

Homework No.1

CS520 / KAIST Fall 2015

Due: October 5, 2015 2:30PM

1. Consider the expressions defined by the following abstract syntax:

$$e \rightarrow 0 \mid 3 \mid e + e \mid e * e$$

where “+” and “*” are addition and multiplication, respectively.

- (a) Define the denotational semantics of e precisely. You need to state what are the domain of meaning and the signature of semantic function.
- (b) Show by structural induction that every expression's value is the multiple of 3.
- (c) Define natural semantics (operational semantics) for e . \square

2. Let \mathbf{P} be the vertical domain of the natural numbers, $\{0, 1, 2, \dots, \infty\}$ where

$$0 \sqsubseteq 1 \sqsubseteq 2 \sqsubseteq 3 \sqsubseteq \dots \sqsubseteq \infty, \text{ and } \mathbf{P}' \text{ the two-element domain } \{\perp, \top\} \text{ where } \perp \sqsubseteq \top.$$

Then the monotone function “ $f x \equiv \text{if } x = \infty \text{ then } \top \text{ else } \perp$ ” is not continuous.

- (a) Why is the function f “not continuous”? Explain.
- (b) Let a function f_k be $f_k x \equiv \text{if } x \leq k \text{ then } \perp \text{ else } \top$. Then is the function f_k continuous? Explain why or why not?
- (c) Does a sequence $f_0 f_1 f_2 \dots$ constitute a chain? Why or why not? What is the least element among f_k where $k = 0, 1, \dots, \infty$?
($f \sqsubseteq g$ iff $\forall x \in \mathbf{P}. f x \sqsubseteq g x$)

3. Define the denotational semantics of the function f that satisfies the following equation. Then describe the function in plain English. Does your intuition match with your denotational semantics?

$$f = \lambda z \in \mathbf{Z}. \text{if } z \geq 0 \text{ then } 1 \text{ else } (1 + f(z+1))$$