Problem 1. Use normal-order reduction.
$Y_t F \rightarrow (\lambda f.\lambda x.x(ffx))(\lambda f.\lambda x.x(ffx))F$
$\rightarrow (\lambda x.x((\lambda f.\lambda x.x(ffx))(\lambda f.\lambda x.x(ffx))x))F$
$\rightarrow F((\lambda f.\lambda x.x(ffx))(\lambda f.\lambda x.x(ffx))F)$
$\rightarrow F(Y_t F)$

Problem 2. Compilation rules from E to C.
$n \rhd n$, $x \rhd x$

$$\frac{e_1 \rhd C_1 \;\; e_2 \rhd C_2}{e_1 + e_2 \rhd C_1.C_2.add}, \quad \frac{e_1 \rhd C_1 \;\; e_2 \rhd C_2 \;\; e_3 \rhd C_3}{if\ e_1\ e_2\ e_3 \rhd C_1.jmp.C_2.C_3}, \quad \frac{e \rhd C}{fn\ x\ e \rhd \lambda x.C}, \quad \frac{e \rhd C}{rec\ f\ e \rhd rec\ f\ \lambda x.C}$$
$$\frac{e_1 \rhd C_1 \;\; e_2 \rhd C_2}{e_1\ e_2 \rhd C_1.C_2.app}, \quad \frac{e_1 \rhd C_1 \;\; e_2 \rhd C_2}{let\ x\ e_1\ e_2 \rhd C_1.x^+.C_2.x^-}$$
$$\frac{e_1 \rhd C_1 \;\; e_2 \rhd C_2}{(e_1, e_2) \rhd C_1.C_2.pair}, \quad \frac{e \rhd C}{fst\ e \rhd C.fst}, \quad \frac{e \rhd C}{snd\ e \rhd C.snd}$$

Transition rules of (S, E, C, D) machine. (* some rules are from lecture notes)
$(S, E, n.C, D) \rightarrow (n.S, E, C, D)$, $(S, E, x.C, D) \rightarrow (x.S, E, C, D)$

$(n_1.n_2.S, E, add.C, D) \rightarrow (n_1 + n_2.S, E, C, D)$
$(0.S, E, jmp.C_1.C_2.C, D) \rightarrow (S, E, C_1.C, D)$
$(n.S, E, jmp.C_1.C_2.C, D) \rightarrow (S, E, C_2.C, D)$ where $n \neq 0$

$(S, E, \lambda x.C_1.C, D) \rightarrow (\langle \lambda x.C_1, E\rangle.S, E, C, D)$
$(S, E, rec\ f\ \lambda x.C_1.C, D) \rightarrow (\langle rec\ f\ \lambda x.C_1, E\rangle.S, E, C, D)$

$(v.\langle \lambda x.C_1, E\rangle.S, E', app.C, D) \rightarrow (S, E[v/x], C_1.return, \langle E', C\rangle.D)$
$(v.\langle rec\ f\ \lambda x.C_1, E\rangle.S, E', app.C, D) \rightarrow (S, E[v/x][\langle rec\ f\ \lambda x.C_1.E\rangle/f], C_1.return, \langle E', C\rangle.D)$
$(S, E', return.C', \langle E, C\rangle.D) \rightarrow (S, E, C, D)$

$(v.S, E, x^+.C, D) \rightarrow (S, E[v/x], C, D)$, $(S, E, x^-.C, D) \rightarrow (S, E|_{dom(E)\setminus\{x\}}, C, D)$

$(v_2.v_1.S, E, pair.C, D) \rightarrow (\langle v_1, v_2\rangle.S, E, C, D)$
$(\langle v_1, v_2\rangle.S, E, fst.C, D) \rightarrow (v_1.S, E, C, D)$, $(\langle v_1, v_2\rangle.S, E, snd.C, D) \rightarrow (v_2.S, E, C, D)$

Prove that $\forall e \in E$. Let $e \rhd C$. Then, $\langle e, \sigma\rangle \rightarrow v \Leftrightarrow \langle \epsilon, \sigma, C, \epsilon\rangle \xrightarrow{*} \langle v, \sigma, \epsilon, \epsilon\rangle$
($\Leftarrow$): by induction on proof trees
($\Rightarrow$): by induction on the length k of transition sequence $\langle \epsilon, \sigma, C, \epsilon\rangle \xrightarrow{k} \langle v, \sigma, \epsilon, \epsilon\rangle$

Problem 3. Two question marks are $(e, S, E[e'/x])$, $(e_1, e_2.S, E)$, respectively.