## Generating Safe Paths in Dynamic Environments By Extracting Minimum Cost Trajectories Using Obstacle Position Probability Distributions and Replanning



CS4099: Major Software Project

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The main text of this project report is NNN words long, including project specification and plan.

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## Chapter 1

## Design

## 1.1 Agents

$$P_a(x, y, t_0, t_m) = \int_{t_0}^{t_m} \mathcal{N}(\zeta_a(t), \alpha \cdot (t - t_0)^2 + \beta, x, y) \cdot (t_m - t)^{\gamma} dt$$
 (1.1)

Where  $\mathcal{N}(\mu, \sigma^2, x, y)$  is the evaluation of a 3D normal distribution centered at  $(\mu_x, \mu_y)$  with a variance of  $\sigma^2$  at (x, y).

$$P_A(x, y, t_0, t_m) = \frac{\sum_{a \in A} P_a(x, y, t_0, t_m)}{|A|}$$
(1.2)

$$\tilde{\zeta}_a(t) = \begin{cases}
\tilde{\zeta}_a(t - \delta t) + \zeta_a'(t) \cdot \delta t + \rho & \text{if } t > 0 \\
\zeta_a^{(0)} & \text{if } t = 0
\end{cases}$$
(1.3)

Where  $\rho \sim \mathcal{U}(-\epsilon, \epsilon)$ ,  $\epsilon > 0$ , and  $\zeta_a^{(0)}$  is the initial position of the obstacle.

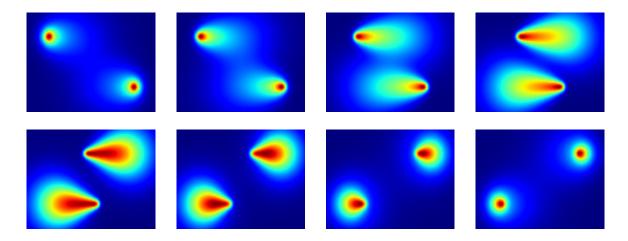


Figure 1.1: Cost distributions indicating the likelihood that an agent will be at a certain location within a given time interval. These figures show how this distribution changes over time (left to right, top to bottom)

## 1.2 Planning Algorithm

$$C_A(i,j) = \int_0^1 \exp\left(P_A(x(\lambda), y(\lambda), i_t, j_t)\right) \cdot ||i - j||_2 d\lambda$$
(1.4)

Where  $x(\lambda) = (j_x - i_x) \cdot \lambda + i_x$  and  $y(\lambda) = (j_y - i_y) \cdot \lambda + i_y$  are the parametric equations of the line from i to j.

```
Algorithm 1 ROADMAP(n, d, w, h, O)
```

```
Input:
```

- n: Maximum number of samples
- d: Maximum distance between neighbouring nodes
- O: Set of obstacles

#### **Output:**

An unweighted graph of points describing the connectivity of the environment

```
1: for i=1 to n do

2: q \leftarrow \text{RANDOMPOINT2D}(w,h)

3: if \bigwedge_{o \in O} \neg \text{COLLISION}(o,q) then

4: V \leftarrow V \cup \{q\}

5: for all q_i \in V do

6: for all q_j \in V do

7: if q_i \neq q_j \wedge ||q_i - q_j|| \leq d then

8: E \leftarrow E \cup \{(q_i, q_j)\}

9: return (V, E)
```

### **Algorithm 2** GetPath $(n, d, w, h, \delta, p, g, O, A, R)$

#### **Input:**

- n: Maximum number of samples for the roadmap
- d: Maximum distance between neighbouring nodes in the roadmap
- w: Width of the scene
- h: Height of the scene

#### **Output:**

```
1: (V, E) \leftarrow \text{ROADMAP}(n, d, w, h, O)
 2: \Pi \leftarrow \text{Set}()
 3: q \leftarrow p
 4: while ||BACK(\Pi) - g||_2 > R do
          \pi \leftarrow \text{SEARCHGRAPH}(V, E, R, A, q, g)
          for all i \in \pi do
              \Pi \leftarrow \Pi \cup \{i\}
 7:
              for all a \in A do
                  STEP(a)
 9:
              \begin{array}{l} \text{if } \bigvee_{a \in A} ||\tilde{\zeta_a}(i_t) - \zeta_a(i_t)|| > \delta \text{ then} \\ \text{ for all } a \in A \text{ do} \end{array}
10:
11:
                      UPDATE(\zeta_a, \zeta_a)
12:
                  q \leftarrow i
13:
                  break
14:
15: return \Pi
```

## $\overline{\mathbf{Algorithm}}$ 3 SearchGraph(V, E, R, A, p, g)

```
1: Q \leftarrow \text{PriorityQueue}()
 2: D \leftarrow \text{Dictionary}()
 3: \Pi \leftarrow \text{Dictionary}()
 4: Insert(Q, p, 0)
 5: while \neg \text{Empty}(Q) do
        q, w \leftarrow \text{Pop}(Q)
        if ||q-g||_2 \leq R then
 7:
           return BACKTRACKPATH(p, q, \Pi)
 9:
        N \leftarrow \text{GetTemporalNeighbours}(V, E, q)
        for all n \in N do
10:
           \Pi_n \leftarrow q
11:
           c \leftarrow \psi \cdot C_A(q, n) + \omega \cdot D_n
12:
           D_n \leftarrow D_n + 1
13:
           if w > c then
14:
15:
              c \leftarrow w
           Q \leftarrow \text{Insert}(Q, n, q)
```

### Algorithm 4 GetTemporalNeighbours(V, E, q)

```
1: S \leftarrow Set()

2: N \leftarrow \text{Neighbours}(V, E, q)

3: for all n \in N do

4: t \leftarrow ||q - n||_2/s + q_t

5: S \leftarrow S \cup \{(n_x, n_y, t)\}
```

### **Algorithm 5** BACKTRACKPATH $(p, g, \Pi)$

```
1: q \leftarrow g

2: S \leftarrow \text{STACK}()

3: while \Pi_q \neq p do

4: S \leftarrow \text{PUSH}(S, q)

5: q \leftarrow \Pi_q

6: S \leftarrow \text{PUSH}(S, p)

7: return S
```

## Chapter 2

# **Experimental Setup**

### Algorithm 6 PF(q, g, O, A, R)

```
1: q_{min} \leftarrow q
2: p_{min} \leftarrow \infty
3: \theta \leftarrow 0
4: while \theta \leq 2\pi do
5: q' \leftarrow q + \delta t \cdot s \cdot \text{ROT}(\theta)
6: p \leftarrow U_{rep}(q', O \cup A) + U_{att}(q', g)
7: if p < p_{min} then
8: p_{min} \leftarrow p
9: q_{min} \leftarrow q'
10: \theta \leftarrow \theta + \delta \theta
11: if ||q_{min} - g|| < R then
12: return \{p_{min}\}
13: return \{q_{min}\} \cup \text{PF}(q_{min}, g, O, R)
```

# Chapter 3

# Discussion