Problem 1. The CONS, CAR, CDR functions are defined as follows.
CONS $= \lambda x.\lambda y.\lambda f.fxy$
CAR $= \lambda p.p(\lambda x.\lambda y.x)$
CDR $= \lambda p.p(\lambda x.\lambda y.y)$

Let $a = \lambda x.x$, $b = \lambda x.\lambda y.x$. Then,
(CONS a b) $= \lambda f.f(\lambda x.x)(\lambda x.\lambda y.x)$,
(CAR (CONS a b)) $= (\lambda x.\lambda y.x)(\lambda x.x)(\lambda x.\lambda y.x) = (\lambda y.(\lambda x.x))(\lambda x.\lambda y.x) = \lambda x.x = a$,
(CDR (CONS a b)) $= (\lambda x.\lambda y.y)(\lambda x.x)(\lambda x.\lambda y.x) = (\lambda y.y)(\lambda x.\lambda y.x) = (\lambda x.\lambda y.x) = b$

Problem 2. if and recursion are defined as follows.
if e_1 e_2 $e_3 = (e_1(\lambda k.e_2)(\lambda k.e_3))(\lambda k.k)$,
fac $= Z(\lambda f.\lambda n.\text{if } n = 0\ 1\ n \times f(n-1))$, where $Z = \lambda f.(\lambda x.f(\lambda z.xx))(\lambda x.f(\lambda z.xx))$

Problem 3. CPS transformation functions of (e_1, e_2) and $e.1$ are defined as follows.
$[(e_1, e_2)] = \lambda\kappa.[e_1](\lambda v_1.[e_2](\lambda v_2.\kappa(\text{CONS } v_1\ v_2)))$,
$[e.1] = \lambda\kappa.[e](\lambda v.\kappa(\text{CAR } v))$ where CONS and CAR defined in problm 1.

Problem 4.a. The constraint generation function is defined as follows.
$V(\Gamma, \text{if0 } e_1\ e_2\ e_3, \tau) = V(\Gamma, e_1, \text{int}) \wedge V(\Gamma, e_2, \tau) \wedge V(\Gamma, e_3, \tau)$

Problem 4.b. $\tau = (\text{int} \to \text{int}) \to \text{int}$, since the constraint equations and its solutions are
$V(\emptyset, \lambda z.\text{let } f = \lambda x.(\text{if0 } x\ 1\ (z\ x)) \text{ in } (f\ 1), \tau)$
$= \exists\alpha_1, \alpha_2.(\tau \doteq \alpha_1 \to \alpha_2) \wedge V(z : \alpha_1, \text{let } f = \lambda x.(\text{if0 } x\ 1\ (z\ x)) \text{ in } (f\ 1), \alpha_2)$
$= \exists\alpha_1, \alpha_2.(\tau \doteq \alpha_1 \to \alpha_2) \wedge (\exists\alpha_3.V(z : \alpha_1, \lambda x.(\text{if0 } x\ 1\ (z\ x)), \alpha_3) \wedge V(\{z : \alpha_1, f : \alpha_3\}, (f\ 1), \alpha_2))$
$= \exists\alpha_1, \alpha_2, \alpha_3.(\tau \doteq \alpha_1 \to \alpha_2) \wedge V(z : \alpha_1, \lambda x.(\text{if0 } x\ 1\ (z\ x)), \alpha_3) \wedge V(\{z : \alpha_1, f : \alpha_3\}, (f\ 1), \alpha_2)$
$= \exists\alpha_1, \alpha_2, \alpha_3.(\tau \doteq \alpha_1 \to \alpha_2) \wedge V(z : \alpha_1, \lambda x.(\text{if0 } x\ 1\ (z\ x)), \alpha_3) \wedge$
$(\exists\alpha_4.V(\{z : \alpha_1, f : \alpha_3\}, f, \alpha_4 \to \alpha_2) \wedge V(\{z : \alpha_1, f : \alpha_3\}, 1, \alpha_4))$, so $\alpha_4 \doteq \text{int}$
$= \exists\alpha_1, \alpha_2, \alpha_3.(\tau \doteq \alpha_1 \to \alpha_2) \wedge V(z : \alpha_1, \lambda x.(\text{if0 } x\ 1\ (z\ x)), \alpha_3) \wedge$
$V(\{z : \alpha_1, f : \alpha_3\}, f, \text{int} \to \alpha_2)$
$= \exists\alpha_1, \alpha_2, \alpha_3.(\tau \doteq \alpha_1 \to \alpha_2) \wedge V(z : \alpha_1, \lambda x.(\text{if0 } x\ 1\ (z\ x)), \alpha_3) \wedge (\alpha_3 \doteq \text{int} \to \alpha_2)$
$= \exists\alpha_1, \alpha_2, \alpha_3.(\tau \doteq \alpha_1 \to \alpha_2) \wedge (\alpha_3 \doteq \text{int} \to \tau)$
$(\exists\alpha_4, \alpha_5.(\alpha_3 \doteq \alpha_4 \to \alpha_5) \wedge V(\{z : \alpha_1, x : \alpha_4\}, \text{if0 } x\ 1\ (z\ x), \alpha_5))$, so $\alpha_4 \doteq \text{int}$
$= \exists\alpha_1, \alpha_2, \alpha_3, \alpha_5.(\tau \doteq \alpha_1 \to \alpha_2) \wedge (\alpha_3 \doteq \text{int} \to \alpha_2)$
$(\exists\alpha_4, \alpha_5.(\alpha_3 \doteq \alpha_4 \to \alpha_5) \wedge V(\{z : \alpha_1, x : \alpha_4\}, \text{if0 } x\ 1\ (z\ x), \alpha_5))$, so $\alpha_4 \doteq \text{int}, \alpha_5 \doteq \alpha_2$
$= \exists\alpha_1, \alpha_2, \alpha_3.(\tau \doteq \alpha_1 \to \alpha_2) \wedge (\alpha_3 \doteq \text{int} \to \alpha_2) \wedge V(\{z : \alpha_1, x : \text{int}\}, \text{if0 } x\ 1\ (z\ x), \alpha_2))$
$= \exists\alpha_1, \alpha_2, \alpha_3.(\tau \doteq \alpha_1 \to \alpha_2) \wedge (\alpha_3 \doteq \text{int} \to \alpha_2) \wedge$
$V(\{z : \alpha_1, x : \text{int}\}, 1, \alpha_2) \wedge V(\{z : \alpha_1, x : \text{int}\}, (z\ x), \alpha_2))$, so $\alpha_2 \doteq \text{int}$
$= \exists\alpha_1, \alpha_3.(\tau \doteq \alpha_1 \to \text{int}) \wedge (\alpha_3 \doteq \text{int} \to \text{int}) \wedge V(\{z : \alpha_1, x : \text{int}\}, (z\ x), \text{int}))$
$= \exists\alpha_1.(\tau \doteq \alpha_1 \to \text{int}) \wedge V(\{z : \alpha_1, x : \text{int}\}, (z\ x), \text{int}))$
$= \exists\alpha_1.(\tau \doteq \alpha_1 \to \text{int}) \wedge$
$(\exists\alpha_2.V(\{z : \alpha_1, x : \text{int}\}, z, \alpha_2 \to \text{int}) \wedge V(\{z : \alpha_1, x : \text{int}\}, x, \alpha_2))$, so $\alpha_2 \doteq \text{int}$
$= \exists\alpha_1.(\tau \doteq \alpha_1 \to \text{int}) \wedge V(\{z : \alpha_1, x : \text{int}\}, z, \text{int} \to \text{int})$, so $\alpha_1 = \text{int} \to \text{int}$
$= \tau \doteq (\text{int} \to \text{int}) \to \text{int})$

Problem 4.c. Please refer to the other sheets.

Problem 5. Please refer to the other sheets.