2016 Spring CS300 Homework #5

Due: May 30. 2016 AM 10:30

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Problem 1.

Suppose that all edge weights in a graph are integers in the range from 1 to |V|. How fast can you make Prim's algorithm run? What if the edge weights are integers in the range from 1 to W for some constant W?

Problem 2.

We are given a directed graph $\mathrm{G} = (\mathrm{V}, \mathrm{E})$ on which each edge $(\mathrm{u}, \mathrm{v}) \in \mathrm{E}$ has an associated value $\mathrm{r}(\mathrm{u}, \mathrm{v})$ which is a real number in the range $0 \leq \mathrm{r}(\mathrm{u}, \mathrm{v}) \leq 1$ that represents the reliability of a communication channel from vertex u to vertex v. We interpret $\mathrm{r}(\mathrm{u}, \mathrm{v})$ as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices.

Problem 3.

Arbitrage is the use of discrepancies in currency exchange rates to transform one unit of a currency into more than one unit of the same currency. For example, suppose that 1 U.S. dollar buys 49 Indian rupees, 1 Indian rupee buys 2 Japanese yen, and 1 Japanese yen buys 0.0107 U.S. dollars. Then, by converting currencies, a trader can start with 1 U.S. dollar and $49 \times 2 \times 0.0107 = 1.0486$ U.S. dollars, thus turning a profit of 4.86 percent.

Suppose that we are given n currencies $c_1, c_2, c_3, \ldots . c_n$ and an $\mathrm{n} \times \mathrm{n}$ table R of exchange rates, such that one unit of currency c_i buys $\mathrm{R}[\mathrm{i}, \mathrm{j}]$units of currency c_j. Give an efficient algorithm to determine whether or not there exists a sequence of currencies $(c_{i1}, c_{i2}, c_{i3}, \ldots . c_{ik})$ such that

$$\mathrm{R}[i_1, i_2] \cdot \mathrm{R}[i_2, i_3] \ldots \ldots \mathrm{R}[i_{k-1}, i_k] \cdot \mathrm{R}[i_k, i_1] > 1.$$

And also PRINT one such a sequence if one exists. Analyze the running time of your algorithm.