

2016 Spring CS300 Homework #6

Due: June 13 AM 10:30

TA in charge of HW6: Heewon Chung

1. Another way to perform topological sorting on a directed acyclic graph $G = (V, E)$ is to repeatedly find a vertex of in-degree 0, output it, and remove it and all of its outgoing edges from the graph. Explain how to implement this idea so that it runs in time $O(V+E)$. What happens to this algorithm if G has cycles?

2. Suppose that $L_1 <_p L_2$. For each of the following statements, determine whether it is true, false, or an open problem. Justify your answers.

(a) If $L_1, L_2 \in \text{NP}$ and $L_2 \in \text{P}$, then $L_1 \in \text{P}$.

(b) If $L_2 \in \text{NP}$, then L_2 is either NP-complete or is in P.

(c) If $\text{SAT} <_p L_1$, then L_2 is NP-complete

(d) If SAT problem is in P, then $\text{co-NP} \neq \text{P}$.

(e) If a problem in NP can be solved in polynomial time, then all problems in NP can be solved in polynomial time.

(f) If an NP-complete problem can be solved in linear time, then all NP-complete problems can be solved in linear time

3. Prove that if $\text{NP} \neq \text{co-NP}$, then $\text{P} \neq \text{NP}$.

4. Prove that $L <_p \bar{L}$ if and only if $\bar{L} <_p L$.