

1. They are not functions as there are two outputs for a single input

2. $y = x^2 + 1$ - function

$$y^2 = x+1 \Rightarrow y = \pm \sqrt{x+1} \Rightarrow \text{not a function}$$

3. $f(x) = 3x$ - no, only the last one

4. first and last

$$5. f(x) = \frac{1}{x+2} \quad y = \frac{1}{x+2} \Rightarrow xy + 2y = 1 \Rightarrow x = \frac{1-2y}{y} = \frac{1}{y} - 2$$
$$f^{-1}(x) = \frac{1}{x} - 2 \quad - \text{true}$$

6. $f(x) = 2 + \sqrt{x-y}$

$$y = 2 + \sqrt{x-y} \Rightarrow (y-2)^2 = x-y \Rightarrow x = y + (y-2)^2 \Rightarrow$$

$$x = y + (y^2 + 4 - 4y) = y + y^2 + 4 - 4y = y^2 - 3y + 4$$

$$f^{-1}(x) = x^2 - 3x + 4$$

$$7. C = \frac{5}{g} (F-32) \quad C = \frac{5}{g} F - \frac{160}{g} \quad F = \left(C + \frac{160}{g}\right) \cdot \frac{g}{5} = \frac{g}{5} C + 32$$

$$F = 1,8C + 32$$

$$8. f(x) = 2\sqrt{x-y} \quad x \geq y \quad \text{domain} \Rightarrow [y; +\infty), \text{range} \Rightarrow [0; +\infty)$$

$$9. h(x) = -2x^2 + 4x - 9 \quad D \in (-\infty; +\infty)$$

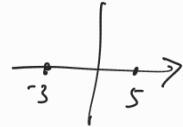
$$\text{Vertex} \Rightarrow -\frac{b}{2a} \quad x = -\frac{4}{-4} = 1 \Rightarrow h(1) = -2 + 4 - 9 = -7$$

$$R \in (-\infty; -7] \quad (1; -7) \quad (2; -3)$$

$$10. f(x) = \frac{x-4}{x^2-2x-15} \quad x^2-2x-15 \neq 0$$

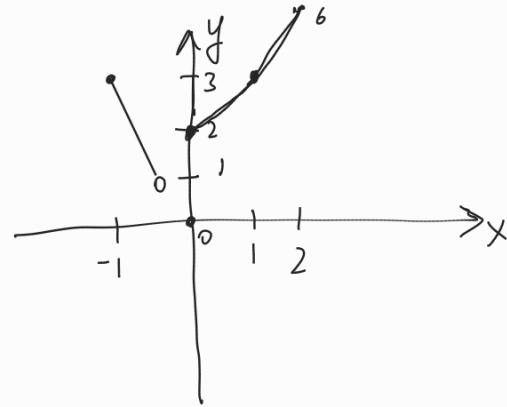
$$D = 4 - 4 \cdot 1 \cdot (-15) = 8^2$$

$$x_{1,2} = \frac{2 \pm 8}{2} \quad x_1 = 5 \quad x_2 = -3$$



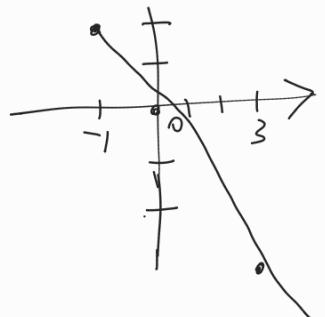
$$D \in (-\infty; +\infty) \setminus \{5, -3\}$$

$$11. f(x) = \begin{cases} -2x+1 & ; -1 \leq x < 0 \\ x^2+2 & ; 0 \leq x \leq 2 \end{cases}$$



$$12. y_1: (-1, 2) \quad \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-4-2}{3+1} = \frac{-6}{4} = -1,5$$

$$y_2: (3, -4)$$

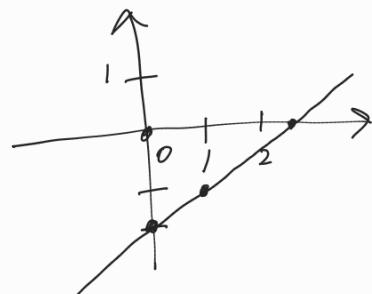


$$13. m = \frac{3}{4} \quad x = (1, -1)$$

$$y = \frac{3}{4}x + b$$

$$-1 = 1 \cdot \frac{3}{4} + b \quad b = -1 - \frac{3}{4}$$

$$b = -1,75$$



$$y = \frac{3}{4}x - 1\frac{3}{4}$$

$$1\frac{3}{4} = \frac{3}{4}x$$

$$\frac{7}{4} \cdot \frac{4}{3}$$

$$14. f(-1) = y \quad ; \quad f(2) = 1$$

$$\text{rate} = \frac{y-1}{-3} = -1$$

$$15. f(x) = x^2 - \frac{1}{x}, \quad [2; 4]$$

$$x_1 = 2 \quad y_1 = 4 - 0,5 = 3,5$$

$$x_2 = 4 \quad y_2 = 16 - 0,25 = 15,75$$

$$\text{rate} = \frac{15,75 - 3,5}{2} = 12\frac{1}{4} \cdot \frac{1}{2} = 6\frac{1}{8} = 6,125$$

$$16. f(t) = t^2 - t, \quad h(x) = 3x + 2 \quad f(h(1))$$

$$h(1) = 5 \quad f(5) = 20$$

$$17. f(x) = \frac{5}{x-1} \quad g(x) = \frac{4}{3x-2} \quad f(g(x)) = \frac{5}{\frac{4}{3x-2} - 1} =$$

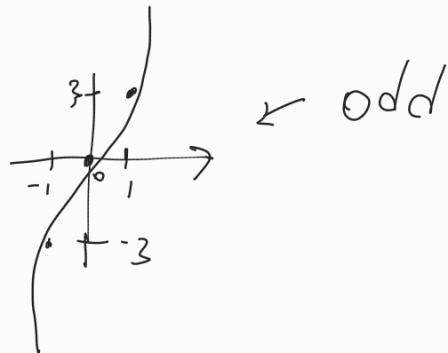
$$= \frac{5(3x-2)}{y-(3x-2)} = \frac{15x-10}{6-3x} ; \quad ; \quad x \neq \frac{2}{3} \quad x \neq 2$$

$$D: (-\infty; \frac{2}{3}) \cup (\frac{2}{3}; 2) \cup (2; +\infty)$$

$$18. \quad f(x) = x-1 \quad (g-f)(x) = x^2 - 1 - x + 1 = x^2 - x = x(x-1)$$

$$g(x) = x^2 - 1 \quad \left(\frac{g}{f}\right)(x) = \frac{x^2 - 1}{x-1} = \frac{(x-1)(x+1)}{x-1} = x+1$$

$$19. \quad f(x) = x^3 + 2x$$



$$f(-x) = -f(x)$$

$$f(-1) = -3$$

$$-f(1) = -3$$

$$20. \quad f(s) = s^4 + 3s^2 + 7 \quad - \text{even}$$

$$21. \quad (5, 1) \quad y = mx + b$$

$$(8, 7)$$

$$1 = 5m + b \quad b = 1 - 5m = 7 - 8m$$

$$7 = 8m + b \quad 8m = 6 \quad m = 2$$

$$y = 2x - 9$$

$$b = 1 - 10 = -9$$

$$22. \quad (3, -2) \quad \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{1+2}{8-3} = \frac{3}{5}$$

23. absolute maxima - 16

minima - (-10)

24. local min - -2

local max - 2

$$25. \quad f(x) = 2x + 3$$

$$h(x) = -2x + 2$$

$$g(x) = \frac{1}{2}x - 4$$

$$j(x) = 2x - 6$$

$$26. \quad \begin{array}{l} 2x + y = 7 \\ x - 2y = 6 \end{array} \quad \begin{array}{l} y = 7 - 2x \\ \frac{x - 6}{2} = y \end{array} \quad \begin{array}{l} 14 - 4x = x - 6 \\ 20 = 5x \quad x = 4 \end{array} \quad y = 7 - 8 = -1$$

$$27. \quad \begin{array}{l} 4x + 2y = 4 \\ 6x - y = 8 \end{array} \quad \begin{array}{l} y = \frac{4 - 4x}{2} = 2 - 2x \\ y = 6x - 8 \end{array} \quad \begin{array}{l} 2 - 2x = 6x - 8 \\ 10 = 8x \quad x = \frac{10}{8} \end{array}$$
$$6 \cdot \frac{10}{8} + \frac{1}{2} = \frac{60}{8} + \frac{1}{2} = \frac{64}{8} = 8 \quad y = 2 - 2 \cdot \frac{10}{8} = \frac{8 - 10}{4} = -\frac{2}{4} = -\frac{1}{2}$$

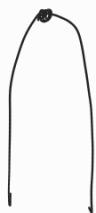
$$28. \quad f(x) = 2x^2 - 6x + 7$$

$$\text{Vertex} \quad \frac{-b}{2a} = \frac{6}{4} = 1,5 \quad x = 1,5$$
$$y = 2 \cdot \frac{9}{4} - \frac{3}{6} \cdot \frac{3}{2} + 7 = 4,5 - 0,75 + 7 = 10,75 - 2 = 8,75$$
$$f(x) = 2 \left(x - \frac{3}{2} \right)^2 + \frac{5}{2}$$

$$29. \quad f(x) = -5x^2 + 8x - 1$$

$$h = -\frac{b}{2a} = \frac{-8}{-10} = \frac{4}{5}$$

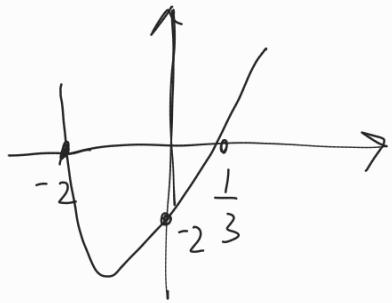
$$f\left(\frac{4}{5}\right) = \frac{61}{20}$$



$$R \in \left(-\infty; \frac{61}{20}\right]$$

$$30. \quad f(x) = 3x^2 + 5x - 2 \quad x_{1,2} = \frac{-5 \pm 7}{6} \quad y_1 = -2$$

$$\mathcal{D} = 25 - 4 \cdot 3 \cdot (-2) = 25 + 24 = 7^2 \quad y_2 = \frac{1}{3}$$

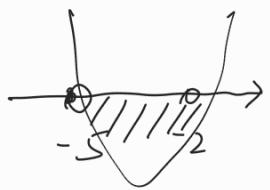


31. $-1 \leq 2x-5 < 7 \quad 4 \leq 2x < 12 \quad 2 \leq x < 6$



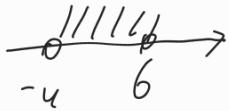
32. $x^2 + 7x + 10 < 0$

$$\Delta = 49 - 4 \cdot 1 \cdot 10 = 3^2 \quad y_{1,2} = \frac{-7 \pm 3}{2} \quad y_1 = -2 \quad y_2 = -5$$



33. $-6 < x - 2 < 4$

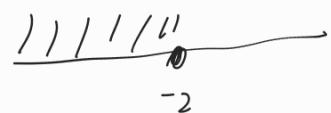
$$-4 < x < 6$$



34. $10 - 2y - 1 \leq -12y - 8 - 3$

$$8 - 2y \leq -12y - 11$$

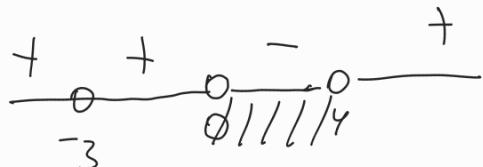
$$20 \leq -10y \quad -2 \geq y$$



$$y \in (-\infty; -2]$$

35. $x(x+3)^2(x-4) < 0$

$$x=0 \quad x=-3 \quad x=4$$



36. $2x^4 > 3x^3 + 8x^2$

$$2x^2 \cancel{x^2} > 3x \cdot \cancel{x^2} + 8 \cdot \cancel{x^2}$$

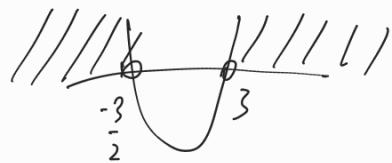
$$3x + 8 < 2x^2$$

$$2x^2 - 3x - 8 > 0$$

$$\vartheta = \varphi + 4 \cdot 2 \circ \varphi = \varphi^2$$

$$x_{1,2} = \frac{3 \pm \sqrt{3}}{4}$$

$$x_1 = 3 \\ x_2 = -\frac{3}{2}$$



$$\left(-\infty; -\frac{3}{2}\right) \cup \left(3; +\infty\right)$$

$$37. \quad f(x) = -\frac{1}{2} |4x - 5| + 3 < 0$$

$$-\frac{1}{2} |4x - 5| < -3$$

$$4x - 5 = 6$$

$$4x = 11 \quad x = \frac{11}{4}$$

$$4x - 5 = -6$$

$$4x = -1 \quad x = -\frac{1}{4}$$

$$|4x - 5| > 6$$

$$\left(-\infty; -\frac{1}{4}\right) \cup \left(\frac{11}{4}; +\infty\right)$$

$$38. \quad 13 - 2|4x - 7| \leq 3$$

$$10 \leq 2|4x - 7|$$

$$|4x - 7| \geq 5$$

$$4x - 7 = 5$$

$$x = 3$$

$$4x - 7 = -5$$

$$x = 0,5$$

