Generating Safe Paths in Dynamic Environments By Extracting Minimum Cost Trajectories Using Obstacle Position Probability Distributions and Replanning



CS4099: Major Software Project

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The main text of this project report is NNN words long, including project specification and plan.

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Chapter 1

Design

$$P_a(x, y, t_0, t_m) = \int_{t_0}^{t_m} \mathcal{N}(\zeta_a(t), \alpha \cdot (t - t_0)^2 + \beta, x, y) \cdot (t_m - t)^{\gamma} dt$$
(1.1)

Where $\mathcal{N}(\mu, \sigma^2, x, y)$ is the evaluation of a 3D normal distribution centered at (μ_x, μ_y) with a variance of σ^2 at (x, y).

$$P_A(x, y, t_0, t_m) = \frac{\sum_{a \in A} P_a(x, y, t_0, t_m)}{|A|}$$
(1.2)

$$C_A(i,j) = \int_0^1 \exp\left(P_A(x(\lambda), y(\lambda), i_t, j_t)\right) \cdot ||i - j||_2 d\lambda$$
(1.3)

Where $x(\lambda) = (j_x - i_x) \cdot \lambda + i_x$ and $y(\lambda) = (j_y - i_y) \cdot \lambda + i_y$ are the parametric equations of the line from i to j.

$$\tilde{\zeta}_a(t) = \begin{cases}
\tilde{\zeta}_a(t - \delta t) + \zeta_a'(t) \cdot \delta t + \rho & \text{if } t > 0 \\
\zeta_a^{(0)} & \text{if } t = 0
\end{cases}$$
(1.4)

Where $\rho \sim \mathcal{U}(-\epsilon, \epsilon)$, $\epsilon > 0$, and $\zeta_a^{(0)}$ is the initial position of the obstacle.

Algorithm 1 ROADMAP(n, d, w, h, O)

Input:

- n: Maximum number of samples
- d: Maximum distance between neighbouring nodes
- O: Set of obstacles

Output:

An unweighted graph of points describing the connectivity of the environment

```
1: for i=1 to n do

2: q \leftarrow \text{RANDOMPOINT2D}(w,h)

3: if \bigwedge_{o \in O} \neg \text{Collision}(o,q) then

4: V \leftarrow V \cup \{q\}

5: for all q_i \in V do

6: for all q_j \in V do

7: if q_i \neq q_j \wedge ||q_i - q_j|| \leq d then

8: E \leftarrow E \cup \{(q_i, q_j)\}

9: return (V, E)
```

Algorithm 2 GetPath $(n, d, w, h, \delta, p, g, O, A, R)$

```
1: (V, E) \leftarrow \text{ROADMAP}(n, d, w, h, O)
 2: \Pi \leftarrow \text{Set}()
 3: q \leftarrow p
 4: while ||\text{BACK}(\Pi) - g||_2 > R do
          \pi \leftarrow \text{SearchGraph}(V, E, R, A, q, g)
          for all i \in \pi do
               \Pi \leftarrow \Pi \cup \{i\}
 7:
               for all a \in A do
 8:
                   STEP(a)
 9:
              \begin{array}{c} \text{if } \bigvee_{a \in A} ||\tilde{\zeta_a}(i_t) - \zeta_a(i_t)|| > \delta \text{ then} \\ \text{ for all } a \in A \text{ do} \end{array}
10:
11:
                       UPDATE(\zeta_a, \tilde{\zeta_a})
12:
                   q \leftarrow i
13:
14: return \Pi
```

Algorithm 3 SearchGraph(V, E, R, A, p, g)

```
1: Q \leftarrow \text{PriorityQueue}()
 2: D \leftarrow \text{Dictionary}()
 3: \Pi \leftarrow \text{Dictionary}()
 4: Insert(Q, p, 0)
 5: while \neg \text{Empty}(Q) do
        q, w \leftarrow \text{Pop}(Q)
 7:
        if ||q-g||_2 \leq R then
          return BACKTRACKPATH(p, g, \Pi)
 8:
        N \leftarrow \text{GetTemporalNeighbours}(V, E, q)
 9:
        for all n \in N do
10:
           \Pi_n \leftarrow q
11:
           c \leftarrow \psi \cdot C_A(q, n) + \omega \cdot D_n
12:
           D_n \leftarrow D_n + 1
13:
           if w > c then
14:
15:
           Q \leftarrow \text{Insert}(Q, n, q)
16:
```

Algorithm 4 GetTemporalNeighbours(V, E, q)

```
1: S \leftarrow Set()
```

- 2: $N \leftarrow \text{Neighbours}(q)$
- 3: for all $n \in N$ do
- 4: $t \leftarrow ||q n||_2/v + q_t$
- $S \leftarrow S \cup \{(n_x, n_y, t)\}$

Algorithm 5 BACKTRACKPATH (p, g, Π)

```
1: q \leftarrow g
```

- 2: $S \leftarrow \text{STACK}()$
- 3: while $\Pi_q \neq p$ do 4: $S \leftarrow \text{Push}(S, q)$
- 5: $q \leftarrow \Pi_q$ 6: $S \leftarrow \text{PUSH}(S, p)$
- 7: \mathbf{return} S

Chapter 2

Discussion