1. $\log_2\left(\dfrac{8\sqrt{2}}{16}\right) + \log_2 32 - 2\log_2 4 = \log_2 2^{\left(3+\frac{1}{2}-4\right)} + \log_2 2^5 - 2\log_2 2^2 =$

$= 3 + \dfrac{1}{2} - 4 + 5 - 4 = \dfrac{1}{2}$

2. $\log_3(x-1) + \log_3(x+1) = 2$

$(x-1)(x+1) = 9$

$x = \pm\sqrt{10}$ $\sqrt{10}$ - answer

3. $20000 = 10000\left(1 + \dfrac{0.06}{4}\right)^{4t}$

$2 = \left(1 + 0,015\right)^{4t} = 1,015^{4t}$

$\ln 2 = 4t \ln 1,015$

$t = \dfrac{\ln 2}{4 \ln 1,015} \approx 11,64$ years

5. $N(t) = N_0 e^{-Kt}$

$70 = 100 e^{-3K}$ $20 = 100 e^{-0,118t}$

$0,7 = e^{-3K}$ $0,2 = e^{-0,118t}$

$\ln 0.7 = -3K$ $\ln 0,2 = -0,118t$

$K = -\dfrac{\ln 0,7}{3} \approx 0,118$ $t = -\dfrac{\ln 0,2}{0,118} \approx 13,5$ hours

6. $\quad A(1,2,3) \qquad \overrightarrow{AB} = (3, 4, 6)$

$\qquad B(4,6,8) \qquad \|\overrightarrow{AB}\| = \sqrt{9 + 16 + 36} = \sqrt{61}$

$$\hat{u} = \frac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|} = \left(\frac{3}{\sqrt{61}}, \frac{4}{\sqrt{61}}, \frac{6}{\sqrt{61}} \right)$$

7. $\quad \vec{a} = (2, -1, 3) \qquad 3\vec{a} - 2\vec{b} = (6, -3, 9) - (-2, 8, 4) =$

$\qquad \vec{b} = (-1, 4, 2) \qquad = (8, -11, 5)$

8. $\quad A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} \qquad\qquad B = \begin{bmatrix} 4 & 5 \\ -2 & 1 \end{bmatrix}$

$2A - 3B = \begin{bmatrix} 4 & -2 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 12 & 15 \\ -6 & 3 \end{bmatrix} = \begin{bmatrix} -8 & -17 \\ 6 & 3 \end{bmatrix}$

9. $\quad C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad D = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

$C \cdot D = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$

10. $\quad \begin{cases} x + y + z = 6 \\ 2x - y + 3z = 14 \\ -3x + 2y - 2z = -10 \end{cases}$

augmented matrix

$$\begin{bmatrix} 1 & 1 & 1 & | & 6 \\ 2 & -1 & 3 & | & 14 \\ -3 & 2 & -2 & | & -10 \end{bmatrix}$$

$R_2 = R_2 - 2R_1 \qquad 0 \quad -3 \quad 1 \quad 2$

$R_3 = R_3 + 3R_1 \qquad 0 \quad 5 \quad 1 \quad 8$

$\downarrow$

$$\begin{bmatrix} 1 & 1 & 1 & | & 6 \\ 0 & -3 & 1 & | & 2 \\ 0 & 5 & 1 & | & 8 \end{bmatrix} \rightarrow$$

$R_3 = R_3 + \left(-\frac{5}{3} R_2 \right)$

$\downarrow$

$$\begin{matrix} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ 0 & 0 & -\frac{2}{3} & \frac{14}{3} \end{matrix}$$

$-\frac{2}{3} \cdot z = \frac{14}{3} \qquad z = -7$

$-3y - 7 = 2$

$y = \frac{9}{-3} = -3$

$x = 6 + 3 + 7 = 16$

11. $B = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$R_1 = R_1 + R_3$

$-1 + 1 = -1 + 1 = 0$

$0 \quad -1 = -1$

$R_1 - [1, 2, 0, -1]$

$R_2 = R_2 - 3R_3$

$R_2 - [0, 1, 0, 8]$

$R_1 = R_1 - 2R_2$

$R_1 - [1, 0, 0, -17]$

$\downarrow$

$\begin{bmatrix} 1 & 0 & 0 & -17 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \qquad A^{-1} = ?$

$$\begin{bmatrix} 2 & 1 & | & 1 & 0 \\ 5 & 3 & | & 0 & 1 \end{bmatrix}$$

$$R_1 = \frac{1}{2}R_1 \quad \rightarrow \begin{bmatrix} 1 & \frac{1}{2} & | & \frac{1}{2} & 0 \\ 5 & 3 & | & 0 & 1 \end{bmatrix}$$

$$R_2 = R_2 - 5R_1 \quad \rightarrow \begin{bmatrix} 1 & \frac{1}{2} & | & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & | & \frac{-5}{2} & 1 \end{bmatrix}$$

$$R_2 = 2R_2 \quad \rightarrow \begin{bmatrix} 1 & \frac{1}{2} & | & \frac{1}{2} & 0 \\ 0 & 1 & | & -5 & 2 \end{bmatrix}$$

$$R_1 = R_1 - \frac{1}{2}R_2 \quad \rightarrow \begin{bmatrix} 1 & 0 & | & 3 & -1 \\ 0 & 1 & | & -5 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$