

# Path Planning for Swarms by Combining Probabilistic Roadmaps and Potential Fields

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# Motion Planning

Proposed approach automatically plans the necessary motions that enable the robot to reach a desired destination while avoiding collisions with obstacles

[movie]

Motivated by applications in

- Navigation
- Exploration
- Search-and-Rescue

# Computational Challenges

Motion planning requires searching a vast high-dimensional state space for *dynamically-feasible motions* that avoid collisions

Taking motion dynamics into account is essential to ensure that the planned motions can be followed in the physical world

- Motion dynamics can be complex
- Often involve non-linear ODEs

# Computational Challenges

[movie]

- Snake-like robot model consists of several links attached to each other  
Continuous state consists of  $s = (x, y, \theta_0, v, \psi, \theta_1, \theta_2, \dots, \theta_N)$
- Motion dynamics modeled as a car pulling trailers

The differential equations of motions are

$$\begin{aligned}\dot{x} &= v \cos(\theta_0) & \dot{y} &= v \sin(\theta_0) & \dot{\theta}_0 &= v \tan(\psi) & \dot{v} &= a & \dot{\psi} &= \omega \\ \dot{\theta}_i &= \frac{v}{d} \left( \prod_{j=1}^{i-1} \cos(\theta_{j-1} - \theta_j) \right) (\sin(\theta_{i-1}) - \sin(\theta))\end{aligned}$$

Robot controls:  $a$  (acceleration);  $\omega$  (rotational velocity of steering wheel)

## Sampling-Based Motion Planning

Expand a tree  $\mathcal{T}$  of collision-free and dynamically-feasible motions

- select a state  $s$  from which to expand the tree
- sample control input  $u$
- generate new trajectory by applying  $u$  to  $s$

## Successful Motion Planners

- RRT [LaValle, Kuffner: IJRR 2001]
- EST [Hsu et al: IJRR 2002]
- PDST [Ladd, Kavraki: RSS 2005]
- SYCLOP [Plaku, Kavraki, Vardi: IEEE TRO 2010]

# Related Work

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# Related Work

On difficult kinodynamic motion-planning problems it has been noted that

- Exploration frequently gets stuck
- Progress slows down
- Exploration guided by limited information, such as distance metrics and nearest neighbors
- Lack of global sense of direction toward goal
- Difficult to discover new promising directions toward goal

[Donald et al., 93; Kim et al., 05; Plaku et al., 05; Yershova et al., 05; Ladd, Kavraki 05; Choset et al., 05; LaValle 06; Hsu et al., 06; Burns, Brock 07; Plaku et al. '10; Sucan, Kavraki '11]

# New Approach

- Treats motion planning with dynamics as a search problem in a *hybrid* state space composed of continuous and discrete components
- Couples sampling-based motion planning with discrete search

discrete layer: guide motion planning

continuous layer: expand tree of feasible motions

interplay: update guide to reflect motion-planning progress

- Significantly improves computational efficiency over related work

# Workspace Decomposition

Workspace decomposition provides discrete layer as adjacency graph

$$G = (R, E)$$

- $R$  denotes the regions of the decomposition
- $E = \{(r_i, r_j) : r_i, r_j \in R \text{ are physically adjacent}\}$

# Backward Discrete Search to Estimate Region Costs

Workspace decomposition provides discrete layer as adjacency graph

$$G = (R, E)$$

- $R$  denotes the regions of the decomposition
- $E = \{(r_i, r_j) : r_i, r_j \in R \text{ are physically adjacent}\}$

$\text{cost}(r)$  estimates the difficulty of reaching the goal region from  $r$   
defined as length of shortest path in  $G = (R, E)$  from  $r$  to goal

$[\text{cost}(r_1), \text{cost}(r_2), \dots, \text{cost}(r_n)]$

computed by running once A\* shortest-path algorithm backwards from goal

# Region Selection based on Cost Estimates

At each iteration, a region  $r_{\text{from}}$  is selected from which to expand the search tree  $T$

Tree vertices are grouped according to the regions that have been reached

$$\text{vertices}(T, r) = \{v : v \in T \wedge \text{region}(v) = r\}$$

$$\Gamma = \{r : r \in R \wedge |\text{vertices}(T, r)| > 0\}$$

$r_{\text{from}}$  is selected from  $\Gamma$  according to the probability distribution

$$\text{prob}(r) = \frac{1}{1 + \text{cost}^2(r)} / \sum_{r' \in R} \frac{1}{1 + \text{cost}^2(r')}$$

selection scheme balances greedy with methodical

# Forward Discrete Search to Compute Discrete Plans

Discrete plan  $\sigma$  consists of a sequence of regions connecting  $r_{\text{from}}$  to goal

Exploitation (with probability  $p$ )

- $\sigma$  computed as the shortest-path in  $G = (R, E)$  from  $r_{\text{from}}$  to goal

Exploration (with probability  $1 - p$ )

- $\sigma$  computed as a random path in  $G = (R, E)$  from  $r_{\text{from}}$  to goal

# Sampling-based Motion Planning to Expand the Search Tree

Expand tree of motions using the discrete plan as a guide

- Select vertices from regions associated with the discrete plan
- Sample input controls
- Generate new trajectories toward next region in the discrete plan
- Allow for deviations from the discrete plan



# Interplay: Update Discrete Plan to Reflect Motion-Planning Progress

# Experimental Results

[movie]

Compare computational efficiency to successful sampling-based motion planners

- RRT [LaValle, Kuffner]
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running time obtained as average of 30 different runs

# Experimental Results (cont.)

[movie]

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# Summary

## Approach

- Treats motion planning with dynamics as a search problem in a *hybrid* state space composed of continuous and discrete components
- Couples sampling-based motion planning with forward and backward discrete search over a workspace decomposition
- Obtains significant computational speedups