

1. Prove the following assertion: For every game tree, the utility obtained by MAX using minimax decisions against a suboptimal MIN will be never be lower than the utility obtained playing against an optimal MIN.

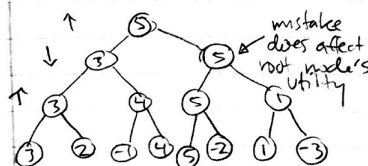
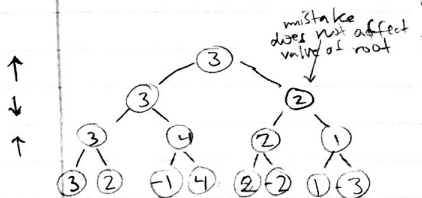
Hw #2

Adam Noack

- ① Let  $T$  denote the set of all possible game tree. Let  $u_{\uparrow\downarrow}(t)$ ,  $u_{\uparrow\downarrow}(t)$  denote the utility for max vs. optim min and max vs. sub-optimal min, respectively. prove the following:  
 $\forall t \in T, u_{\uparrow\downarrow}(t) \leq u_{\uparrow\downarrow}(t)$

$u_{\uparrow\downarrow}(t) \Rightarrow$  at every node in  $t$  max and min choose the action that will lead to the child node with the greatest and least minimax utilities, respectively.

$u_{\uparrow\downarrow}(t) \Rightarrow$  at every node in  $t$  max chooses the action that will lead to the child node with the greatest possible minimax utility but min, at some node, will choose an action that does not lead to a child node with the least possible minimax utility.



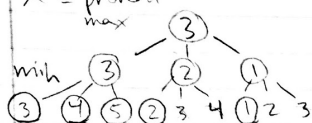
Because max is playing optimally, min's necessary "mistake" has the chance to rise to the root node and therefore this root node's utility  $u_{\uparrow\downarrow}(t)$  can never be less than  $u_{\uparrow\downarrow}(t)$ .

2. Prove that alpha-beta pruning takes time  $O(b^{(m/2)})$  with optimal move ordering, where  $m$  is the maximum depth of the game tree and  $b$  is the branch factor.

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○ = examined  
X = pruned



$$\max(\min(3, 4, 5), \min(2, 3, 4), \min(1, 2, 3))$$

When move ordering is optimal, each player's best move at any given game state (tree node) is examined first.

Whenever it is a given player's turn, all of the child nodes must be evaluated ( $b$  moves). However, because the next player's next possible states are ordered optimally (as in the example above), only the first, leftmost node must be visited in order to prove that the original player should never move down that branch. This effectively prunes every node in the subtree besides the leftmost node for that player at each level.

The result of this optimal ordering is that at every other level,  $l$ , of the tree, only one node is examined instead of  $b^l$ . So instead of  $b_1 * b_2 * \dots * b_m$  nodes being visited only  $b_1 * 1 * b_2 * 1 * \dots * b_m$ , where every other term is a 1 instead of  $b$ .

This effectively collapses to  $b^{m/2}$ .

(This formulation is true only if  $m$  is odd.

If  $m$  is even, the last term will be a 1)

3. Programming assignment:

I did not end up implementing arc consistency or forward checking.

Total assignments to find all 32 solutions: 758.

OUTPUT FROM PROGRAM:

Enter the minuend (5 letters): eight

Enter the subtrahend (4 letters): four

Enter the difference (4 letters): four

EIGHT

-FOUR

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FOUR

IMPLIES THAT...

FOUR

+FOUR

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EIGHT

Solution #1

Cumulative assignments: 99

Variable assignments: {'F': 7, 'I': 4, 'O': 2, 'R': 9, 'c2': 1, 'T': 8, 'c3': 0, 'G': 5, 'H': 3, 'c1': 1, 'E': 1, 'c4': 1, 'U': 6}

Solution #2

Cumulative assignments: 131

Variable assignments: {'F': 6, 'I': 2, 'O': 4, 'R': 5, 'c2': 1, 'T': 0, 'c3': 0, 'G': 9, 'H': 7, 'c1': 1, 'E': 1, 'c4': 1, 'U': 8}

Solution #3

Cumulative assignments: 156

Variable assignments: {'F': 9, 'I': 8, 'O': 2, 'R': 7, 'c2': 1, 'T': 4, 'c3': 0, 'G': 5, 'H': 3, 'c1': 1, 'E': 1, 'c4': 1, 'U': 6}

Solution #4

Cumulative assignments: 187

Variable assignments: {'F': 5, 'I': 0, 'O': 4, 'R': 6, 'c2': 1, 'T': 2, 'c3': 0, 'G': 9, 'H': 7, 'c1': 1, 'E': 1, 'c4': 1, 'U': 8}

Solution #5

Cumulative assignments: 210

Variable assignments: {'F': 8, 'I': 7, 'O': 6, 'R': 5, 'c2': 0, 'T': 0, 'c3': 1, 'G': 2, 'H': 9, 'c1': 1, 'E': 1, 'c4': 1, 'U': 4}

Solution #6

Cumulative assignments: 222

Variable assignments: {'F': 8, 'I': 7, 'O': 5, 'R': 6, 'c2': 0, 'T': 2, 'c3': 1, 'G': 0, 'H': 9, 'c1': 1, 'E': 1, 'c4': 1, 'U': 4}

Solution #7

Cumulative assignments: 276

Variable assignments: {'F': 6, 'I': 3, 'O': 9, 'R': 7, 'c2': 0, 'T': 4, 'c3': 1, 'G': 8, 'H': 5, 'c1': 1, 'E': 1, 'c4': 1, 'U': 2}

Solution #8

Cumulative assignments: 288

Variable assignments: {'F': 6, 'I': 3, 'O': 7, 'R': 9, 'c2': 0, 'T': 8, 'c3': 1, 'G': 4, 'H': 5, 'c1': 1, 'E': 1, 'c4': 1, 'U': 2}

Solution #9

Cumulative assignments: 323

Variable assignments: {'F': 8, 'I': 6, 'O': 2, 'R': 5, 'c2': 0, 'T': 0, 'c3': 0, 'G': 4, 'H': 7, 'c1': 1, 'E': 1, 'c4': 1, 'U': 3}

Solution #10

Cumulative assignments: 343

Variable assignments: {'F': 7, 'I': 4, 'O': 3, 'R': 9, 'c2': 0, 'T': 8, 'c3': 0, 'G': 6, 'H': 5, 'c1': 1, 'E': 1, 'c4': 1, 'U': 2}

Solution #11

Cumulative assignments: 363

Variable assignments: {'F': 6, 'I': 2, 'O': 4, 'R': 5, 'c2': 0, 'T': 0, 'c3': 0, 'G': 8, 'H': 7, 'c1': 1, 'E': 1, 'c4': 1, 'U': 3}

Solution #12

Cumulative assignments: 383

Variable assignments: {'F': 9, 'I': 8, 'O': 3, 'R': 7, 'c2': 0, 'T': 4, 'c3': 0, 'G': 6, 'H': 5, 'c1': 1, 'E': 1, 'c4': 1, 'U': 2}

Solution #13

Cumulative assignments: 395

Variable assignments: {'F': 9, 'I': 8, 'O': 2, 'R': 5, 'c2': 0, 'T': 0, 'c3': 0, 'G': 4, 'H': 7, 'c1': 1, 'E': 1, 'c4': 1, 'U': 3}

Solution #14

Cumulative assignments: 425

Variable assignments: {'F': 5, 'I': 0, 'O': 4, 'R': 6, 'c2': 0, 'T': 2, 'c3': 0, 'G': 8, 'H': 7, 'c1': 1, 'E': 1, 'c4': 1, 'U': 3}

Solution #15

Cumulative assignments: 437

Variable assignments: {'F': 5, 'I': 0, 'O': 2, 'R': 8, 'c2': 0, 'T': 6, 'c3': 0, 'G': 4, 'H': 7, 'c1': 1, 'E': 1, 'c4': 1, 'U': 3}

Solution #16

Cumulative assignments: 439

Variable assignments: {'F': 5, 'I': 0, 'O': 2, 'R': 9, 'c2': 0, 'T': 8, 'c3': 0, 'G': 4, 'H': 7, 'c1': 1, 'E': 1, 'c4': 1, 'U': 3}

Solution #17

Cumulative assignments: 462

Variable assignments: {'F': 8, 'I': 7, 'O': 6, 'R': 2, 'c2': 1, 'T': 4, 'c3': 1, 'G': 3, 'H': 0, 'c1': 0, 'E': 1, 'c4': 1, 'U': 5}

Solution #18

Cumulative assignments: 482

Variable assignments: {'F': 7, 'I': 5, 'O': 6, 'R': 2, 'c2': 1, 'T': 4, 'c3': 1, 'G': 3, 'H': 8, 'c1': 0, 'E': 1, 'c4': 1, 'U': 9}

Solution #19

Cumulative assignments: 502

Variable assignments: {'F': 6, 'I': 3, 'O': 7, 'R': 2, 'c2': 1, 'T': 4, 'c3': 1, 'G': 5, 'H': 8, 'c1': 0, 'E': 1, 'c4': 1, 'U': 9}

Solution #20

Cumulative assignments: 514

Variable assignments: {'F': 6, 'I': 3, 'O': 8, 'R': 2, 'c2': 1, 'T': 4, 'c3': 1, 'G': 7, 'H': 0, 'c1': 0, 'E': 1, 'c4': 1, 'U': 5}

Solution #21

Cumulative assignments: 539

Variable assignments: {'F': 8, 'I': 6, 'O': 3, 'R': 2, 'c2': 1, 'T': 4, 'c3': 0, 'G': 7, 'H': 0, 'c1': 0, 'E': 1, 'c4': 1, 'U': 5}

Solution #22

Cumulative assignments: 579

Variable assignments: {'F': 7, 'I': 4, 'O': 2, 'R': 3, 'c2': 1, 'T': 6, 'c3': 0, 'G': 5, 'H': 8, 'c1': 0, 'E': 1, 'c4': 1, 'U': 9}

Solution #23

Cumulative assignments: 608

Variable assignments: {'F': 6, 'I': 2, 'O': 3, 'R': 4, 'c2': 1, 'T': 8, 'c3': 0, 'G': 7, 'H': 0, 'c1': 0, 'E': 1, 'c4': 1, 'U': 5}

Solution #24

Cumulative assignments: 642

Variable assignments: {'F': 9, 'I': 8, 'O': 3, 'R': 2, 'c2': 1, 'T': 4, 'c3': 0, 'G': 7, 'H': 0, 'c1': 0, 'E': 1, 'c4': 1, 'U': 5}

Solution #25

Cumulative assignments: 653

Variable assignments: {'F': 9, 'I': 8, 'O': 2, 'R': 3, 'c2': 1, 'T': 6, 'c3': 0, 'G': 5, 'H': 4, 'c1': 0, 'E': 1, 'c4': 1, 'U': 7}

Solution #26

Cumulative assignments: 672

Variable assignments: {'F': 5, 'I': 0, 'O': 3, 'R': 2, 'c2': 1, 'T': 4, 'c3': 0, 'G': 7, 'H': 8, 'c1': 0, 'E': 1, 'c4': 1, 'U': 9}

Solution #27

Cumulative assignments: 678

Variable assignments: {'F': 5, 'I': 0, 'O': 3, 'R': 4, 'c2': 1, 'T': 8, 'c3': 0, 'G': 7, 'H': 2, 'c1': 0, 'E': 1, 'c4': 1, 'U': 6}

Solution #28

Cumulative assignments: 684

Variable assignments: {'F': 5, 'I': 0, 'O': 3, 'R': 2, 'c2': 1, 'T': 4, 'c3': 0, 'G': 7, 'H': 6, 'c1': 0, 'E': 1, 'c4': 1, 'U': 8}

Solution #29

Cumulative assignments: 728

Variable assignments: {'F': 8, 'I': 7, 'O': 5, 'R': 2, 'c2': 0, 'T': 4, 'c3': 1, 'G': 0, 'H': 6, 'c1': 0, 'E': 1, 'c4': 1, 'U': 3}

Solution #30

Cumulative assignments: 734

Variable assignments: {'F': 8, 'I': 7, 'O': 5, 'R': 3, 'c2': 0, 'T': 6, 'c3': 1, 'G': 0, 'H': 4, 'c1': 0, 'E': 1, 'c4': 1, 'U': 2}

Solution #31

Cumulative assignments: 752

Variable assignments: {'F': 7, 'I': 5, 'O': 9, 'R': 2, 'c2': 0, 'T': 4, 'c3': 1, 'G': 8, 'H': 6, 'c1': 0, 'E': 1, 'c4': 1, 'U': 3}

Solution #32

Cumulative assignments: 758

Variable assignments: {'F': 7, 'I': 5, 'O': 9, 'R': 3, 'c2': 0, 'T': 6, 'c3': 1, 'G': 8, 'H': 4, 'c1': 0, 'E': 1, 'c4': 1, 'U': 2}