1 = f_1 + f_2$

As medium on both sides are same,

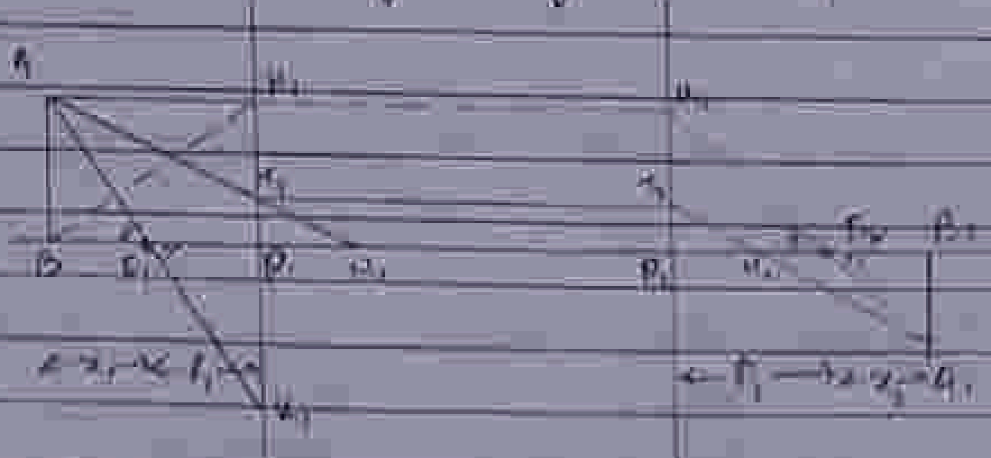
$-f_1 = f_2 = F$

$\Rightarrow P_1 N_1 = 0$

Hence P_1 coincides to N_1

Similarly P_2 coincides to N_2

(iii) Construction of the Image using cardinal points



In ΔABF_1 & $\Delta K_1 P_1 F_1$

$\angle AFB = \angle K_1 F_1 P_1$ & $AB = K_1 P_1$, Hence similar

$$\frac{AB}{K_1P_1} = \frac{BF_1}{P_1F_1} = \frac{x_1}{f_1} \quad \text{— (i)}$$

In $\Delta A_1B_1F_2$ & $\Delta H_2P_2F_2$

$\angle H_2F_2P_2$ & $\angle A_1F_2B_1$ (equal)

$A_1B_1 = H_2P_2$

Hence similar

$$\frac{A_1B_1}{H_2P_2} = \frac{B_1F_2}{P_2F_2} = \frac{x_2}{f_2} \quad \text{— (ii)}$$

Taking reciprocal

$$\frac{H_2P_2}{A_1B_1} = \frac{f_2}{x_2} \quad \text{— (iii)}$$

Equating (iii) & (i)

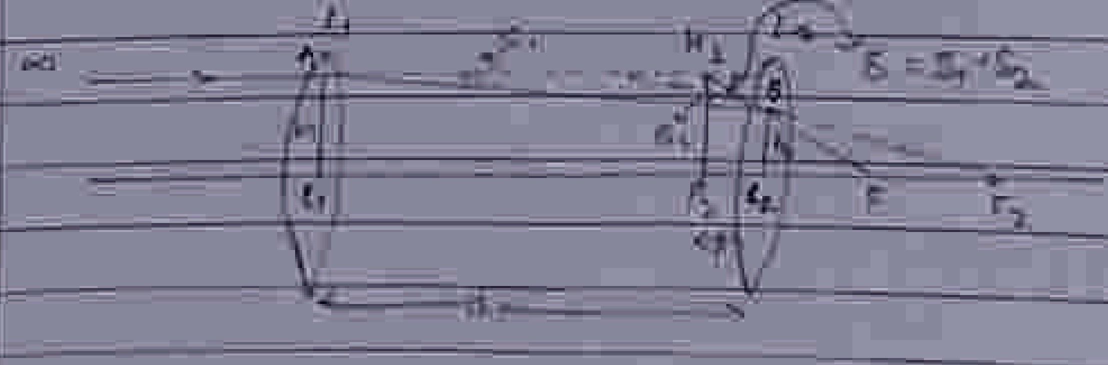
$$\Rightarrow \frac{f_2}{x_1} = \frac{x_2}{f_1}$$

$$\Rightarrow x_1x_2 = f_1f_2$$

→ where x_1 & x_2 are the distance of image & object from respective focal points

→ f_1 & f_2 are first and second focal length respectively

(4) Location of cardinal points in a coaxial system of two thin lenses



$L_2F_2 = v$

$P_2F_2 = f$

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$$

$\delta = \delta_1 + \delta_2$

$\Rightarrow \frac{h}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2}$

$\Delta A L_1 F_1 \sim \Delta B L_2 F_1$

$\Rightarrow \frac{AL_1}{BL_2} = \frac{L_1F_1}{L_2F_1}$

$\Rightarrow \boxed{\frac{h_1}{h_2} = \frac{f_1}{f_1 - d}}$ — (i)

$h_2 = h_1 \frac{(f_1 - d)}{f_1}$

as $\Delta H_2 P_2 F$ & $\Delta B L_2 F$ are similar

$\frac{H_2P_2}{BL_2} = \frac{P_2F}{L_2F}$

$\Rightarrow \frac{h_1}{h_2} = \frac{f}{(f - L_2P_2)}$ — (ii)

Equating (i) & (ii)

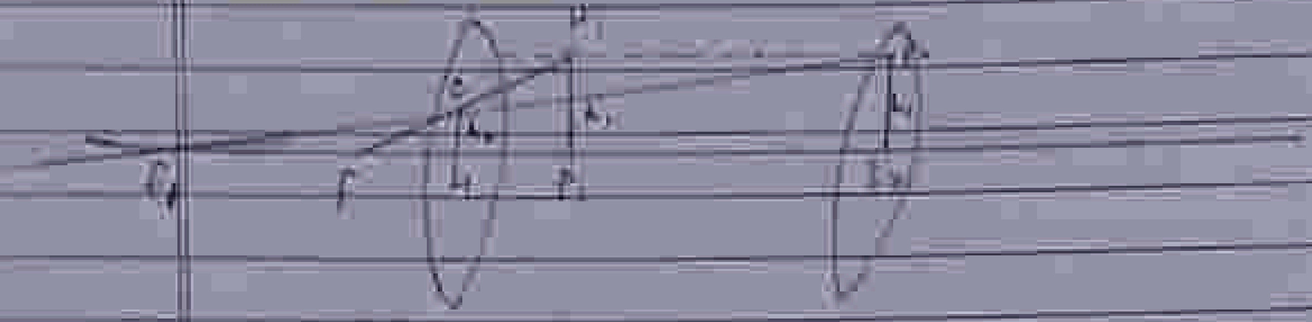
$\frac{f_1}{(f_1 - d)} = \frac{f}{(f - L_2P_2)}$

$\Rightarrow \cancel{f_1 f} - f_1(L_2P_2) = \cancel{f f_1} - df$

$\Rightarrow L_2P_2 = \frac{-fd}{f_1}$, $L_2P_2 = -\beta$

$\Rightarrow \boxed{\beta = \frac{fd}{f_1}}$ & $L_2F = P_2F - P_2L_2$

$$\boxed{L_2F = f - \frac{fd}{f_1} = f\left(1 - \frac{d}{f_1}\right)}$$



$\Delta P_1H_1F \sim \Delta CL_1F$

$\Rightarrow \dfrac{P_1F}{L_1F} \Rightarrow \dfrac{H_1P_1}{CL_1}$

$\Rightarrow \dfrac{f}{f - L_1P_1} = \dfrac{h_1}{h_2}$ — (i)

ΔBL_2F_1 & ΔCL_1F_1 are similar.

$\Rightarrow \dfrac{BL_2}{CL_1} = \dfrac{L_2F_1}{L_1F_1}$

$\Rightarrow \dfrac{h_1}{h_2} = \dfrac{f_2}{f_2 - d}$ — (ii)

Equating (i) & (ii)

$\Rightarrow \dfrac{f}{f - L_1P_1} = \dfrac{f_2}{f_2 - d}$

$\Rightarrow \cancel{ff_2} - df = \cancel{ff_2} - f_2 L_1P_1$

$\Rightarrow L_1P_1 = \dfrac{fd}{f_2} \Rightarrow \boxed{\alpha = \dfrac{fd}{f_2}}$

Now, $L_1F = P_1F - L_1P_1$

$\Rightarrow -f - (-\alpha)$

$L_1F \Rightarrow -f + \alpha = -f + \dfrac{fd}{f_1} = -f\left(1 - \dfrac{d}{f_2}\right)$

(8) Two thin convex lenses of focal length 20 cm & 5 cm are kept coaxially separated by a distance of 10 cm. Plot the position of the cardinal points for the combination.

Sol → $f_1 = 20\,cm$, $f_2 = 5\,cm$, $d = 10\,cm$

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{20 \times 5}{20 + 5 - 10} = \frac{100}{15}\,cm = \frac{20}{3}\,cm$$

$$= 6.67\,cm$$

$$\alpha = \frac{fd}{f_2} = \frac{20 \times 10}{3 \times 5} = \frac{40}{3} = 13.33\,cm$$

$$\beta = -\frac{fd}{f_1} = -\frac{20 \times 10}{3 \times 20} = -\frac{10}{3} = -3.33\,cm$$

$$L_1F = P_1F - P_1L_1$$

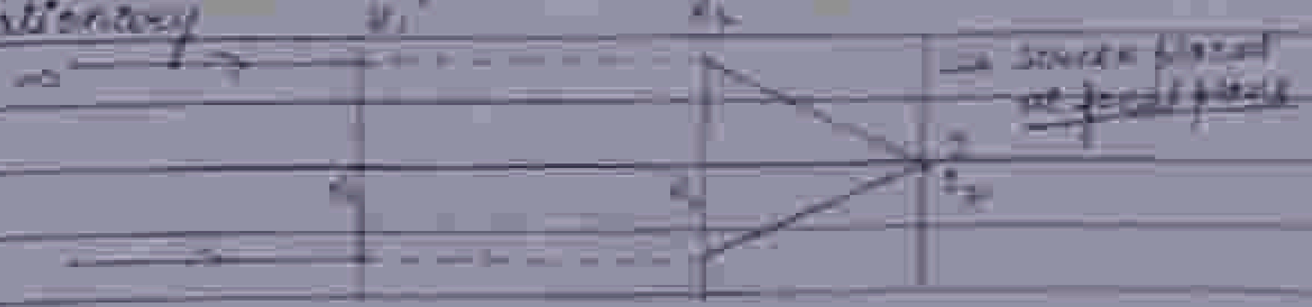
$$= -f\left(1 - \frac{d}{f_2}\right) = -\frac{20}{3}\left(1 - \frac{10}{5}\right) = \frac{20}{3} = 6.67\,cm$$

$$L_2F = P_2F - L_2P_2$$

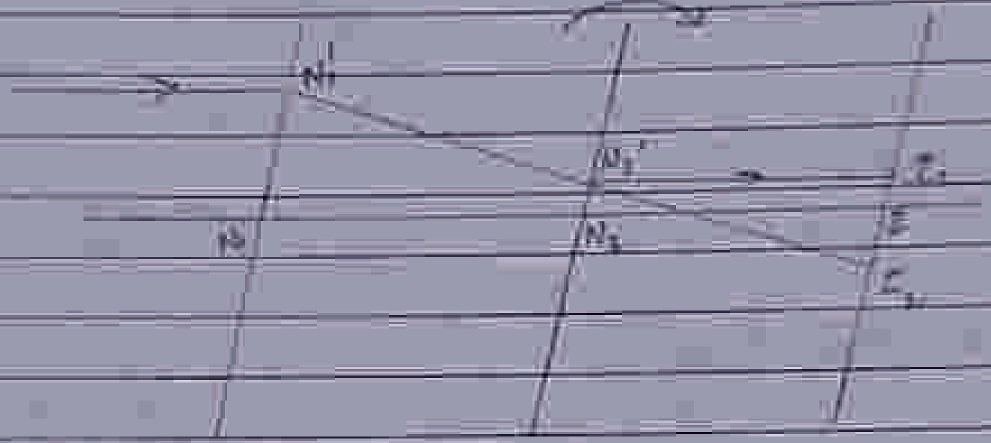
$$= f - \frac{fd}{f_1} = f\left(1 - \frac{d}{f_1}\right) = \frac{20}{3}\left(1 - \frac{10}{20}\right) = 3.33\,cm$$

(4) Nodal slide :- It is a particular type of horizontal metal support for a lens system that provides a method for locating the focal & nodal points and determining the focal length of a lens system.

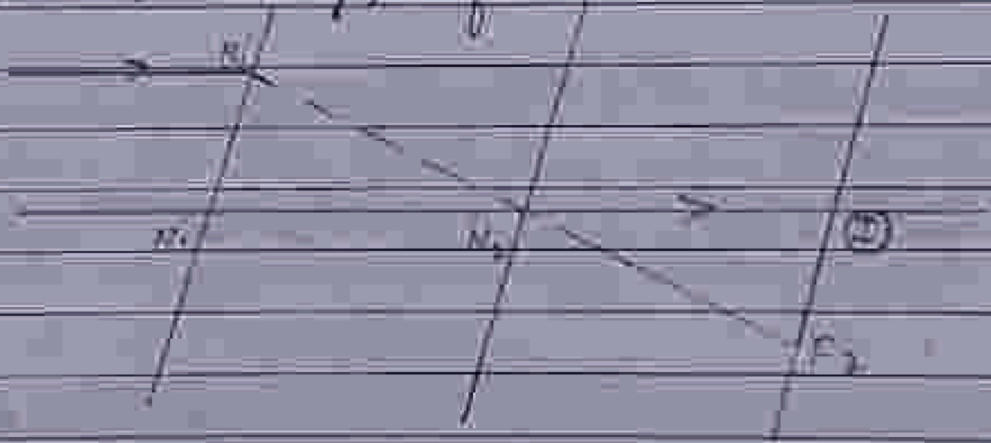
Principle :- It is a parallel beam of light is incident on a convergent lens system it forms an image on the screen field at its second focal plane. When the lens system is rotated through a small angle about a vertical axes through its second nodal point, the image doesn't shift laterally and remains stationary.



(1) Nodal slide is rotated about an axis ⊥ to the principal axis and passing through O. I shifts to I_1.

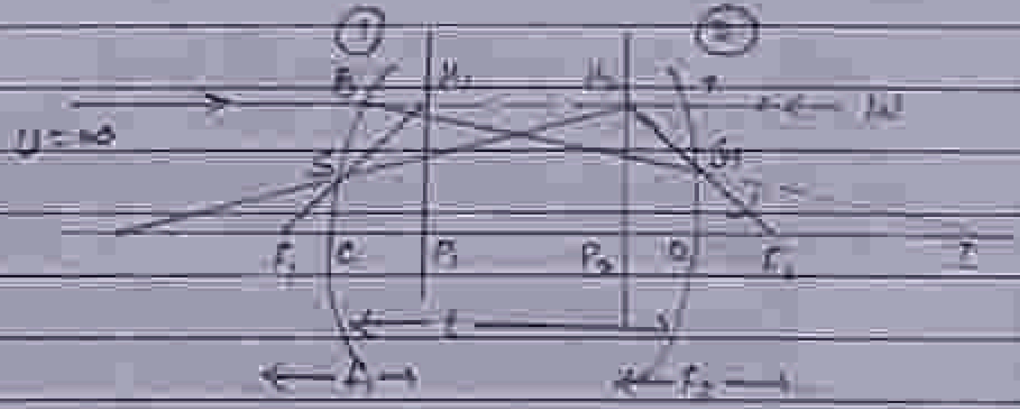


(2) When nodal slide system is rotated through a small angle about an axis passing through N_2, the image remains stationary, N_2 fixed.



(3) The axis of rotation lies before N_1, the image position changes.

(4) Thick Lens



$P_1F_1 = -f_1$, $P_2F_2 = f_2$, $f_2 = -f_1 = F$

$CI = V_1$

The refraction at the surface 1 is governed through the following refraction expression:-

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \frac{\mu}{V_1} - \frac{1}{\infty} = \frac{\mu - 1}{R_1}$$

$$\Rightarrow \frac{1}{V_1} = \frac{\mu - 1}{\mu R_1} \quad \text{— (1)}$$

$$\frac{1}{DF_2} - \frac{\mu}{DI} = \frac{\mu - 1}{-R_2} \quad \text{— (2)}$$

The triangles $H_2P_2F_2$, G_1DF_2 and BCI, G_1DI are similar.

From, $\triangle H_2P_2F_2$ & G_1DF_2

$$\frac{H_2P_2}{P_2F_2} = \frac{G_1D}{DF_2}$$

$$\Rightarrow H_2P_2 = \frac{G_1D\,(P_2F_2)}{DF_2}$$

From $\triangle BCI$ & G_1DI

$$\frac{BC}{CI} = \frac{G_1D}{DI}$$

$$BC = \frac{G_1D}{DI}(CI)$$

$$H_2P_2 = BC \Rightarrow \frac{G_1D}{DF_2}(P_2F_2) = \frac{G_1D}{DI}(CI)$$

$$\Rightarrow \frac{P_2F_2}{DF_2} = \frac{CI}{DI}$$

$$\Rightarrow \frac{DF_2}{P_2F_2} = \frac{DI}{CI} \quad \text{— (2)}$$

$$\Rightarrow \frac{DF_2}{P_2F_2} = \frac{DI}{CI}$$

✓

$$\frac{1}{P_2F_2} = \frac{DI}{CI}\,\frac{1}{DF_2}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{DF_2}\left(\frac{DI}{v_1}\right) \quad \text{— (4)}$$

Multiplying Eq^n (ii) by DI, we get

$$\Rightarrow \frac{DI}{DF_2} - \mu = \frac{\mu - 1}{-R_2}DI$$

$$\Rightarrow \frac{DI}{DF_2} = \mu + \left(\frac{\mu - 1}{-R_2}\right)DI$$

$$\Rightarrow \frac{DI}{DF_2} = \mu - \left(\frac{\mu - 1}{R_2}\right)DI$$

Substituting the value of $\frac{DI}{DF_2}$ in Eq^n (4) we get

$$\frac{1}{f} = \frac{1}{v_1}\left[\mu - \left(\frac{\mu - 1}{R_2}\right)DI\right]$$

$$\Rightarrow \frac{1}{f} = \frac{1}{v_1}\left(\mu - \left(\frac{\mu - 1}{R_2}\right)(CI - CD)\right)$$

$CI = v_1$; $CD = t$

$$\Rightarrow \frac{1}{f} = \frac{1}{v_1}\left\{\mu - \left(\frac{\mu - 1}{R_2}\right)(v_1 - t)\right\}$$

$$\Rightarrow \frac{1}{f} = \frac{\mu}{v_1} - \frac{(\mu - 1)}{R_2} + \frac{\mu - 1}{R_2}\,\frac{t}{v_1}$$

Substituting the value of $\frac{1}{V_1}$ from Eq^n (1) in the above Eq^n we get

$$\frac{1}{f} = \frac{\mu(\mu-1)}{\mu R_1} - \frac{(\mu-1)}{R_2} + \frac{\mu-1}{R_2} \cdot \frac{t(\mu-1)}{\mu R_1}$$

$$\Rightarrow \boxed{\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{(\mu-1)^2 t}{\mu R_1 R_2}}$$

When $t = 0$ (thin lens)

$$\boxed{\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

(iii) Cardinal points of a thick lens :

(1) Second focal point :- From Eq^n (1)

$$\frac{DF_2}{P_2F_2} = \frac{DI}{CI}$$

$$\Rightarrow DF_2 = \frac{DI\,(P_2F_2)}{CI}$$

$$\Rightarrow DF_2 = \left(\frac{CI - CD}{CI}\right) P_2F_2$$

$$\Rightarrow DF_2 = f\left(1 - \frac{CD}{CI}\right)$$

$$\Rightarrow DF_2 = f\left(1 - \frac{t}{V_1}\right)$$

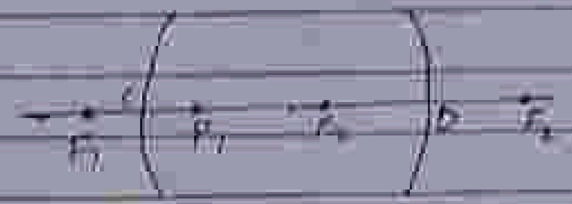
From Eq^n (1) $\frac{1}{V_1} = \frac{\mu-1}{\mu R_1}$, Putting the value of $\frac{1}{V_1}$ in above Eq^n

$$\boxed{DF_2 = f\left(1 - \frac{(\mu-1)t}{\mu R_1}\right)}$$

② Second principal point

$P_2F_2 = f$

$DF_2 = f\left(1 - \frac{(\mu-1)t}{\mu R_1}\right)$



$P_2D = -\beta = P_2F_2 - DF_2$

$$\Rightarrow -\beta = f - f\left(1 - \frac{(\mu-1)t}{\mu R_1}\right)$$

$$\Rightarrow -\beta = f\left(1 - 1 + \frac{(\mu-1)t}{\mu R_1}\right)$$

$$\Rightarrow -\beta = \frac{f(\mu-1)t}{\mu R_1}$$

$$\Rightarrow \boxed{\beta = -\frac{f(\mu-1)t}{\mu R_1}}$$

③ First focal point → If we consider the incident ray is coming from the right, the first focal point is located at F_1 and its distance from C is CF_1. CF_1 is obtained by changing R_1 by $-R_2$ and putting -ve sign in expression of DF_2

$$CF_1 = -f\left[1 + \frac{(\mu-1)t}{\mu R_2}\right]$$

④ First principal point :-

$CP_1 = \alpha = P_1F - CF_1$

$$\alpha = -f + f\left[1 + \frac{(\mu-1)t}{\mu R_2}\right]$$

$$\alpha = f\left[-1 + 1 + \frac{(\mu-1)t}{\mu R_2}\right]$$

$$\boxed{\alpha = \frac{f(\mu-1)t}{\mu R_2}}$$

(5) Glass sphere as a lens:

$$\frac{1}{f} = (\mu-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(\mu-1)t}{\mu R_1 R_2}\right]$$

$\Rightarrow$ In this case $R_1 = R$ and $R_2 = -R$

$$\frac{1}{f} = (\mu-1)\left[\frac{1}{R} + \frac{1}{R} + \frac{(\mu-1)2R}{\mu R^2}\right]$$

$$\Rightarrow \frac{1}{f} = (\mu-1)\left[\frac{2}{R} + \frac{(\mu-1)}{\mu}\frac{2}{R}\right]$$

$$\Rightarrow \frac{1}{f} = (\mu-1)\frac{2}{R}\left[1 + \frac{(\mu-1)}{\mu}\right]$$

$$\Rightarrow \frac{1}{f} = (\mu-1)\frac{2}{R}\left[\frac{\mu-\mu+1}{\mu}\right]$$

$$\Rightarrow \boxed{f = \frac{\mu R}{2(\mu-1)}}$$

Cardinal points:

(i) Principal points:

$$O_1P_1 = \alpha = f\frac{(\mu-1)t}{\mu R_2}$$

$$\Rightarrow \alpha = \frac{\mu R}{2(\mu-1)} \frac{(\mu-1)(2R)}{\mu R}$$

$$\Rightarrow \boxed{\alpha = R}$$

$$O_2P_2 = \beta = -\frac{f(\mu-1)t}{\mu R_1}$$

$$\Rightarrow -\frac{\mu R}{2(\mu-1)} \frac{(\mu-1)2R}{\mu R}$$

$$\boxed{\beta = -R}$$

(ii) Focal points :-

$$O_1F_1 = -f\left(1 + \frac{(\mu-1)t}{\mu R_2}\right)$$

$$t = 2R,\ R_2 = -R$$

$$O_1F_1 = -f\left(1 + \frac{(\mu-1)2R}{-\mu R}\right)$$

$$O_1F_1 = \frac{-\mu R}{2(\mu-1)}\left[1 + \frac{(\mu-1)2}{-\mu}\right]$$

$$= \frac{-\mu R}{2(\mu-1)} + \frac{\mu R(\mu-1)2}{2(\mu-1)\mu}$$

$$\Rightarrow \frac{-\mu R}{2(\mu-1)} + R$$

$$\Rightarrow \frac{-\mu R + 2\mu R - 2R}{2(\mu-1)} = \frac{\mu R - 2R}{2(\mu-1)}$$

$$\boxed{O_1F_1 = \frac{R(\mu-2)}{2(\mu-1)}}$$

Similarly, $O_2F_2 = f\left(1 - \frac{(\mu-1)t}{\mu R_1}\right)$

$$\Rightarrow O_2F_2 = \frac{\mu R}{2(\mu-1)}\left[1 - \frac{(\mu-1)2R}{\mu R}\right]$$

$$= \frac{\mu R}{2(\mu-1)} - \frac{\mu R^2}{\mu R}$$

$$\Rightarrow \frac{\mu R}{2(\mu-1)} - R = \frac{\mu R - 2\mu R + 2R}{2(\mu-1)}$$

$$\boxed{O_2F_2 = \frac{R(2-\mu)}{2(\mu-1)}}$$

Huygens Eyepiece :- It is the combination of two lenses used to magnifying the power of the lens system. This eye piece eliminates the chromatic & spherical aberration. In this Eyepiece converging beam enters the field lens & form the virtual image before the eye lens

$f_1 = F_1$
$f_2 = F_2$
d = distance b/w the lenses

To avoid chromatic abberation,

$$d = \frac{f_1 + f_2}{2} \quad -(1)$$

To avoid spherical abberation,

$$d = f_1 - f_2 \quad -(2)$$

From Eqⁿ (1) & (2)

$$\frac{f_1 + f_2}{2} = f_1 - f_2$$

$$\Rightarrow f_1 + f_2 = 2f_1 - 2f_2$$

$$\Rightarrow 3f_2 = f_1$$

$$\Rightarrow \frac{f_1}{f_2} = \frac{3}{1}$$ ⇒ The ratio of focal length should be 3:1 and $d = 2f$

Thus $f_1 : f_2 = 3 : 1$
and $d = f_1 - f_2$
$f_1 = 3f$, $f_2 = f$ and $d = 2f$

The objective lens forms real and inverted $I' I_1'$. This image is situated at the principal focus of the eye lens, and the final image is formed at infinity.

Equivalent focal length:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad , \; f_1 = 3f, \; f_2 = f, \; d = 2f$$

$$\Rightarrow \frac{1}{f_{eq}} = \frac{1}{3f} + \frac{1}{f} - \frac{2f}{3f^2}$$

$$\Rightarrow \frac{1}{f_{eq}} = \frac{f + 3f - 2f}{3f^2}$$

$$\Rightarrow f_{eq} = \frac{3f^2}{2f} = \frac{3f}{2}$$

Equivalent lens is at a distance $= \frac{3f}{2} - 2f = -\frac{f}{2}$

$\frac{f}{2}$ distance behind the Eye lens

Cardinal points

$$\alpha = \frac{f \times d}{f_2} = \frac{3f}{2} \times \frac{2f}{f} = 3f$$

$$\beta = -\frac{fd}{f_1} = -\frac{3f}{2} \times \frac{2f}{3f} = -f$$

$$L_1F_1 = -f\left(1 - \frac{d}{f_2}\right) \quad , \quad L_2F_2 = f\left(1 - \frac{d}{f_1}\right)$$

$$= -\frac{3f}{2}\left(1 - \frac{2f}{f}\right) \qquad L_2F_2 = \frac{3f}{2}\left(1 - \frac{2f}{3f}\right)$$

$$= \frac{3f}{2} \qquad L_2F_2 = \frac{f}{2}$$

→ The rays coming from the objective lens are intercepted at F_1 & it is the place where crosswire should be placed. If $u = -\frac{3f}{2}$ ⇒ $v = f$ ⇒ b/w the lens.

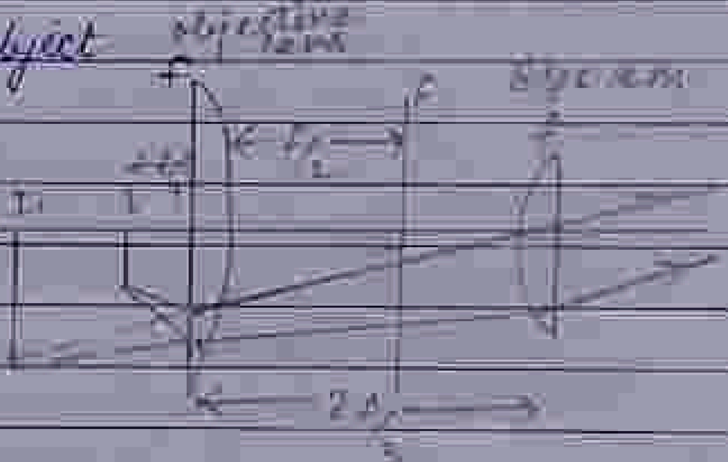
→ Since the image formed by the objective lens lies behind the field lens, it is called negative eye piece.

Ramsden Eyepiece:- It consists of two plano convex lenses each of focal length f separated by a distance of $\frac{2}{3}f$

$f_1 = f,\ f_2 = f$

$d = \frac{2}{3}f$

I = image of the distant object



I act as an object for the objective lens which gives rise the virtual image at I_1. I_1 acts as the object for the eye lens which gives final image at infinity as I_1 is at principal focus

Equivalent focal length:-

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\Rightarrow \frac{1}{f_{eq}} = \frac{1}{f} + \frac{1}{f} - \frac{2f}{3f^2} = \frac{2}{f} - \left(\frac{2}{3f}\right) = \frac{6-2}{3f}$$

$$\boxed{f_{eq} = \frac{3f}{4}}$$

Cardinal point $\Rightarrow$ (I) $\alpha = \frac{fd}{f_2} = \frac{3f}{4} \times \frac{2f}{3 \times f} = \frac{f}{2}$

$$L_1 F_1 = \left(\frac{3f}{4} - \frac{f}{2}\right) = \frac{f}{4}$$

(II) $\beta = -\frac{fd}{f_1} = -\frac{3f}{4} \times \frac{2f}{3 \times f} = -\frac{f}{2}$

$$L_2 F_2 = \frac{3f}{4} - \frac{f}{2} = \frac{f}{4}$$

The cross wire should be placed at I. The position of I can be found as

$$AI = \frac{3f}{4} \quad AL_1 = f/2$$

$$L_1 I = f/4$$

⇒ Cross wire should be kept at $f/4$ distance from the objective lens.

(7)

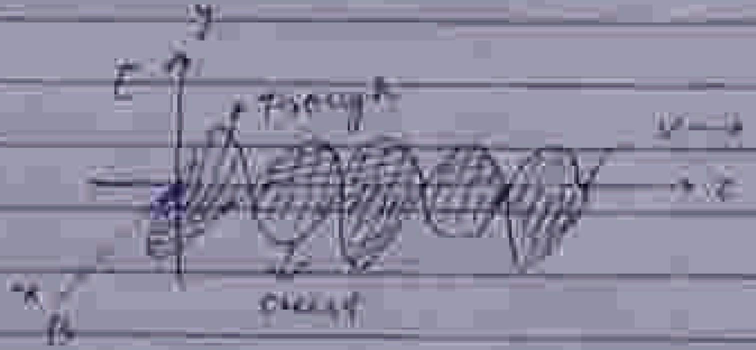
Ramsden Eyepiece	Huygen's Eyepiece
① It is a positive Eyepiece. The image formed by the objective lens lies in the front. Cross wire used	① It is a negative Eyepiece as the image formed by the objective lens lies b/w the lenses. Cross wire cannot be used
② The condition for chromatic & spherical aberration is not satisfied	② Condition for spherical & chromatic aberration is satisfied
③ It is achromatic for only two chosen colours	③ It is achromatic for all the colours.

Date: 18 2 21

Interference

Light : It is an electromagnetic wave consisting of periodically varying electric field & magnetic field oscillating ⊥ to each other and also to the direction of the propagation of wave.

$E = E_0 \sin(\underbrace{kz - \omega t}_{\text{Phase}})$



E_0 = Amplitude

$k = \frac{2\pi}{\lambda}$ (Angular wave number)

λ - Wavelength

$\boxed{v = \nu\lambda}$

$T = \frac{2\pi}{\omega}$

$\boxed{E = CB}$

Coherent waves :- If two waves maintain a constant phase difference over a long distance & time then the waves are called coherent. It is possible when the frequencies of both the wave are same.

Optical path :- The distance travelled by a light in medium of refractive index μ in time t is given by $d = vt$.
The distance d is called geometric path length.

$\boxed{\Delta = \mu d}$

Δ = optical path length.

Phase difference $= \frac{2\pi}{\lambda} \times$ (Path difference Δx)

Principle of Superposition:- When two or more waves overlap, the resultant displacement at any point and at any instant may be found by adding the instantaneous displacement that would be produced by the individual waves.

This interference is redistribution of light energy due to the superposition of light waves from two or more coherent sources is known as interference.

Theory of Interference:-

$$E_A = E_1 \sin(\omega t) \quad \text{--- (1)}$$
$$E_B = E_2 \sin(\omega t + \delta) \quad \text{--- (2)}$$

$E_R = E_A + E_B$

(Resultant wave) $= E_1 \sin(\omega t) + E_2 \sin(\omega t + \delta)$

$= E_1 \sin\omega t + E_2 \sin\omega t \cos\delta + E_2 \cos\omega t \sin\delta$

$= (E_1 + E_2 \cos\delta) \sin\omega t + E_2 \sin\delta \cos\omega t$

$$E_1 + E_2 \cos\delta = E \cos\phi \quad \text{--- (3)}$$
$$E_2 \sin\delta = E \sin\phi \quad \text{--- (4)}$$

$$E_R = E \sin\omega t \cos\phi + E \cos\omega t \sin\phi$$
$$\Rightarrow \boxed{E_R = E \sin(\omega t + \phi)}$$

Resultant wave has same frequency but with different phase difference and amplitude

dividing eqn (4) by eqn (3)

$$\boxed{\tan\phi = \frac{E_2 \sin\delta}{E_1 + E_2 \cos\delta}}$$

By squaring eq^n ③ and eq^n ④ and adding -

$$E^2(\sin^2\phi + \cos^2\phi) = (E_1 + E_2\cos\delta)^2 + (E_2\sin\delta)^2$$

$$\Rightarrow \boxed{E^2 = E_1^2 + E_2^2 + 2E_1E_2\cos\delta} \text{ --- } ⑤$$

The term $2E_1E_2\cos\delta$ is called interference term

Eq^n ⑤ can also be written as

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\delta$$

For I_{max}, $\cos\delta = 1$, $\boxed{\delta = 2n\pi}$ --- ⑥

$n = 0, 1, 2, 3, 4, ---$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}$$

$$= (\sqrt{I_1} + \sqrt{I_2})^2 \text{ --- } ⑦$$

when $I_1 = I_2 = I_0$, $I_{max} = 4I_0$

For I_{min}, $\cos\delta = -1$, $\boxed{\delta = (2n+1)\pi}$ --- ⑧

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2}$$

$$\Rightarrow I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 \text{ --- } ⑨$$

when $I_1 = I_2 = I_0$, $I_{min} = 0$

$$\boxed{\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}}$$

$$I_1 = I_2 = I_0$$

$$I = I_0 + I_0 + 2I_0\cos\delta$$

$$\Rightarrow I = 2I_0(1 + \cos\delta)$$

$$\Rightarrow I = 2I_0 \cdot 2\cos^2\frac{\delta}{2} = 4I_0\cos^2\frac{\delta}{2}$$

$$\Rightarrow I = 4I_0\cos^2\delta/2$$

$\Rightarrow$ The resultant intensity varies according to the cosine square.

④ <u>Condition for max. & min in terms of path difference</u>

Phase difference $= \frac{2\pi}{\lambda}$ (Path difference)

for I_{max}, $\delta = 2n\pi$

$$2n\pi = \frac{2\pi}{\lambda}(\Delta x)$$

$$\Rightarrow \Delta x = n\lambda$$

for I_{min}, $\delta = (2n+1)\pi$

$$\Rightarrow (2n+1)\pi = \frac{2\pi}{\lambda}(\Delta x)$$

$$\Rightarrow \Delta x = (2n+1)\frac{\lambda}{2}$$

$$I_{avg} = \frac{I_{max}+I_{min}}{2} = \frac{4I_0+0}{2} = 2I_0$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$$I_1 = I_2 = I_0$$

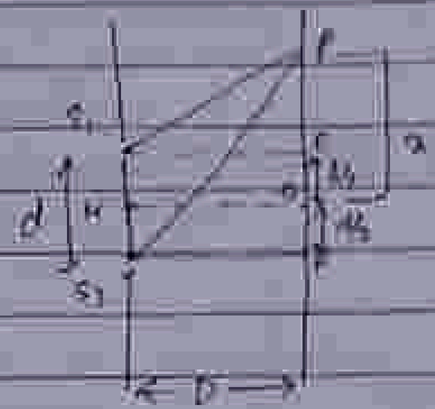
$I = 2I_0 + 2I_0\cos\delta$

$I_{av} = 2I_0 + 2I_0\langle\cos\delta\rangle$

$\boxed{I_{av} = 2I_0}$, $\langle\cos\delta\rangle = 0$

The energy in the process of interference is constant

(1) Determination of fringe width



$S_1S_2 = d$

$S_1M = d/_2 = OE$

$S_2M = d/_2 = OF$

$PE = x - d/_2$

$PF = x + d/_2$

$S_2F = S_1E = D$

In ΔS_1EP

$(S_1P)^2 = (PE)^2 + (S_1E)^2$, $(S_2P)^2 = (S_2F)^2 + (PF)^2$

$\Rightarrow (S_1P)^2 = \left(x - d/_2\right)^2 + D^2$ $\Rightarrow (S_2P)^2 = (D)^2 + \left(x + d/_2\right)^2$

$(S_2P)^2 - (S_1P)^2 = \left(x + d/_2\right)^2 - \left(x - d/_2\right)^2$

$= x^2 + d^2/_4 + dx - \left[x^2 + d^2/_4 - dx\right]$

$(S_2P)^2 - (S_1P)^2 = 2xd$

$(S_2P - S_1P)(S_2P + S_1P) = 2xd$

$S_2P \simeq S_1P \simeq D$

$\Rightarrow (S_2P - S_1P) = \dfrac{2xd}{2D} = \dfrac{xd}{D}$

Path difference $= \dfrac{xd}{D}$

For n^{th} Bright fringe at point P,

Path difference $= n\lambda$

$\Rightarrow n\lambda = \dfrac{x_n d}{D}$

$\Rightarrow x_n = \dfrac{n\lambda D}{d}$ —— (1)

For $(n+1)^{th}$ bright fringe,

$$x_{n+1} = \frac{(n+1)\lambda D}{d}$$

Fringe width $(\beta) = x_{n+1} - x_n$

$$= \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d}$$

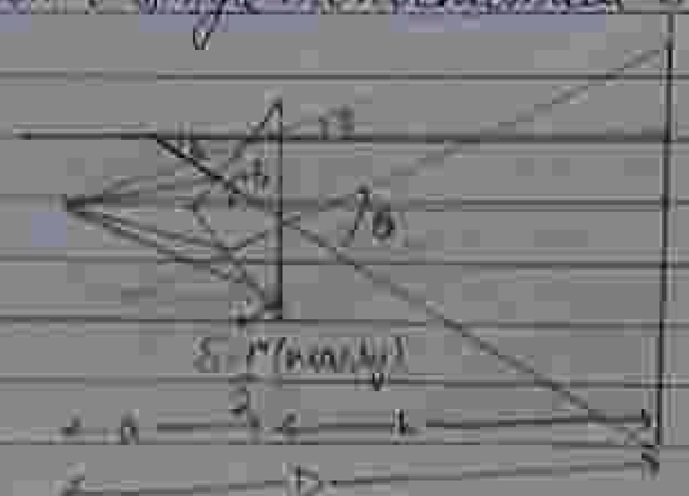
$$\boxed{\beta = \frac{\lambda D}{d}}$$

It implies that

1. β is independent of order of fringe.
2. $\beta \propto \lambda$
3. $\beta \propto D$
4. $\beta \propto \frac{1}{d}$

(4) <u>Fresnel's Biprism Experiment</u> :- <u>It is an experiment that works on the principle of interference & used to determine the wavelength of light</u>

<u>Biprism</u> :- It consists of two prisms with very small refracting angle joined base to base. It produces interference fringes by deriving two coherent sources from a single monochromatic sources.



$$\beta = \frac{\lambda D}{d} \Rightarrow \boxed{\lambda = \frac{\beta \cdot d}{D}}$$

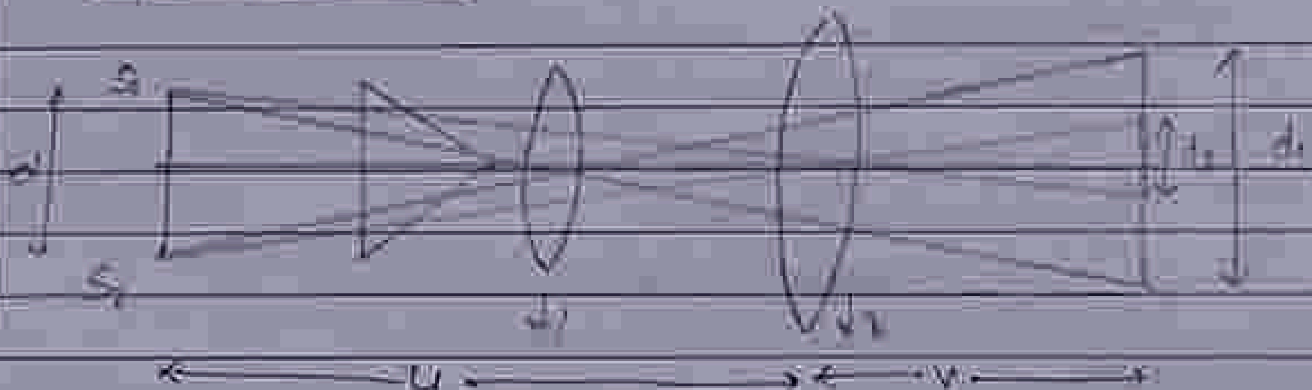
β = known, D = known, d = ?

The distance between the two virtual sources ($2d$) is determined by any one of the following two methods.

① Displacement method

② Deviation method

① Displacement method



From lens L_1, $\frac{v}{u} = \frac{d_1}{d}$ — (I)

From lens L_2, $\frac{u}{v} = \frac{d_2}{d}$ — (II)

From eq^n (I) (II) $\frac{d_1}{d} = \frac{d}{d_2} \Rightarrow d^2 = d_1 d_2$

$\Rightarrow \boxed{d = \sqrt{d_1 d_2}}$

② Deviation method

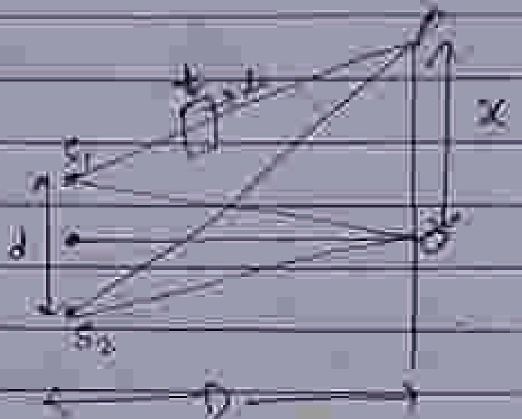
$\delta = (\mu - 1)\alpha$

deviation — angle of prism

$d = a\theta$

$\delta = \theta/2$ — using these $\Rightarrow \frac{d}{2a} = (\mu - 1)\alpha \Rightarrow \boxed{d = 2a(\mu - 1)\alpha}$

Lateral shift in fringes:



$S_1O - S_2O = 0$

$\Rightarrow \boxed{S_1O = S_2O}$

O = central maxima

When a glass of thickness t and refractive index μ is placed in the path of a light ray say, so the position of central maxima is shifted by distance x.

$$\Delta S_1P = (S_1P) + \mu t - t$$

$$\Delta S_2P = S_2P$$

For central maxima,

$$\Delta S_1P = \Delta S_2P$$

$$S_1P - t + \mu t = S_2P$$

$$\Rightarrow S_2P - S_1P = (\mu - 1)t$$

Path difference $= (\mu - 1)t$

We know that,

$$S_2P - S_1P = \frac{xd}{D}$$

$$\Rightarrow \frac{xd}{D} = (\mu - 1)t$$

$$\Rightarrow x = \frac{Dt(\mu - 1)}{d}$$

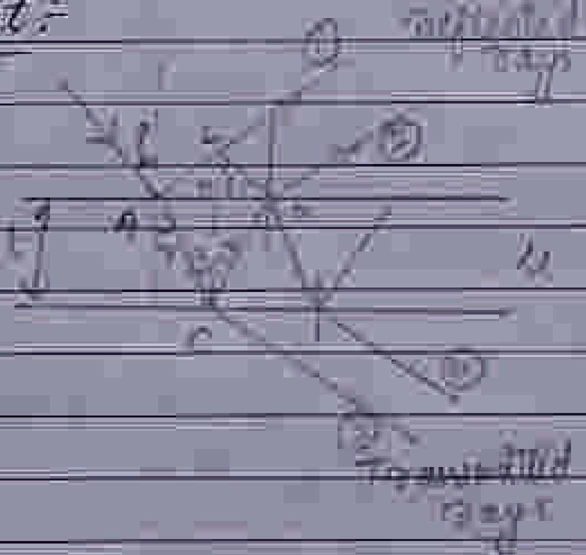
or $\boxed{t = \frac{xd}{D(\mu - 1)}}$

Interference in thin films :-

Thin film: An optical medium having thickness in the order of one wavelength of light in visible region or in the range of 0.5 μm to 10 μm is called thin film

When a thin film of oil spreads on the surface of water and is exposed to white light beautiful colours are seen. This phenomena can be explained on the basis of interference between the light reflected from the upper and lower surfaces of a thin film

Interference due to reflected light :-



The optical path difference between rays ① & ② is

$$\Delta = \mu(AC+CD) - AL$$

ΔACN,

$$\frac{CN}{AC} = \cos r$$

$$\Rightarrow AC = \frac{CN}{\cos r} = \frac{t}{\cos r}$$

Similarly,

ΔCND

$$\Rightarrow \frac{CN}{CD} = \cos r$$

$$\Rightarrow CD = \frac{CN}{\cos r} = \frac{t}{\cos r}$$

$$AC + CD = \frac{t}{\cos r} + \frac{t}{\cos r} = \frac{2t}{\cos r}$$

From ΔANC

$$\frac{AN}{CN} = \tan r \Rightarrow AN = CN \tan r = t \tan r$$

$$\frac{ND}{CN} = \tan r \Rightarrow ND = CN \tan r = t \tan r$$

$AD = AN + ND = t\tan r + t\tan r = 2t\tan r$

From ΔALD,

$$\frac{AL}{AD} = \sin i$$

$$\Rightarrow AL = AD\sin i$$

$$\Rightarrow AL = 2t\tan r\sin i$$

$$\Rightarrow \frac{\sin i}{\sin r} = \mu$$

$$\Rightarrow \sin i = \mu\sin r$$

$$AL = 2t\tan r \times \mu\sin r$$

$$\Rightarrow \frac{2\mu t\sin^2 r}{\cos r}$$

$$\Delta = \mu(AC + CD) - AL$$

$$\Rightarrow \mu\left(\frac{2t}{\cos r}\right) - \frac{2\mu t\sin^2 r}{\cos r}$$

$$\Rightarrow \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$\Delta \Rightarrow \frac{2\mu t(\cos^2 r)}{\cos r} \Rightarrow 2\mu t\cos r$$

As the ray ① is reflected from a denser medium an additional path difference of $\lambda/2$ or phase difference of π is therefore,

The effective path difference $= \Delta - \frac{\lambda}{2}$

$$\boxed{\Delta_t = 2\mu t\cos r - \frac{\lambda}{2}}$$

Condition for max & min intensities:-

(1) For constructive Interference

$$\Rightarrow 2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow \boxed{2\mu t \cos r = n\lambda + \frac{\lambda}{2} = \frac{(2n+1)\lambda}{2}}, \quad n = 0, 1, 2, \ldots$$

(2) For destructive Interference.

$$\Rightarrow 2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \boxed{2\mu t \cos r = n\lambda}, \quad n = 0, 1, 2, 3, \ldots$$

when $t = 0$, the path difference is $\lambda/2$ and the condition of minimum intensity is satisfied and the film will appear dark.

(4) Interference due to transmitted light:-

The optical path difference between rays (3) & (4)

$$\Delta = \mu(BC + CD) - BN \quad \text{--- (1)}$$

In ΔBMC

$$\Rightarrow \frac{MC}{BC} = \cos r$$

$$\Rightarrow BC = \frac{MC}{\cos r} = \frac{t}{\cos r}$$

Similarly ΔMCD

$$\Rightarrow \frac{MC}{CD} = \cos r \Rightarrow CD = \frac{MC}{\cos r} = \frac{t}{\cos r}$$

$$BC + CD = \frac{2t}{\cos r} \quad \text{--- (2)}$$

$$BD = BM + MD$$

In $\Delta BMC \Rightarrow \frac{BM}{MC} = \tan r \Rightarrow BM = MC \tan r = t \tan r$ --- (3)

Similarly from ΔDMC, $MD = t \tan r$

$BD = BM + MD = 2t \tan r$

ΔDNB,

$\sin i = \frac{BN}{BD} \Rightarrow BN = BD \sin i$

$\Rightarrow BN = 2t \tan r \sin i$

$\Rightarrow \frac{\sin i}{\sin r} = \mu$

$\Rightarrow \sin i = \mu \sin r$

$BN = 2t \tan r \, \mu \sin r$

$\Rightarrow BN = 2\mu t \frac{\sin^2 r}{\cos r}$ —— (3)

From Eq^n (2) & (3) in eq^n (1)

$\Delta = \mu(BC + CD) - BN$

$= \mu \left[\frac{2t}{\cos r}\right] - \frac{2\mu t \sin^2 r}{\cos r}$

$\Delta = \frac{2\mu t}{\cos r}(\cos^2 r)$

$\boxed{\Delta = 2\mu t \cos r}$

Condition for constructive interference,

$\boxed{2\mu t \cos r = n\lambda}$, $n = 0, 1, 2, \text{——}$

Condition for destructive interference,

$\boxed{2\mu t \cos r = (2n+1)\frac{\lambda}{2}}$, $n = 0, 1, 2, -$

Colours in thin films:- When a beam of white light is incident normally ($r = 0$) on a thin film and seen in reflected light then coloured fringes will be observed. The path difference between the interfering rays depends on the thickness of the film and $\angle r$. At a particular t, $\angle r$ the waves of only certain wavelength which satisfy the condition of minima ($2\mu t \cos r = n\lambda$) will be absent from the reflected system. Hence the point of the film appeared coloured.

The colours visible in reflected system will be complementary to the colours visible in the transmitted system as the condition of constructive & destructive interference are reversed.

No colours in thick and thin films:- A thick film shows no colours in reflected system when illuminated with a white light. In case of white light if t and r made constant, μ varies with wavelength. Due to large thickness, large number of wavelength satisfy the condition of constructive interference $[2\mu t \cos r = (2n+1)\frac{\lambda}{2}]$ and on the other hand some wavelength satisfy the condition of destructive interference. $[2\mu t \cos r = n\lambda]$ at the same point. The max. and min. intensities overlap and produce uniform illumination. Thus thick films shows no colour but appears white in reflected system.

In case of thin film, when light is observed in reflected mode, it appears black. It is due to the fact that as $t \to 0$ the path difference b/w reflected rays becomes $\lambda/2$. This is a condition of min. intensity for all wavelengths.

In view of the above discussion, we can say that interference occurs only when the optical path difference Δ between the interfering rays is less than the coherence length

$$\Delta << l_{coh}$$

$$\Rightarrow 2\mu t\cos r - \frac{\lambda}{2} << l_{coh}$$

$$l_{coh} = \frac{\lambda^2}{\Delta\lambda}$$

$$\Rightarrow 2\mu t\cos r - \frac{\lambda}{2} << \frac{\lambda^2}{\Delta\lambda}$$

$$\Rightarrow t < \frac{\lambda\left[\frac{\lambda}{\Delta\lambda} + \frac{1}{2}\right]}{2\mu\cos r}$$

when $\frac{\lambda}{\Delta\lambda} >> \frac{1}{2}$ and $\cos r = 1$

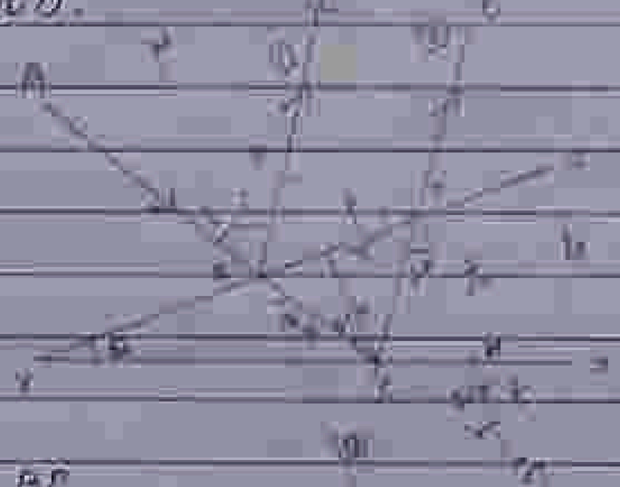
$$\Rightarrow \boxed{t < \frac{\lambda^2}{2\mu\Delta\lambda}}$$

(ii) Interference due to wedge shaped film :-

A thin film having zero thickness at one end and progressively increasing to a particular thickness at the other end is called a wedge

Let wedge shaped thin film XYZ of refractive index μ and angle of wedge θ be considered within air medium.

AB - Incident light
BK - light reflected from the upper surface
BC - light transmitted into the film through upper surface
CD - light reflected from the lower surface of the film.
DE - light transmitted outside the film through the upper surface
MF - Normal to XY
CF and DG Normal to XZ.
t - thickness of the film at point D.



The optical path difference between rays (1) and (2)

$$\Delta = \mu(BC + CD) - BP$$

$$\Delta = \mu(BN + NC + CD) - \mu BN$$

$$\Delta = \mu(NC + CD)$$

$\triangle BPD$

$$\sin i = \frac{BP}{BD}$$

$\triangle DNB$

$$\sin r = \frac{BN}{BD} \Rightarrow \mu = \frac{\sin i}{\sin r} = \frac{BP}{BN}$$

$$\boxed{BP = \mu BN}$$

$$\frac{\sin(\theta + r)}{CD} = \frac{\sin(\theta + r)}{CG} \Rightarrow CD = CG$$

$$\Delta = \mu(NC + CG)$$

$$\Delta = \mu NG \qquad \{ DH = HG = t \quad (DG = 2t)$$

$\triangle DNG$

$$\cos(\theta + r) = \frac{NG}{DG} \Rightarrow NG = 2t\cos(\theta + r)$$

$$\boxed{\Delta = \mu\, 2t\cos(\theta + r)}$$

Due to reflection from ~~denser to rarer~~ rarer to denser medium, an extra path difference of $\lambda/2$ occurs between rays (1) and (2)

The effective path difference:

$$\Delta = 2\mu t\cos(\theta+\alpha) - \frac{\lambda}{2}$$

Condition for max.

$$2\mu t\cos(\theta+\alpha) - \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow \boxed{2\mu t\cos(\theta+\alpha) = (2n+1)\frac{\lambda}{2},\ n=0,1,2\ldots}$$

Condition for minima.

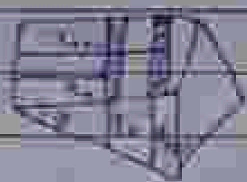
$$2\mu t\cos(\theta+\alpha) - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \boxed{2\mu t\cos(\theta+\alpha) = n\lambda,\ n=0,1,2,\ldots}$$

Fringe width

For n^{th} bright fringe,

$$2\mu t\cos(\alpha+\theta) = (2n+1)\frac{\lambda}{2} \quad \text{— (i)}$$



For $(n+1)^{th}$ bright fringe

$$2\mu t\cos(\theta+\alpha) = (2n+3)\frac{\lambda}{2} \quad \text{— (ii)}$$

$$\tan\theta = \frac{t_n}{x_n} \Rightarrow t_n = x_n\tan\theta, \quad t_{n+1} = x_{n+1}\tan\theta$$

$$2\mu(x_n\tan\theta)\cos(\alpha+\theta) = (2n+1)\frac{\lambda}{2}$$

$$2\mu(x_{n+1}\tan\theta)\cos(\alpha+\theta) = (2n+3)\frac{\lambda}{2}$$

Taking difference,

$$2\mu(x_{n+1}-x_n)\tan\theta\cos(\alpha+\theta) = \lambda$$

For normal incidence ⇒ $i = r = 0$

$\cos(r+\theta) = \cos\theta$

$x_{n+1} - x_n = \beta$

$\Rightarrow 2\mu\beta \tan\theta \cos\theta = \lambda$

$$\boxed{\beta = \frac{\lambda}{2\mu\sin\theta}}$$

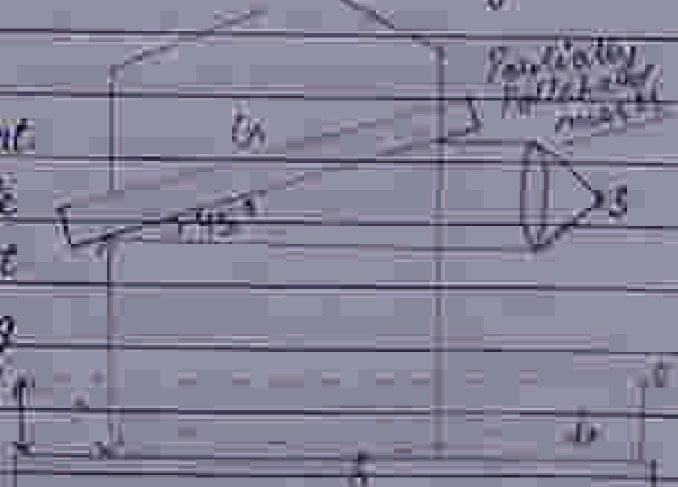
For small angle θ, $\sin\theta = \theta$

$$\boxed{\beta = \frac{\lambda}{2\mu\theta}}$$

Newton's ring

→ It is used to measure the wavelength of a given light

→ It works on the principle of interference due to wedge shaped film



Beams of light from S are incident on G. Being a half silvered plate G will reflect half of the incident light and transmit the remaining. As G is inclined at an angle of 45° to the horizontal. Hence the incident light will suffer 90° deviation on reflection and fall on C normally. The light reflected from the upper and lower surfaces of the film being coherent. Interfere on superposition. Since the upper surface of the thin film is circular so for a particular order the locus of points having constant thickness will be circular and therefore bright and dark will be formed in the form of concentric circle as 'O' as centre

The condition for max,

$$2\mu t\cos(\theta+r) = (2n+1)\frac{\lambda}{2} \quad - \text{max}$$

$$2\mu t\cos(\theta+r) = n\lambda \quad - \text{min}$$

For normal incidence, $r = 0$ and due to large radius of curvature of the convex surface of the lens, $\theta \to 0$

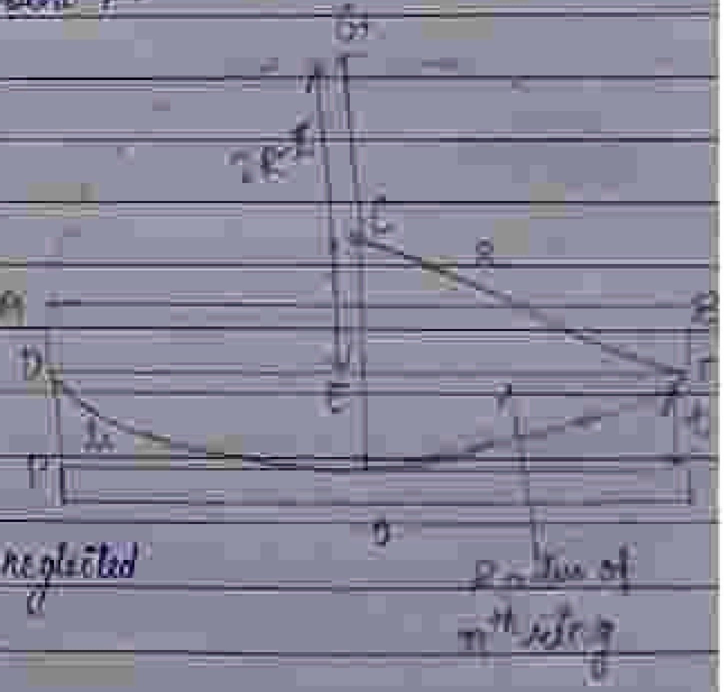
$\Rightarrow \cos(r+\theta) = 1$

$$2\mu t = (2n+1)\frac{\lambda}{2} \quad - \text{max} \; ; n = 0,1,2,3$$

$$2\mu t = n\lambda \quad - \text{min} \; (n = 0,1,2,3)$$

$R \rightarrow$ Radius of curved surface AOB

$t \rightarrow$ thickness of film at point F



From the property of circle,

$DE \times EF = OE \times EG$

$EF = ED = r$ (radius of ring passes through point F)

$r_n \cdot r_n = t(2R-t)$

$\Rightarrow r_n^2 = 2tR - t^2$

$\Rightarrow r_n^2 = 2Rt$, $t \ll R$, t^2 is neglected

$\Rightarrow r_n = \sqrt{2Rt}$

$r_n \propto \sqrt{R}$ and $r_n \propto \sqrt{t}$

<u>For bright fringe</u>, $2\mu t = (2n+1)\frac{\lambda}{2}$

$$\Rightarrow 2\mu \frac{r_n^2}{2R} = \frac{(2n+1)\lambda}{2}$$

$$\Rightarrow r_n^2 = \frac{(2n+1)\lambda R}{2\mu}$$

If D_n is the diameter of the n^{th} bright ring

$$\left(\frac{D_n}{2}\right)^2 = \frac{(2n+1)\lambda R}{2\mu}$$

$$\Rightarrow (D_n)^2 = \frac{2(2n+1)\lambda R}{\mu}$$

$$\Rightarrow D_n = \sqrt{\frac{2(2n+1)\lambda R}{\mu}}$$

For air, $\mu = 1$

$$D_n^2 = 2(2n+1)\lambda R$$

$$\Rightarrow D_n = \sqrt{2\lambda R}\sqrt{2n+1}$$

$$K = \sqrt{2\lambda R}$$

$$D_n = K\sqrt{2n+1}$$

$$\boxed{D_n \propto \sqrt{2n+1}}, \; n = 0, 1, 2, 3$$

$\Rightarrow$ The diameters of the bright rings are proportional to the square root of odd natural numbers

For dark rings

$$2\mu t = n\lambda$$

$$\Rightarrow 2\mu \frac{r_n^2}{2R} = n\lambda \text{, where } r_n = \text{Radius of } n^{th} \text{ dark ring}$$

If D_n is the diameter of n^{th} dark ring

$$\left(\frac{D_n}{2}\right)^2 = \frac{Rn\lambda}{\mu}$$

$$\Rightarrow D_n^2 = \frac{4Rn\lambda}{\mu}$$

For air film, $\mu = 1$

$$\Rightarrow D_n^2 = 4n\lambda R \Rightarrow D_n = \sqrt{4\lambda R}\sqrt{n} = K\sqrt{n}$$

$D_n \propto \sqrt{n}$, $n = 0, 1, 2, 3, \ldots$

The diameter of dark rings are proportional to the square root of the natural numbers

If D_{n+1} & D_n are the diameters of the n^{th} & $(n+1)^{th}$ dark rings, then

$$\Rightarrow D_{n+1} - D_n = k\left[\sqrt{n+1} - \sqrt{n}\right]$$

$$\Rightarrow D_2 - D_1 = k\left[\sqrt{2} - 1\right]$$

$$D_3 - D_2 = k\left(\sqrt{3} - \sqrt{2}\right)$$

$\Rightarrow$ The spacing decreases with the order of the ring & fringes get closer and closer as their order increases

<u>Newton's rings by transmitted light</u>:

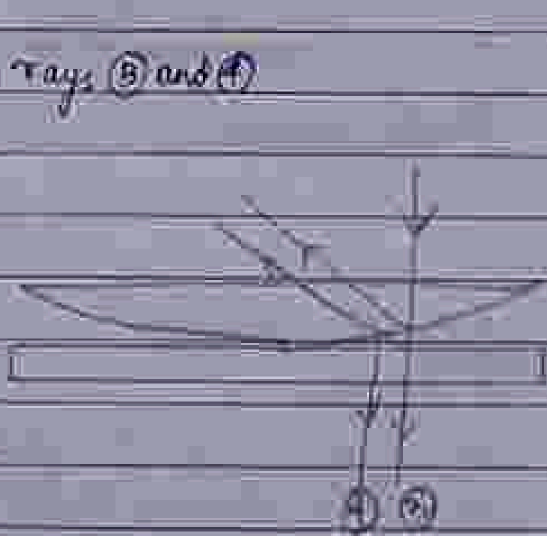
optical path difference between rays ③ and ④

$2\mu t = n\lambda$, $n = 0, 1, 2, 3$

for bright fringe

$2\mu t = (2n+1)\frac{\lambda}{2}$, $n = 0, 1, 2, 3$

for dark fringe



As we know that,

$r_n^2 = 2Rt$

$\Rightarrow t = \frac{r_n^2}{2R}$ (n^{th} order ring radius)

If D_n is the diameter of the n^{th} order ring

for bright ring, $2\mu\left(\frac{D_n}{2}\right)^2 = 2Rn\lambda$

for air film $\mu = 1$ $\Rightarrow$ $D_n^2 = 4Rn\lambda$

$$\Rightarrow D_n = \sqrt{4R\lambda}\sqrt{n} = K\sqrt{n}$$

$$\boxed{D_n \propto \sqrt{n}}$$

For diameter of dark rings:

$$2\mu t = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow 2\mu\left(\frac{D_n}{2}\right)^2 = (2n+1)\lambda R$$

$$\Rightarrow D_n^2 = 2\lambda R\,(2n+1) \quad \{\text{for air film as } \mu = 1\}$$

$$D_n = \sqrt{2\lambda R}\,\sqrt{2n+1}$$

$$D_n = k\sqrt{2n+1}, \quad k = \sqrt{2\lambda R}$$

⇒ In case of transmitted light the diameters of bright rings are proportional to the square root of natural number while the diameter of dark rings are proportional to the square root of the odd natural number and the central ring is bright.

(4) Determination of wavelength of sodium light using Newton's Ring

Let D_n & D_{n+p} are the diameters of the n^{th} & $(n+p)^{th}$ dark rings respectively.

$$D_n^2 = 4n\lambda R, \quad (D_{n+p})^2 = 4(n+p)\lambda R$$

$$(D_{n+p})^2 - (D_n)^2 = 4(n+p)\lambda R - 4n\lambda R = 4p\lambda R$$

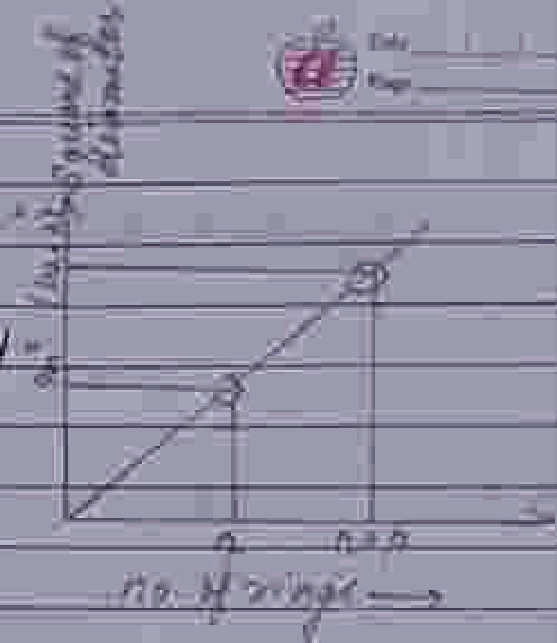
$$\boxed{\lambda = \frac{(D_{n+p})^2 - D_n^2}{4pR}}$$

A graph is plotted between D^2 and no. of rings n the graph is a straight line.

The slope of the line gives, $\dfrac{(D_{n+p})^2 - (D_n)^2}{p}$

$$\lambda = \frac{(D_{n+p})^2 - D_n^2}{4pR}$$

→ If R is known, λ can be calculated using the above Equation.



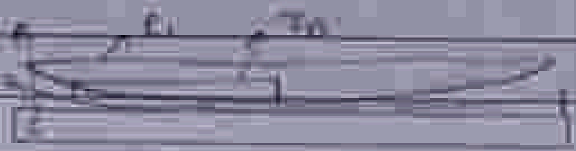
Newton's Rings formed by two curved surface

$R_2 > R_1$

$t = AB = AC - BC$

$AC = \frac{r_n^2}{2R_1}$, $BC = \frac{r_n^2}{2R_2}$

$t = \frac{r_n^2}{2R_1} - \frac{r_n^2}{2R_2}$



$\Rightarrow t = \frac{r_n^2}{2}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

For bright rings,

$$\Delta = 2\mu t - \frac{\lambda}{2} = n\lambda$$

$$2\mu t = (2n+1)\frac{\lambda}{2}$$

$$2\mu \frac{r_n^2}{2}\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (2n+1)\frac{\lambda}{2}$$

⇒ $\mu = 1$ for air film

$\left(r_n = \frac{D_n}{2}\right)$

$$\boxed{D_n^2 = \frac{2(2n+1)\lambda}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}}$$

Similarly for dark ring,

$$\boxed{D_n^2 = \frac{4n\lambda}{\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}}$$

(3) When both the lenses are plano-convex

$t = AB = AC + CB$

$$t = \frac{r_n^2}{2R_1} + \frac{r_n^2}{2R_2}$$

For,

bright fringe,

$$2\mu t = (2n+1)\frac{\lambda}{2}$$

$\Rightarrow$ for air film $\mu = 1$, $r_n = \frac{D_n}{2}$

$$2\left(\frac{D_n^2}{8R_1} + \frac{D_n^2}{8R_2}\right) = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \frac{D_n^2}{4R_1} + \frac{D_n^2}{4R_2} = \frac{(2n+1)\lambda}{2}$$

$$\Rightarrow D_n^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = 2(2n+1)\lambda$$

$$* \boxed{D_n^2 = \frac{2(2n+1)\lambda}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}}$$

For dark ring, $$\boxed{D_n^2 = \frac{4n\lambda}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}}$$

When air is replaced by the liquid

$$(D_n^2)_{liquid} = \frac{4n\lambda R}{\mu} \quad (n^{th} \text{ dark ring})$$

$$(D_n^2)_{air} = 4n\lambda R$$

$$\frac{(D_n^2)_{liq}}{(D_n^2)_{air}} = \frac{1}{\mu} \Rightarrow (D_n^2)_{liq} = \frac{1}{\mu}(D_n^2)_{air}$$

$$\Rightarrow \boxed{(D_n)_{liq} = \frac{(D_n)_{air}}{\sqrt{\mu}}}$$

$$\boxed{(D_n)_{liq} < (D_n)_{air}}$$

Date 28.2.21

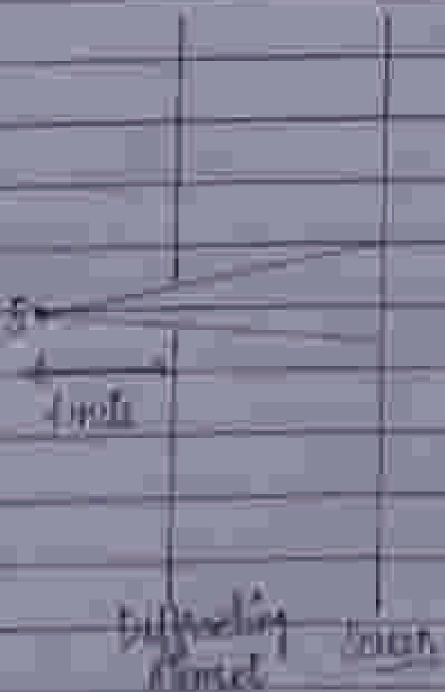
Diffraction

Diffraction: The phenomenon of bending of waves around the corners of an obstacle and their spreading into geometrical shadows is called diffraction and the distribution of light intensity resulting in dark and bright fringes, called diffraction pattern. Diffraction phenomena are caused by the interference of secondary wavelets originated from the undisturbed portion of the wavefront.

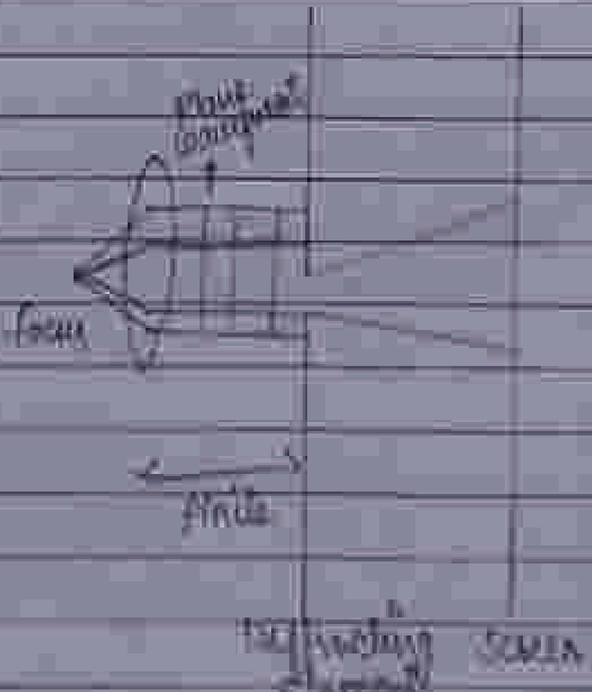
It is divided in two categories:
1. Fresnel's diffraction
2. Fraunhoffer diffraction

Fresnel's diffraction: In this class of diffraction, the source of light and diffracting element are at finite distance.

Fraunhoffer diffraction: In this class of diffraction the source of light and diffracting element are at infinite distance.



Fresnel's diffraction

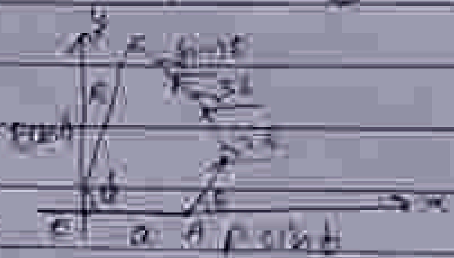


Fraunhoffer diffraction

Resultant of n simple harmonic motions of equal amplitude & period, and phases increasing in arithmetic progression.

$$R\cos\phi = a + a\cos\delta + a\cos 2\delta + \dots + a\cos(n-1)\delta \quad \text{— (1)}$$

$$R\sin\phi = 0 + a\sin\delta + a\sin 2\delta + \dots + a\sin(n-1)\delta \quad \text{— (2)}$$



Multiplying by $2\sin\frac{\delta}{2}$ in eqn (1)

$$2R\cos\phi\sin\frac{\delta}{2} = 2a\sin\frac{\delta}{2} + 2a\sin\frac{\delta}{2}\cos\delta + \dots + 2a\sin\frac{\delta}{2}\cos(n-1)\delta$$

$$2\sin A\cos B = \sin(A+B) - \sin(A-B)$$

$$2R\cos\phi\sin\frac{\delta}{2} = 2a\sin\frac{\delta}{2} + \left(\sin\frac{3\delta}{2} - \sin\frac{\delta}{2}\right) + \left(\sin\frac{5\delta}{2} - \sin\frac{3\delta}{2}\right) + \dots + \left(\sin\left(n-\frac{1}{2}\right)\delta - \sin\left(n-\frac{3}{2}\right)\delta\right)$$

$$= a\left[\sin\frac{\delta}{2} + \sin\left(n-\frac{1}{2}\right)\delta\right]$$

$$2R\cos\phi\sin\frac{\delta}{2} = 2a\sin\frac{n\delta}{2}\cos\frac{(n-1)\delta}{2}$$

$$R\cos\phi = \frac{a\sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}}\cos\frac{(n-1)\delta}{2} \quad \text{— (3)}$$

Similarly we can also obtain

$$R\sin\phi = \frac{a\sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}}\sin\frac{(n-1)\delta}{2} \quad \text{— (4)}$$

Squaring eqⁿ (7) and (8) and adding we get

$$R^2(\cos^2\phi + \sin^2\phi) = \frac{a^2\sin^2\frac{n\delta}{2}}{\sin^2\frac{n\delta}{2}}\left[\cos^2\frac{(n-1)\delta}{2} + \sin^2\frac{(n-1)\delta}{2}\right]$$

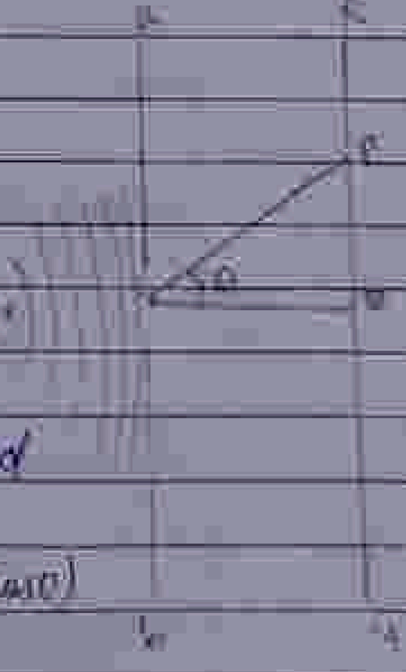
$$R^2 = \frac{a^2\sin^2\frac{n\delta}{2}}{\sin^2\frac{\delta}{2}}$$

$$\boxed{R = \frac{a\sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}}} \quad , \quad \tan\phi = \tan\frac{(n-1)\delta}{2}$$

$$\boxed{\phi = \frac{(n-1)\delta}{2}}$$

Fresnel's assumption to explain diffraction :-

A/c to fresnel's the resultant effect on point P due to a wavefront will depend on the following factors.



① A wavefront can be divided into a large number of zones called fresnel's zone and the resultant effect at any point will depend on the combined effect of all the secondary waves originated from the various zones

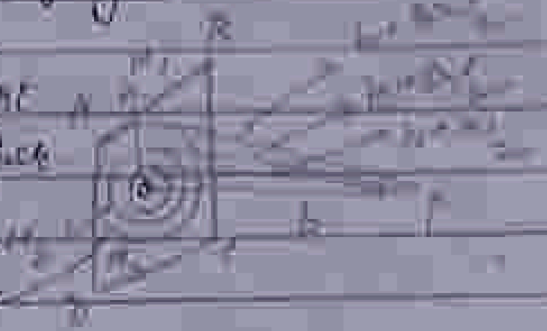
② The effect at point due to any zone will depend on the distance from the zones

③ The effect at a point P is proportional to $(1+\cos\theta)$ called obliquity factor

⇒ The effect is maximum at point O as $\theta = 0°$

(7) <u>Rectilinear propagation of light</u>

ABCD is a plane wavefront $\perp^r$ to the plane of paper



Half period zone

$OP = b$, $OM_1 = r_1$, $OM_2 = r_2$, $OM_3 = r_3$

$M_1P = b + \frac{\lambda}{2}$, $M_2P = b + \frac{2\lambda}{2}$, $M_3P = b + \frac{3\lambda}{2}$

The area of the first half period zones

$$\pi (OM_1)^2 = \pi \left[(M_1P)^2 - (OP)^2 \right]$$

$$= \pi \left[\left(b + \frac{\lambda}{2}\right)^2 - (b)^2 \right]$$

$$= \pi \left[\cancel{b^2} + \frac{\lambda^2}{4} + b\lambda - \cancel{b^2} \right] = \pi b \lambda \quad \text{as } \lambda^2 \text{ neglected}$$

Radius of half period zone $r_1 = OM_1 = \sqrt{b\lambda}$

The radius of 2^{nd} half period zone is $OM_2 = \left[(M_2P)^2 - (OP)^2 \right]^{\frac{1}{2}}$

$$= \left[(b + \lambda)^2 - b^2 \right]^{\frac{1}{2}}$$

$$= \sqrt{2b\lambda}$$

The area of 2^{nd} half period zone is:

$$= \pi \left[(OM_2)^2 - (OM_1)^2 \right]$$

$$= \pi (2b\lambda - b\lambda)$$

$$= \pi b \lambda$$

The area of half period zone is equal to $\pi b \lambda$

$$\boxed{A \propto b}, \boxed{A \propto \lambda}$$

$$\boxed{r_n \propto \sqrt{n}}$$

The effect at point P depends upon
(a) The distance of P from the wavefront
(b) The area of the zone
(c) The obliquity factor

Let M_1, M_2, M_3 represents the amplitude of vibrations of ether particle at point P due to secondary waves from the 1st and 2nd, 3rd ..... The zones are moving from O, the obliquity factor increases and hence the quantities $M_1, M_2 ... M_n$ are of decreasing order.

$$M_2 = \frac{M_1 + M_3}{2}$$

The resultant intensity at P at any instant is given by

$$A = m_1 - m_2 + m_3 - m_4 + ... + m_n \text{ (if n is odd)}$$

&

$$A = m_1 - m_2 + m_3 - m_4 + ... - m_n \text{ (if n is Even)}$$

For n = odd

$$A = \frac{m_1}{2} + \frac{m_1}{2} - m_2 + \frac{m_3}{2} + \frac{m_3}{2} - m_4 - ...... + \frac{m_n}{2}$$

$$= \frac{m_1}{2} + \left(\frac{m_1}{2} - m_2 + \frac{m_3}{2}\right) + \left(\frac{m_3}{2} - m_4 + \frac{m_5}{2}\right) + ... + \frac{m_n}{2}$$

$$= \frac{m_1}{2} + 0 + 0 + 0 + ... \frac{m_n}{2}$$

$$\boxed{A = \frac{m_1 + m_n}{2}}$$

For n = Even: $\boxed{A = \frac{m_1}{2} + \frac{m_{n-1}}{2} - m_n}$

If the complete wavefront taken is unobstructed, the no. of half period zones that can be constructed will be infinite $\Rightarrow n \to \infty$

$\Rightarrow$ Contribution of m_n & m_{n+1} ... $\to 0$

The resultant amplitude at point P due to the complete wave front,

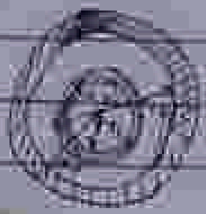
$$A = \frac{m_1}{2}$$

$$\Rightarrow \quad I \propto A^2 \propto \frac{m_1^2}{4}$$

The intensity at point P is only ¼ of that due to the first half period zone alone. It means that when a small obstacle of the size of half of the area of 1st half period zone placed at P will screen the effect of whole wavefront and the intensity at point P due to the rest of the wavefront will be zero while considering the rectilinear propagation of light, if the size of obstacle is greater than the area of 1st half period zone then bending effect in light ray at the edge of the slit can't be observed. When the size of obstacle is comparable is comparable to the wavelength of light, then it is possible to observe illumination in the geometrical shadow region.

(4) Zone plate :- A zone plate is especially constructed screen such that light is obstructed from every alternate zone. For constructing zone plate concentric circles are drawn on white paper such that the radii are proportional to the square root of the natural number

$r_1 = \sqrt{b\lambda}$
$r_2 = \sqrt{2b\lambda}$
$r_3 = \sqrt{3b\lambda}$
⋮



$r_n = \sqrt{nb\lambda}$

$$\boxed{\frac{r_n^2}{n\lambda} = b} \Rightarrow \boxed{f_n = \frac{r_n^2}{n\lambda}}$$

= Zone plate has different foci for different wavelengths.

When source is very far from the plate, the even zones cut-off the light and intensity at P will be.

$A = m_1 + m_3 + m_5 + \dots$ (n = odd)

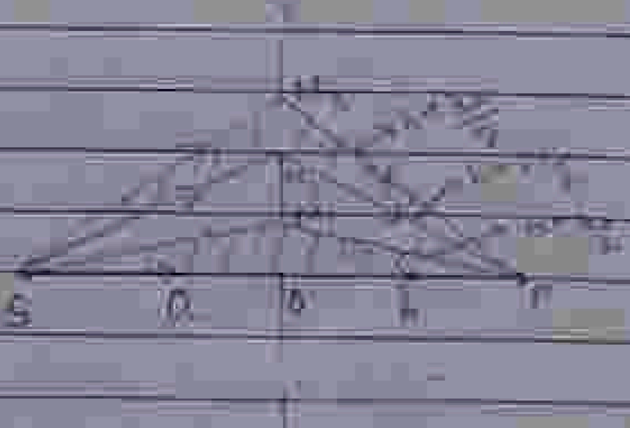
When the complete wavefront is obstructed

$A = m_1 - m_2 + m_3 - m_4 + \dots + m_n$

$A = \frac{m_1}{2}$ (when n is large and odd)

⇒ The function of zone plate is similar to that of convex lens.

Action of a zone plate :-



$SO + OP = a + b$

$SM_1 + M_1P = a + b + \frac{\lambda}{2}$

$SM_2 + M_2P = a + b + \frac{2\lambda}{2}$

from ΔSM_1O

$$SM_1 = (SO^2 + OM_1^2)^{1/2}$$
$$= (a^2 + r_1^2)^{1/2}$$

$$M_1P = (OP^2 + OM_1^2)^{1/2}$$
$$= (b^2 + x_1^2)^{1/2}$$

$$(a^2 + x_1^2)^{1/2} + (b^2 + x_1^2)^{1/2} = a + b + \lambda/2$$

$$\Rightarrow a\left(1 + \frac{x_1^2}{a^2}\right)^{1/2} + b\left(1 + \frac{x_1^2}{b^2}\right)^{1/2} = a + b + \lambda/2$$

$$\Rightarrow a + \frac{x_1^2}{2a} + b + \frac{x_1^2}{2b} = a + b + \frac{\lambda}{2}$$

$$\Rightarrow \frac{x_1^2}{2}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{\lambda}{2}$$

$$\Rightarrow x_n^2\left(\frac{1}{a} + \frac{1}{b}\right) = n\lambda$$

$$\Rightarrow \boxed{\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{n\lambda}{x_n^2}}$$

By applying the sign convention, i.e. $a \rightarrow -a$

$$\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{x_n^2} = \frac{1}{f_n}$$

$$\Rightarrow \boxed{\frac{1}{f_n} = \frac{1}{b} - \frac{1}{a}} \quad \left\{ f_n = \frac{x_n^2}{n\lambda} \right\}$$

The above Eq^n is similar to the Eq^n $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ and therefore zone plate acts as a converging lens.

Difference between Interference and diffraction

Interference	Diffraction
→ It is result of interaction of light coming from different wavefronts originating from the source	→ It is result of interaction of light coming from the different points of same wavefronts
→ The width of fringes is same	→ Fringes are not of the same width

→ The regions of minimum intensity are perfectly dark	→ Regions of minimum intensity are perfectly dark

(B) Fraunhoffer diffraction of Single slit

S = Source
L_1 = Convex lens
AB = Slit = a
XY = screen
L_2 = Converging

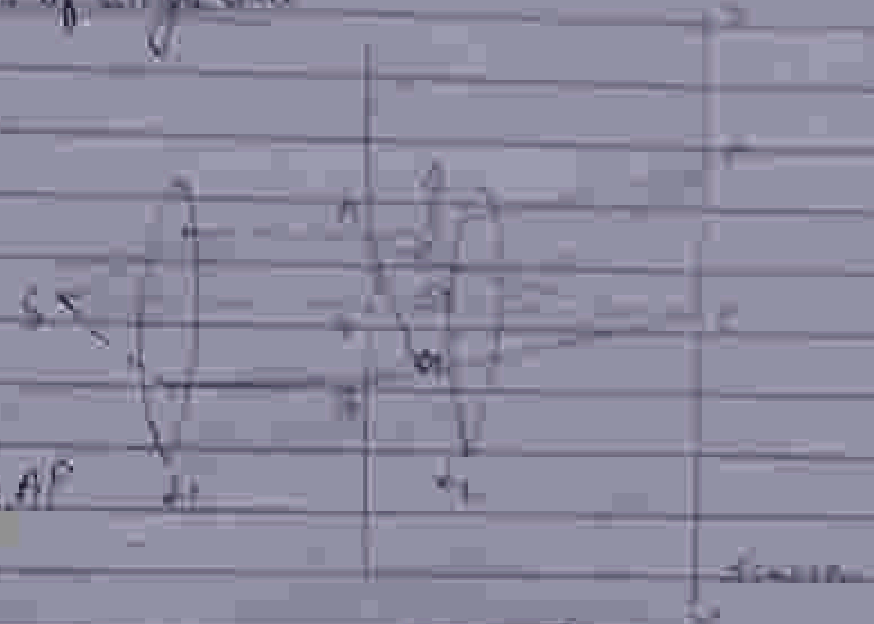


Fig 1

The path difference b/w AP and BP is BN

From $\triangle ABN$

$$\frac{BN}{AB} = \sin\theta$$

$$\Rightarrow BN = AB \sin\theta = a\sin\theta$$

$$\text{Phase difference} = \frac{2\pi}{\lambda}(\text{Path difference})$$

$$= \frac{2\pi}{\lambda}(a\sin\theta)$$

The width AB is supposed to be divided into a large no. of n equal parts. each part acts as source of secondary wavelets the amplitude of the wave due to each part is equal to a but their phases will vary gradually from 0 to $\frac{2\pi}{\lambda}(a\sin\theta)$

→ The phase difference b/w the waves from any two successive parts of the slit AB would be $\frac{1}{n}\left(\frac{2\pi}{\lambda} a\sin\theta\right) = \delta$

⇒ n. no of waves having equal amplitude (a) and a common phase difference are meeting at point P. the resultant amplitude at point P is given by

$$R = \frac{a \sin\left(\frac{n\delta}{2}\right)}{\sin\frac{\delta}{2}} = \frac{a \sin\left\{\frac{\pi a \sin\theta}{\lambda}\right\}}{\sin\left\{\frac{\pi a \sin\theta}{n\lambda}\right\}}$$

$$\alpha = \frac{\pi a \sin\theta}{\lambda}$$

$$R = \frac{a \sin\alpha}{\sin(\alpha/n)} \quad ; \quad \frac{\alpha}{n} \to \text{small} \quad \sin(\alpha/n) = \alpha/n$$

$$R = \frac{na \sin\alpha}{\alpha} = \frac{A \sin\alpha}{\alpha} \quad \boxed{A = na} \quad \text{— (i)}$$

$$\boxed{I = R^2 = \frac{A^2 \sin^2\alpha}{\alpha^2}}$$

Positions of maxima & minima:

$$I = \frac{A^2 \sin^2\alpha}{\alpha^2}$$

As $R = \frac{A \sin\alpha}{\alpha}$

$$= \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} \ldots\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \ldots\right]$$

For max R, $\alpha = 0$, ⇒ $\frac{\pi}{\lambda}(a \sin\theta) = 0$

⇒ $\boxed{\theta = 0}$

$\Rightarrow$ $I_{max} = A^2$ at point c where $\theta = 0$

Position of minima

$$R = \frac{A \sin\alpha}{\alpha}$$

for $R = 0$, $\frac{A \sin\alpha}{\alpha} = 0$

$\Rightarrow \sin\alpha = 0$, $\alpha = \pm n\pi$

$n = 1, 2, 3 \ldots$

$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi$

$$\frac{\pi a \sin\theta}{\lambda} = \pm n\pi$$

$\boxed{a \sin\theta = \pm n\lambda}$, $n = 1$ first minima

$n = 2$, Second minima

$n = 3$, 3^{rd} minima

Secondary maxima :-

$$I = \frac{A^2 \sin^2\alpha}{\alpha^2}$$

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left(\frac{A^2 \sin^2\alpha}{\alpha^2}\right)$$

$$= A^2 \left[\frac{2 \sin\alpha}{\alpha}\left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right)\right] = 0$$

Either, $2A^2 \neq 0$, Hence $\frac{\sin\alpha}{\alpha} = 0$ or $\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$

when $\sin\alpha = 0$, corresponding to minima except $\theta = 0$

When $\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$

$\Rightarrow \boxed{\alpha = \tan\alpha}$

Positions of secondary max is given by the equation

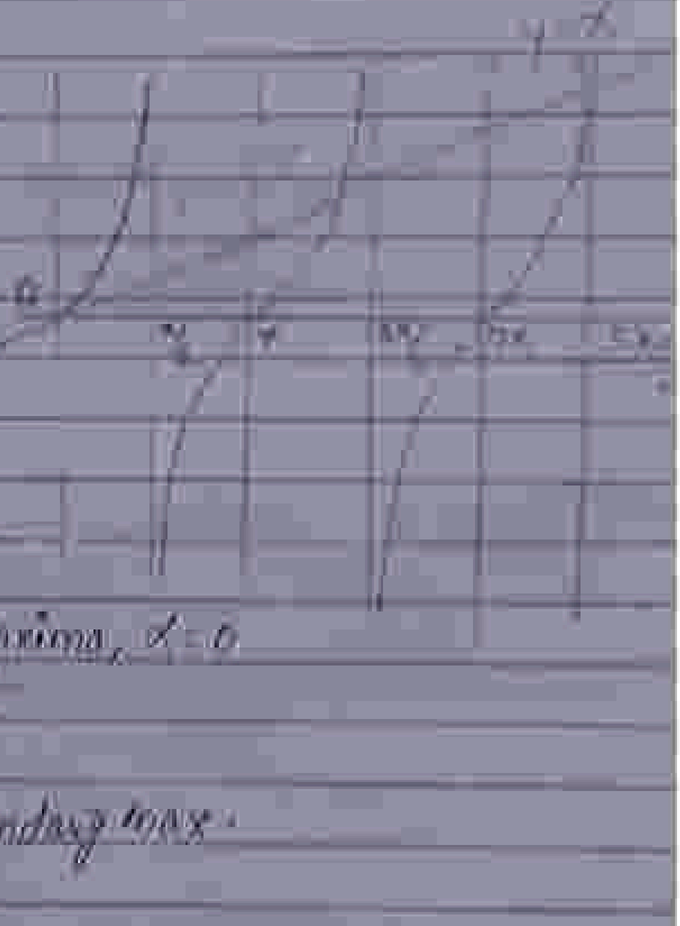
$$\boxed{\alpha = \tan\alpha}$$

It can be solved graphically by plotting the curves of $y = \alpha$ and $y = \tan\alpha$

The point of intersection of these two curves give the value of α that satisfy $\boxed{\alpha = \tan\alpha}$

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$$

$$\alpha = (2n+1)\frac{\pi}{2}$$



$$\frac{\pi a \sin\theta}{\lambda} = (2n+1)\frac{\pi}{2}$$

$$\boxed{a \sin\theta = \pm(2n+1)\frac{\lambda}{2}}$$

For principal or central maxima, $\alpha = 0$

$$\boxed{I = I_0} = A^2$$

The intensity of 1st secondary max:

$$\alpha = \frac{3\pi}{2}$$

$$I_1 = A^2\left(\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right)^2 = \frac{4}{9\pi^2}A^2$$

$$\Rightarrow \frac{I_1}{I_0} = A^2 \times \frac{4}{9\pi^2} \times \frac{1}{A^2} = \frac{4}{9\pi^2} = \frac{1}{22}$$

$$\boxed{I_1 = \frac{I_0}{22}}$$

The intensity of 2nd secondary max.

$$\alpha = \frac{5\pi}{2}$$

$$I_2 = \frac{A^2\left(\sin\frac{5\pi}{2}\right)^2}{\left(\frac{5\pi}{2}\right)^2} = \frac{4A^2}{25\pi^2}$$

$$\frac{I_2}{I_0} = \frac{4}{25\pi^2} \times \frac{A^2}{A^2} = \frac{4}{25\pi^2} = \frac{1}{60}$$

$$\boxed{I_2 = \frac{I_0}{60}}$$

If $I_0 = 1$, then relative intensity of central, first and second max are :-

$$1, \frac{4}{9\pi^2}, \frac{4}{25\pi^2}, \frac{4}{49\pi^2}, \ldots$$

(iii) Width of central maxima

Condition for 1st minima is given by

$$a\sin\theta = \pm\lambda$$

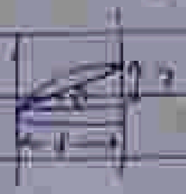
$$\sin\theta = \pm\lambda/a$$

when L_2 is very closed to slit

with reference to fig (1)

slit ...

$$\sin\theta = \theta = \frac{x}{f}$$



$$\Rightarrow \frac{x}{f} = \frac{\lambda}{a}$$

$$\Rightarrow x = \frac{\lambda f}{a}$$

Width of central maxima $= 2x = \frac{2f\lambda}{a}$

Diffraction at a double slit :-

S_1 and S_2 – Slits

S – Source

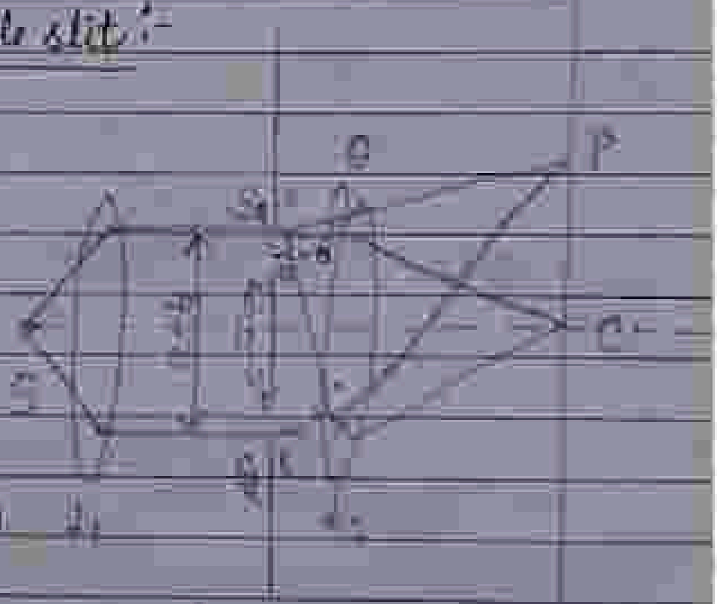
L_1, L_2 = Convex lens

$AC = a+b$

$BD = b$

C = Central max

P = any point of the screen



The resultant amplitude at point P due to all the wavelets diffracted from each slit is given by

$$R = \frac{A\sin\alpha}{\alpha} \quad \text{where } \alpha = \frac{\pi a \sin\theta}{\lambda} \quad \text{—(2)}$$

Central maximum occurs at $\theta = 0$

The path difference b/w two waves originating from S_1 & S_2 and reaching at point P is equal to

$$S_2M = (a+b)\sin\theta \quad \text{—(3)}$$

The phase difference b/w them is

$$\phi = \frac{2\pi}{\lambda}(a+b)\sin\theta \quad \text{—(4)}$$

The resultant amplitude at point P can be obtained by the following diagram.

$$(R')^2 = R^2 + R^2 + 2R^2\cos\phi$$
$$(R')^2 = 2R^2 + 2R^2\cos\phi$$
$$(R')^2 = 2R^2(1+\cos\phi)$$
$$\Rightarrow (R')^2 = 2R^2 \times 2\cos^2\phi/2$$

$$(R')^2 = 4R^2\cos^2\phi/2 \quad \text{—} \quad (5)$$

The values of R and ϕ from eq^n (3) and (4) in (5) we get

$$\boxed{I = (R')^2 = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta}$$

$$\beta = \frac{\phi}{2} = \frac{2\pi}{2\lambda}(a+b)\sin\theta$$

$$\boxed{\beta = \frac{\phi}{2} = \frac{\pi}{\lambda}(a+b)\sin\theta}$$

$$I = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

due to diffraction   due to interference

for central maximum $\theta = 0$. Angular position of minima can be obtained by

$\sin\alpha = 0$ but $\alpha \neq 0$

$\alpha = \pm m\pi$, $m = 1, 2, 3, \ldots$

$\frac{\pi a\sin\theta}{\lambda} = \pm m\pi$

$a\sin\theta = \pm m\lambda \Rightarrow \sin\theta = \pm\left(\frac{m\lambda}{a}\right)$, $m = 1, 2, 3, \ldots$

The positions of secondary maxima are:-

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots$$

A/c to the interference term, the intensity will be max when

$$\cos^2\beta = 1$$

$$\beta = \pm n\pi, \quad n = 0, 1, 2, \ldots$$

$$\frac{\pi(a+b)\sin\theta}{\lambda} = \pm n\pi$$

$$\boxed{(a+b)\sin\theta = \pm n\lambda}$$

The intensity of the interference will be minimum when

$$\cos^2\beta = 0, \quad \beta = \pm(2n+1)\frac{\pi}{2}$$

$$\frac{\pi(a+b)\sin\theta}{\lambda} = \pm(2n+1)\frac{\pi}{2}$$

$$\Rightarrow \boxed{(a+b)\sin\theta = \pm(2n+1)\frac{\lambda}{2}}$$

$$n = 0, 1, 2, \ldots$$

Fringe separation:-

$$\sin\theta_1 = \frac{3\lambda}{2(a+b)}$$

$$\sin\theta_2 = \frac{5\lambda}{2(a+b)}$$

$$\sin\theta_2 - \sin\theta_1 = \frac{2\lambda}{2(a+b)} = \frac{\lambda}{a+b}$$

→ when slit width 'a' is increased the central peak becomes sharper and therefore the number of interference max. falling under the central maxima decreases.

⇒ If the slit separation 'b' is increased the fringe spacing decrease and thus no. of interference maxima fall under the central maxima increase.

(4) Missing order interference max:-

The direction of interference maxima is given by

$$(a+b)\sin\theta = \pm n\lambda, \quad n = 0,1,2,3 \ldots$$

The direction of diffraction minima is given by

$$a\sin\theta = \pm m\lambda, \quad m = 1,2,3 \ldots$$

Since 'θ' is same in both the cases, thus in this condition interference max fall on diffraction minima and therefore that particular order of maxima will be missing in the pattern

$$\frac{(a+b)\sin\theta}{a\sin\theta} = \frac{\pm n\lambda}{\pm m\lambda}$$

$$\boxed{\frac{a+b}{a} = \frac{n}{m}}$$

(i) If $b = a$, $n = 2m$

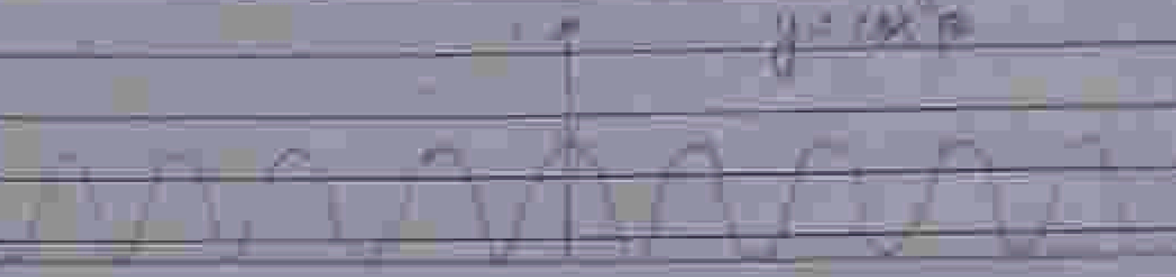
⇒ $2^{nd}, 4^{th}, 6^{th}, \ldots$ order max will be missing

For $m=1$, $n=2$ ⇒ There should be five interference max ($n = 0, \pm1, \pm2$) in the central max. but only $(0, \pm1)$ max will appear in the pattern and 2^{nd} order max will be missing

(ii) $b = 2a$, $n = 3m$, $m = 1,2,3,4,$

$n = 3, 6, 9, 12, \ldots$

⇒ $3^{rd}, 6^{th}, 9^{th}$ order max will be missing in the pattern

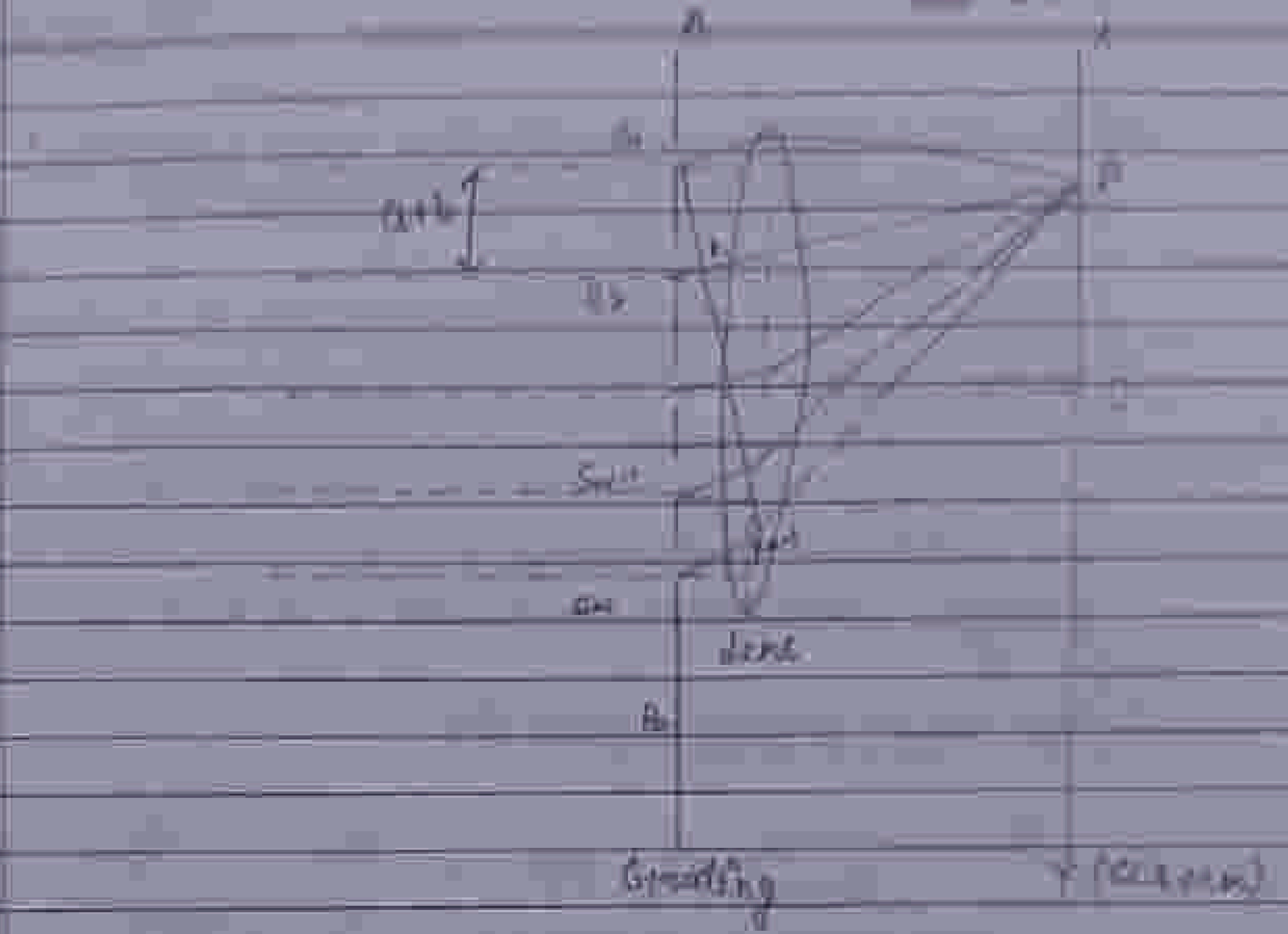


(#) Diffraction Grating (N-slits diffraction)

Diffraction is an arrangement consisting of a large number of transparent and equidistant slits each of equal width 'a' and with neighbouring slits being separated an opaque region of width 'b'. The spacing between the two consecutive slits (a+b) is called diffraction element or grating element.

$S_1, S_2, S_3, \ldots S_n \rightarrow$ No. of slits on grating

AB – Grating, XY → Screen, P → Point where intensity has to be determined.



A/c to theory of single slit diffraction the resultant amplitude "R" due to all wavelets originated from each slit in the direction θ, is given by

$$R = \frac{A \sin\alpha}{\alpha} \quad \text{where } \alpha = \frac{\pi a \sin\theta}{\lambda}$$

Thus, the waves diffracted from all the N slits in direction θ, are equivalent to N parallel waves, each starting from middle point of $S_1, S_2, S_3, \ldots S_n$.

$S_1 K_{n-1}$ = perpendicular from S_1 on $S_2 P$

$\therefore S_1 K_1 = S_1 S_2 \sin\theta = (a+b)\sin\theta$ (path diff. b/w S_1P & S_2P)

$S_2 K_2 = S_2 S_3 \sin\theta = (a+b)\sin\theta$

|

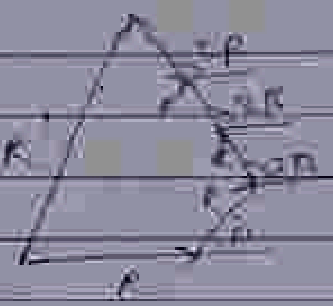
|

$S_n K_{n-1} = S_{n-1} S_n \sin\theta = (a+b)\sin\theta$

⇒ The phase difference b/w the rays coming from two consecutive slits is $\frac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta$

⇒ As we go from one vibration to another the phase difference goes on increasing by an amount of $\frac{2\pi}{\lambda}(a+b)\sin\theta$. The increase in phase difference is in arithmetic progression. Thus the resultant find resultant amplitude of N waves of equal amplitude $\left(\frac{A\sin\alpha}{\alpha}\right)$ and phase is increasing in arithmetic progression.

Thus the amplitude at point P is :-



$$R' = R\frac{\sin N\beta}{\sin\beta}$$

where $\beta = \frac{\pi}{\lambda}(a+b)\sin\theta$

$$R' = \frac{A\sin\alpha}{\alpha}\cdot\frac{\sin N\beta}{\sin\beta}$$

⇒ The intensity at P is given by

$$I = (R')^2 = \underbrace{\frac{A^2\sin^2\alpha}{\alpha^2}}_{\text{Due to diffraction}}\;\underbrace{\frac{\sin^2 N\beta}{\sin^2\beta}}_{\text{Due to interference}}$$

Principal maxima :- For principal max.

$\sin\beta = 0$

$\Rightarrow \beta = \pm n\pi,\ n = 0, 1, 2, 3, \ldots$

⇒ $\sin N\beta = 0 \Rightarrow \frac{\sin N\beta}{\sin\beta}$ becomes indeterminate

$$\lim_{\beta \to n\pi} \frac{\sin N\beta}{\sin \beta} = \lim_{\beta \to n\pi} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin \beta)}$$

$$= \lim_{\beta \to n\pi} \frac{N \cos N\beta}{\cos \beta} = N$$

Intensity of principal maxima.

$$I = \frac{A^2 \sin^2 \alpha \, N^2}{\alpha^2}$$

Direction for principal max —

$$\sin \beta = 0 \Rightarrow \beta = \pm n\pi, \ n = 0, 1, 2, \ldots$$

$$\frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi$$

$$(a+b)\sin\theta = \pm n\lambda$$

$$n = 0, 1, 2, 3, \ldots$$

For $n = 0$, $\theta = 0$ (Zeroth order principal maxima)
$n = 1$, $\theta = \theta_1$ (1st order principal maxima)
$n = 2$, $\theta = \theta_2$ (2nd order principal maxima)

$\Rightarrow$ we have many number of central max.

Minima :- I will be minimum when

$$\sin N\beta = 0 \text{ but } \sin\beta \neq 0$$

$$\Rightarrow N\beta = \pm m\pi$$

$$\Rightarrow N\frac{\pi}{\lambda}(a+b)\sin\theta = \pm m\pi$$

$$\Rightarrow \boxed{N(a+b)\sin\theta = \pm m\lambda}$$

m can have all integral values except 0, N, 2N, 3N, ... because these values of m the $\sin\beta$ will be such that it gives the positions of principal maxima.

$m = N$, Nth order maxima

$\therefore m = 1, 2, 3, \ldots, N-1$, we get

$1^{st}, 2^{nd}, 3^{rd}, \ldots (N-1)^{th}$ minima

$\Rightarrow$ There are $(N-1)$ equispaced minima b/w zero and first order maxima

$\Rightarrow$ $(N-1)$ minima between two successive principal maxima.

(2) <u>Secondary Maxima</u>:-

$$I = \frac{A^2 \sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta}$$

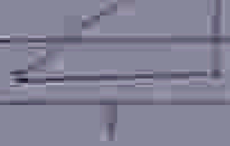
$$\frac{dI}{d\beta} = \frac{A^2\sin^2\alpha}{\alpha^2}\frac{d}{d\beta}\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right) = 0$$

$$\frac{dI}{d\beta} = \frac{A^2\sin^2\alpha}{\alpha^2}\, 2\left[\frac{\sin N\beta}{\sin\beta}\right]\frac{N\cos N\beta \sin\beta - \sin N\beta\cos\beta}{\sin^2\beta}$$

$$\Rightarrow N\cos N\beta\sin\beta - \sin N\beta\cos\beta = 0$$

$$\Rightarrow \tan N\beta = N\tan\beta$$

$$\sin N\beta = \frac{N\tan\beta}{\sqrt{1+N^2\tan^2\beta}}$$



$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2\tan^2\beta}{(1+N^2\tan^2\beta)\sin^2\beta}$$

$$= \frac{N^2}{(\cos^2\beta + N^2\sin^2\beta)}$$

$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{\left(\frac{\cos^2\beta}{\sin^2\beta} + N^2\right)\sin^2\beta}$$

$$= \frac{N^2}{(\cos^2\beta + N^2\sin^2\beta)}$$

$$= \frac{N^2}{(1 - \sin^2\beta + N^2\sin^2\beta)}$$

$$= \frac{N^2}{[1 + (N^2-1)\sin^2\beta]}$$

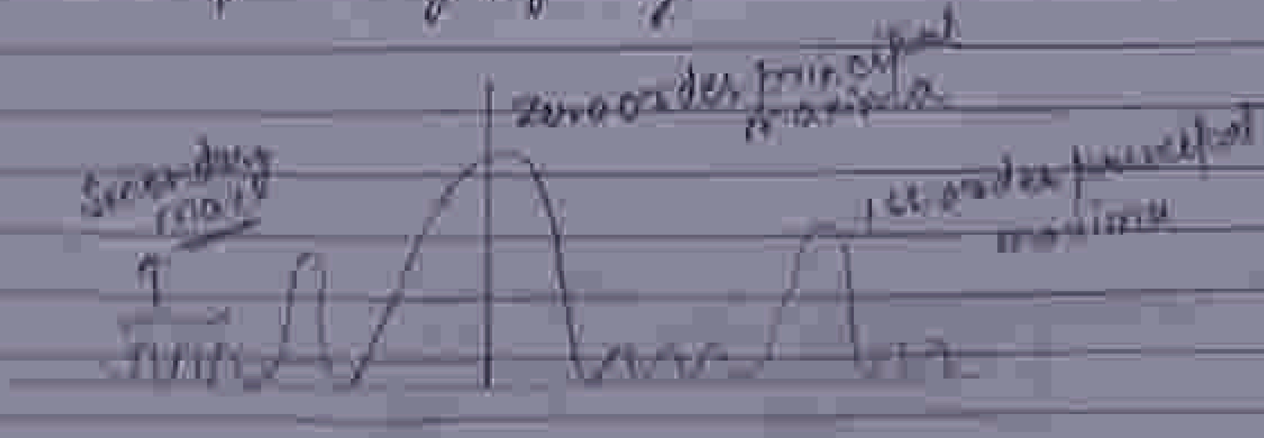
$$I' = \frac{A^2\sin^2\alpha}{\alpha^2}\,\frac{N^2}{[1+(N^2-1)\sin^2\beta]}$$

$$\frac{I'}{I} = \frac{1}{1+(N^2-1)\sin^2\beta}$$

$$\therefore \boxed{I' = \frac{I}{1+(N^2-1)\sin^2\beta}}$$

In actual grating, N is very large, hence secondary maxima are not visible in the grating spectrum.

The distribution of Intensity of grating is



Width of principal maximum

→ The direction of N^{th} order principal maximum.

$$(a+b)\sin\theta = n\lambda \quad \text{--- (1)}$$

→ $(\theta_n + d\theta_n)$ and $(\theta_n - d\theta_n)$ are the directions of 1^{st} order and upper sided minima adjacent to the n^{th} order principal maximum

The direction of minima are

$$N(a+b)\sin\theta = m\lambda \quad \text{--- (2)}$$

For $m = (Nn \pm 1)$

$$N(a+b)\sin(\theta_n \pm d\theta_n) = (nN \pm 1)\lambda$$

$d\theta_n \rightarrow$ Small $\quad \cos d\theta_n \rightarrow 1$

$\sin d\theta_n \rightarrow d\theta_n$

$$N(a+b)(\sin\theta_n \cos d\theta_n \pm \cos\theta_n \sin d\theta_n) = (nN \pm 1)\lambda$$

$$N(a+b)\sin\theta_n \pm N(a+b)\cos\theta_n\, d\theta_n = nN\lambda \pm \lambda$$

$$\cancel{Nn\lambda} \pm N(a+b)\cos\theta_n\, d\theta_n = \cancel{nN\lambda} \pm \lambda$$

$$N(a+b)\cos\theta_n\, d\theta_n = \lambda$$

$$d\theta_n = \frac{\lambda}{N(a+b)\cos\theta_n}$$

The width of n^{th} order principal max. $= \boxed{2d\theta_n = \dfrac{2\lambda}{N(a+b)\cos\theta_n}}$

Missing order with a diffraction grating:-

Max $-$ $(a+b)\sin\theta = n\lambda$, $n = 0, 1, 2, \ldots$

Min $-$ $a\sin\theta = m\lambda$, $m = 1, 2, 3, \ldots$

$$\frac{a+b}{a} = \frac{n}{m}$$

$$\boxed{n = \left(\frac{a+b}{a}\right) m}$$

when $a = b$

$$n = 2m \; [m = 1, 2, 3 \ldots]$$

$\Rightarrow n = 2^{nd}, 4^{th}, 6^{th}$ ... order max will be missing

(iv) Dispersive power of a plane transmission

Spreading: The rate of change of angle of diffraction with the change of wavelength is called dispersion

$$(a+b) \sin\theta = n\lambda$$

$$(a+b) \cos\theta \, d\theta = n \, d\lambda$$

$$(a+b) \cos\theta \frac{d\theta}{d\lambda} = n$$

$$\frac{d\theta}{d\lambda} = \text{dispersive power} = \frac{n}{(a+b)\cos\theta}$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)(1-\sin^2\theta)^{1/2}} \quad \left\{ \because \sin\theta = \frac{n\lambda}{a+b} \right\}$$

$$\Rightarrow \frac{d\theta}{d\lambda} = \frac{n}{(a+b)\left(1 - \frac{n^2\lambda^2}{(a+b)^2}\right)^{1/2}}$$

$$\Rightarrow \boxed{\frac{d\theta}{d\lambda} = \frac{1}{\sqrt{\left(\frac{a+b}{n}\right)^2 - \lambda^2}}}$$

Resolving power of an optical instrument :-

The capacity of an optical instrument to show two close object separately is called resolution and the ability of the instrument to resolve the images of two close point objects is called resolving power.

Rayleigh criterion of resolution

The equally intense spectral lines are just resolved by an optical instruments when the central max. of the diffraction pattern due to one source exactly falls on the 1st minima of the diffraction pattern of other.

$$\text{Resolving power} = \frac{\lambda}{d\lambda}$$

Resolving power of a grating

$(a+b)\sin\theta = n\lambda$. Let 1st min adjacent to the n^{th} order max be formed in the direction $(\theta + d\theta)$

$N(a+b)\sin\theta = Nn\lambda$

$N(a+b)\sin(\theta+d\theta) = (Nn+1)\lambda$

For $\lambda + d\lambda$

$$(a+b)\sin(\theta+d\theta) = n(\lambda+d\lambda)$$

$$N(a+b)\sin(\theta+d\theta) = Nn(\lambda+d\lambda)$$

$$\boxed{\frac{\lambda}{d\lambda} = nN}$$

, n = order of diffraction
N = slits on grating.