1. $O(n) \cdot O(n) = O(n^2)$

2. $f(n) = \Theta\left(f\left(\frac{n}{2}\right)\right)$ – False

$f(n) = 2^{2n}$ $\quad 2^{2n} \neq O(2^n)$

3. a) $T(n) = 5T\left(\frac{n}{3}\right) + n\lg n$

$a = 5$

$b = 3$

$f(n) = n\lg n$

$n^{\log_b a} = n^{\log_3 5} \approx n^{1,47}$

$n\lg n = O(n^{\log_3 4 - \varepsilon})$ $\quad 0 < \varepsilon \leq 0,47$

By MT $T(n) = \Theta(n^{\log_3 5})$

b) $T(n) = 3T\left(\frac{n}{3}\right) + n\lg n$ $\quad n^{\log_b a} = n^{\log_3 3} = n$

$a = 3$

$b = 3$

$f(n) = n\lg n$

c) $T(n) = 8T\left(\frac{n}{2}\right) + n^3\sqrt{n}$

$f(n) = n^3\sqrt{n} = n^{\frac{7}{2}}$

$n^{\log_b a} = n^{\log_2 8} = n^3$

$n^{\frac{7}{2}} = \Omega(n^{3+\varepsilon})$, for $\varepsilon = \frac{1}{2}$

$af\left(\frac{n}{b}\right) = 8\left(\frac{n}{2}\right)^3\sqrt{\frac{n}{2}} = \frac{n^{\frac{7}{2}}}{\sqrt{2}} \leq c\,n^{\frac{7}{2}} \Rightarrow$

$T(n) = \Theta(n^3\sqrt{n})$

d) $T(n) = 2T\left(\frac{n}{2} - 2\right) + \frac{n}{2}$

$f(n) = \frac{n}{2}$ $\quad n^{\log_b a} = n^{\log_2 2} = n$

$T(n) = \Theta(n^{\log_2 2} \lg n) = \Theta(n \lg n)$

e) $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\lg n}$

$f(n) = \frac{n}{\lg n}$ $\quad n^{\log_b a} = n^{\log_2 2} = n$

$T(n) = \Theta(n \lg n \lg n)$

f) $T(n) = T(n/2) + T(n/4) + T(n/8) + n$

$T(n) = 4n + 2n + n + n = 8n$

$T(n) = \Theta(n)$

g) $T(n) = T(n-1) + 1/n$

$T(n) = H(n)$

$H(n) = \Theta(\lg n) \Rightarrow T(n) = \Theta(\lg n)$

h) $T(n) = T(n-1) + \lg n$

$T(n) = \Theta(n \lg n)$

i) $T(n) = T(n-2) + 1/\lg n$

$T(n) = \Theta(n/\lg n)$

j) $T(n) = \sqrt{n}\, T(\sqrt{n}) + n$

$T(n) = \sqrt{n}\, T(\sqrt{n}) + n = \sqrt{n}(\sqrt{n} \lg\lg\sqrt{n}) + n =$

$= n\lg\lg n^{\frac{1}{2}} + n = n\lg((1/2)\lg n) + n = n(\lg(\frac{1}{2}) + \lg\lg n) + n =$

$= -n + n\lg\lg n + n = n\lg\lg n$

4. $T(n) = T(i) + T(n-1-i) + 1$

$nT(n) = 2(T(0) + T(1) + \dots + T(n-1)) + n$

$nT(n) - (n-1)T(n-1) = 2T(n-1) + 1$

$T(n) = n$

5. $2^{2^{n+1}}$

2^{2^n}

$(n+1)!$

$n!$

$n2^n$

e^n

2^n

$\left(\frac{3}{2}\right)^n$

$(\lg n)!$

$n^{\lg(\lg n)}$ $\lg(n)^{\lg n}$

n^3

n^2 $4^{\lg(n)}$

$n\lg n$ $\lg(n!)$

$2^{\lg n}$ n

$(\sqrt{2})^{\lg n}$

$2^{\sqrt{2\lg n}}$

$\lg^2 n$

$\ln n$

$\sqrt{\lg n}$

$\ln(\ln n)$

$2^{\lg^* n}$

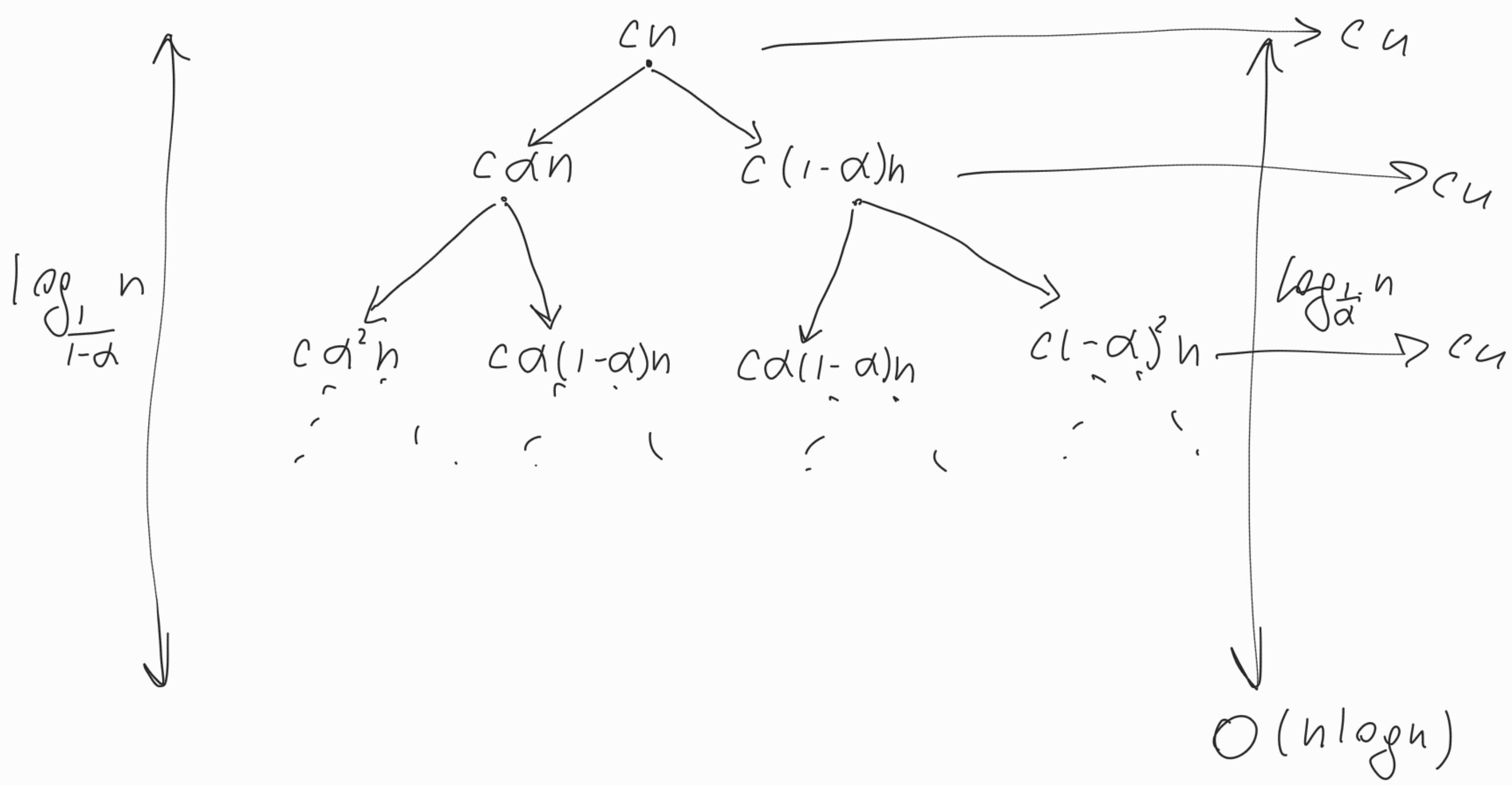
$\lg^* n$ $\lg^* \lg n$

$\lg(\lg^* n)$

1 $n^{\frac{1}{\lg n}}$

6) $T(n) = T(\alpha n) + T((1-\alpha)n) + \Theta n$ $0 < \alpha < 1$

let $\alpha \geq 1-\alpha$, so $0 < 1-\alpha \leq \frac{1}{2}$ and $\frac{1}{2} \leq \alpha < 1$



guess $\Omega(n \log_{\frac{1}{1-\alpha}} n) = \Omega(n \lg n)$

$$T(n) = T(\alpha n) + T((1-\alpha)n) + cn$$

$$\leq d\alpha n \lg(\alpha n) + d(1-\alpha)n \lg((1-\alpha)n) + cn$$

$$= d\alpha n \lg\alpha + d\alpha n \lg n + d(1-\alpha) n \lg(1-\alpha) + d(1-\alpha) n \lg n + cn$$

$$= dn\lg n + dn(\alpha\lg\alpha + (1-\alpha)\lg(1-\alpha)) + cn$$

$$\leq dn\lg n$$