1. e - irrational

let's say e - rational $e = \frac{a}{b}$

Define an $x = b!\left(\frac{a}{b} - \sum_{n=0}^{b} \frac{1}{n!}\right) = b!\frac{a}{b} - \sum_{n=0}^{b} \frac{b!}{n!} =$

$= a(b-1)! - \sum_{n=0}^{b} \frac{b!}{n!}$, so x - integer

$x = b!\left(\sum_{n=0}^{\infty} \frac{1}{n!} - \sum_{n=0}^{b} \frac{1}{n!}\right) = \sum_{b+1}^{\infty} \frac{b!}{n!} > 0$

$\frac{b!}{n!} = \frac{\cancel{b(b-1)(b-2)\cdots}}{n(n-1)(n-2)\cdots(b+1)\cancel{\cdot b\cdot(b-1)\cdots}} = \frac{1}{n(n-1)(n-2)\cdots(b+1)} \leq$

since n starts from $b+1$

$\leq \frac{1}{(b+1)^{n-b}}$

$0 < x = \sum_{n=b+1}^{\infty} \frac{b!}{n!} < \boxed{\sum_{n=b+1}^{\infty} \frac{1}{(b+1)^{n-b}}}$ $\sum_{k=1}^{\infty} \frac{1}{(b+1)^k} =$

$= \frac{\frac{1}{b+1}}{1 - \frac{1}{b+1}} = \frac{1}{b} < 1$ there is no integer between 0 and 1, so e - is irrational

2. every odd integer is diff of two squares

$|x^2 - y^2| = z$ (odd) if $z = x+y$ and $x-y=1$ then $z = 1+2y$

$x^2 - y^2 = (x-y)(x+y)$ $z = 2k+1$ = odd

$= (k+1)^2 - k^2 = k^2 + 2k + 1 - k^2 = 2k+1 = n = z$

3. $r + t = n$ r - rational
t - irrational
n - irrational

$r = \frac{a}{b}$, $b \neq 0$ - rational number

$\underset{\text{rat}}{S} = \underset{\text{rat}}{r} + \underset{\text{ir}}{i}$ $S + (-r)$ - rational since both are rational
$S - r = \cancel{r} + i - \cancel{r}$ $\underset{\text{real}}{S - r} = \underset{\text{rat.}}{i}$ - contradiction

4. $\underset{\text{irrat.}}{a} \times \underset{\text{irrat.}}{b} = \underset{\text{irrat.}}{p}$ we know for sure that $\sqrt{2}$ - irrat.
so $\sqrt{2} \times \sqrt{2} = 2$ - which is rational. So, it's a counterexample, so it's disproved.

5. if $x + y \geq 2$ (x, y - real numbers), then $x \geq 1$ or $y \geq 1$

if not $x \geq 1$ or $y \geq 1$ then it's not $x + y \geq 2$

$x < 1 \land y < 1$ $x + y < 2$ - negation of $x + y \geq 2$

6. n - int, $n^3 + 5$ - odd, then n - even

1) $n^3 + 5$ - odd, n - odd   if $n \times n$ - odd then n^3 - odd

odd + 5 = odd
not even

2) if n - odd then $n^3 + 5$ - even

$n = 2k + 1$ $n^3 + 5 = (2k+1)^3 + 5 = 8k^3 + 12k^2 + 6k + 6 =$

$= 2(4k^3 + 6k^2 + 3k + 3) = 2n$

7. U - set of all men in the community.

$S(u)$ - propositional function

$\forall x\, (S(x) \to B(x))$

$\forall x \in U : (\neg S(x)) \Leftrightarrow B(x)$

$B(b) \Leftrightarrow S(b)$

$S(b) \Leftrightarrow B(b) \Leftrightarrow (\neg S(b))$

$\neg ((p \Leftrightarrow q) \wedge (q \Leftrightarrow r)) \Rightarrow (p \Leftrightarrow r)$

8. if x and y - integers

xy - even

$x + y$ - even

then x - even and y - even

x, y - not even

x - odd or y - odd

$x = 2m + 1$

y - even $\quad y = 2n \quad x + y = 2m + 1 + 2n = 2(m + n) + 1$ odd

y - odd $\quad y = 2n + 1 \quad xy = (2m+1)(2n+1) = 2(2mn + m + n) + 1$

odd