Developing Topological Representations for Robotic Exploration of Known Environments

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***Abstract*— test**

I. Introduction

In the context of mobile robotics, it is vital for a robot or a planner to understand the topological and metric connectivity of the environment it is operating in. In order to conduct higher level planning such as monitoring locations of interest or harvesting resources, a map of the environment is needed that is able to represent through which paths two areas in the environment are connected. There has been a lot of work in persistent coverage and autonomous surveillance which solve variations of the travelling salesman problem on graphical representations of an environment that assume knowledge of the paths between areas of interest in their search [1], [2], [3], [4] but do not provide a method of determining these paths. This work seeks to provide an simple graphical representation such that there are a low number of nodes in the representation in order for solutions to the travelling salesman problem and its variants are still feasible and a user of the presented algorithm will be able to simply look-up the path between areas of interest.

There has been a lot of work done in generating roadmaps of the environment (i.e. graphs that represent the connectivity of the environment). Kavraki et al developed the very popular probabilistic roadmap which randomly samples points in the configuration space of a robot and connects them in a graphical structure if it is feasible for the robot to move from one configuration two another avoiding collisions [5]. LaValle developed the rapidly-exploring random tree which incrementally develops a tree like structure of paths that would lead a robot from an initial to a goal configuration [6]. However, these approaches do not provide a concise graphical representation of the environment.

There has also been a lot of work in thinning free space to provide a more concise representation. Beenson et al developed a topological thinning algorithm using an Extended Voronoi Graph (EVG) that can show topologically distinct paths in free space [7]. However, the work lacks a concise graphical structure. Instead of providing a graph representing the connectivity, the work has provided a list of points which lie on the thinned representation of free space. Similarly, work has been done using morphology operations to produce a thin skeleton of the original volume [8], [9] but do not provide information about the connectivity of regions. Portugal and Rocha were able to use the work done by Beenson et al to determine the location of critical points that can be used as nodes in a graph [10], however this work did not provide a means of determining edges between the discovered nodes.

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This paper presents an algorithm that is able to provide a graphical structure that represents both the topological and metric connectivity of the environment with a smaller number of nodes than work done previously by Portugal and Rocha and a simple look-up procedure to determine the path between nodes. Our method consists of three steps:

1) Produce a Voronoi graph of the environment using boundary points along the obstacles as centers of the Voronoi cells
2) Reduce the number of nodes by pruning redundant nodes that are not critical in defining the topological or metric connectivity of the environment
3) Prune the redundant leaf branches left over from the Voronoi graph

These steps are discussed in more detail in sections II, III, and IV respectively.

II. Developing the Initial Voronoi Diagram

We develop the initial graphical representation of the environment by determining points along the edge of all the obstacles in the environment. We use these points as centers of the Voronoi cells and provide them to the Voronoi diagram algorithm to determine the initial topological representation. This is the technique used by Choset and Burdick when developing generalized Voronoi graphs of occupancy grids [11]. Algo. 1 shows how these boundary points are chosen from the occupancy grid.

In order to extract a graphical structure from the Voronoi diagram, the points along the ridges are used as vertices. The vertices are connected to other points along the same ridge. Ridges that intersect obstacles and ridges totally contained by obstacles are discarded. This connected structure provides the first graphical representation of the environment.

III. Pruning Redundant Nodes

In order to produce a graph with a relatively small number of vertices, we need to prune vertices from the graph generated by the Voronoi diagram that are not critical to defining the metric and topological structure of the environment. A critical node is defined as any node with one edge or with three or more edges. The initial pruning of redundant nodes is done by first determining which nodes in the original graph are critical and connecting them through a

Algorithm 1 BOUNDARYPOINTS($\mathcal{O}$)

Input:

- $\mathcal{O}$: $M \times N$ binary matrix representing obstacles in the environment, where if $\mathcal{O}_{i,j}$ is 1, (i, j) is inside an obstacle

Output:

- A set of pairs representing boundary points in the occupancy grid

```
S ← {}
for i = 1 to M do
  for j = 1 to N do
    if O_{i,j} = 1 then
      for k = −1 to 1 do
        for l = −1 to 1 do
          if O_{i+k,j+l} == 0 then
            S ← S ∪ {(i, j)}
return S
```

series a of non-critical points. This chain of non-critical points will develop the path between the critical nodes. After pruning these nodes, a directed multi-graph structure, $G = (V, E, \Pi)$, which contains a set of critical nodes, a set of pairs representing the topological connectivity between nodes, and a set of sequences representing the geometric paths between the nodes is produced. Note that this graph structure is able to have multiple paths between the same pair of nodes.

To develop this initial reduced graph, we iterate through all of the critical nodes in the original graph determined by the Voronoi diagram. Then for each neighbour of each critical node, we determine if that node is also critical. If so, an edge is created and the iteratively stored path between them is added to the edge. If the neighbour is not a critical node, it gets added to path stemming from the critical node until another critical node is discovered. This process is shown in Algo. 2.

IV. PRUNING REDUNDANT LEAF BRANCHES

Once redundant nodes have been initially pruned from the Voronoi graph, we need to remove leaf branches that have been generated which do not actually provide any additional information about the environment. A heuristic is used to prune these branches from the Voronoi graph. If a node has only one edge and it is within a certain distance from any obstacle, it is removed from the graph. Once leaf branches meeting this criteria are removed, we traverse the resulting graph and remove nodes that have become redundant and are no longer critical (i.e. they only have two edges). This is done by iterating through all of the current critical nodes and traversing their neighbours. If the neighbour is not critical, the node itself and the path connecting the critical node to the neighbour are stored as the current path and all of it's neighbours are added to the search. Once another critical node is reached, an edge is created between them and the accumulated path is stored in the graphical structure. This is

Algorithm 2 RTVD($\mathcal{O}$)

Input:

- $\mathcal{O}$: $M \times N$ binary matrix representing obstacles in the environment

Output:

- A graph representing the topological connectivity of the environment, $\mathcal{O}$, with redundant leaf branches.

```
P ← BOUNDARYPOINTS(O)
V ← VORONOI(P)
G = (V, E) ← DIGRAPH()
for i ∈ CRITICALNODES(V) do
  for j ∈ NEIGHBOURS(i) do
    Π ← {i}
    S ← {j}
    while |S| > 0 do
      n ← POP(S)
      Π ← Π ∪ {n}
      if n ∈ CRITICALNODES(V) then
        V ← V ∪ i
        E ← E ∪ (i, n, Π)
        break
      else
        for m ∈ NEIGHBOURS(n) do
          if m ∉ Π then
            S ← S ∪ {m}
return H
```

shown more formally in Algo. 3. It is necessary to remove these redundant leaf branches to provide a more concise representation of the environment and reduce the number of nodes generated. Fig. IV shows where redundant leaf branches are generated and provides a motivation for their removal.

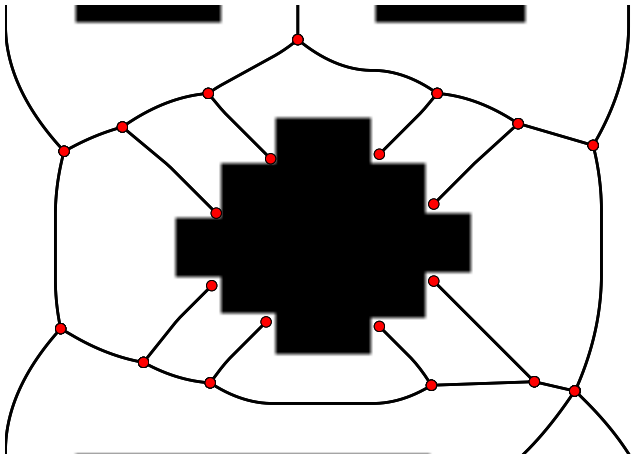

Fig. 1. Redundant leaves generated by the Voronoi diagram which are pruned by the propsed algorithm

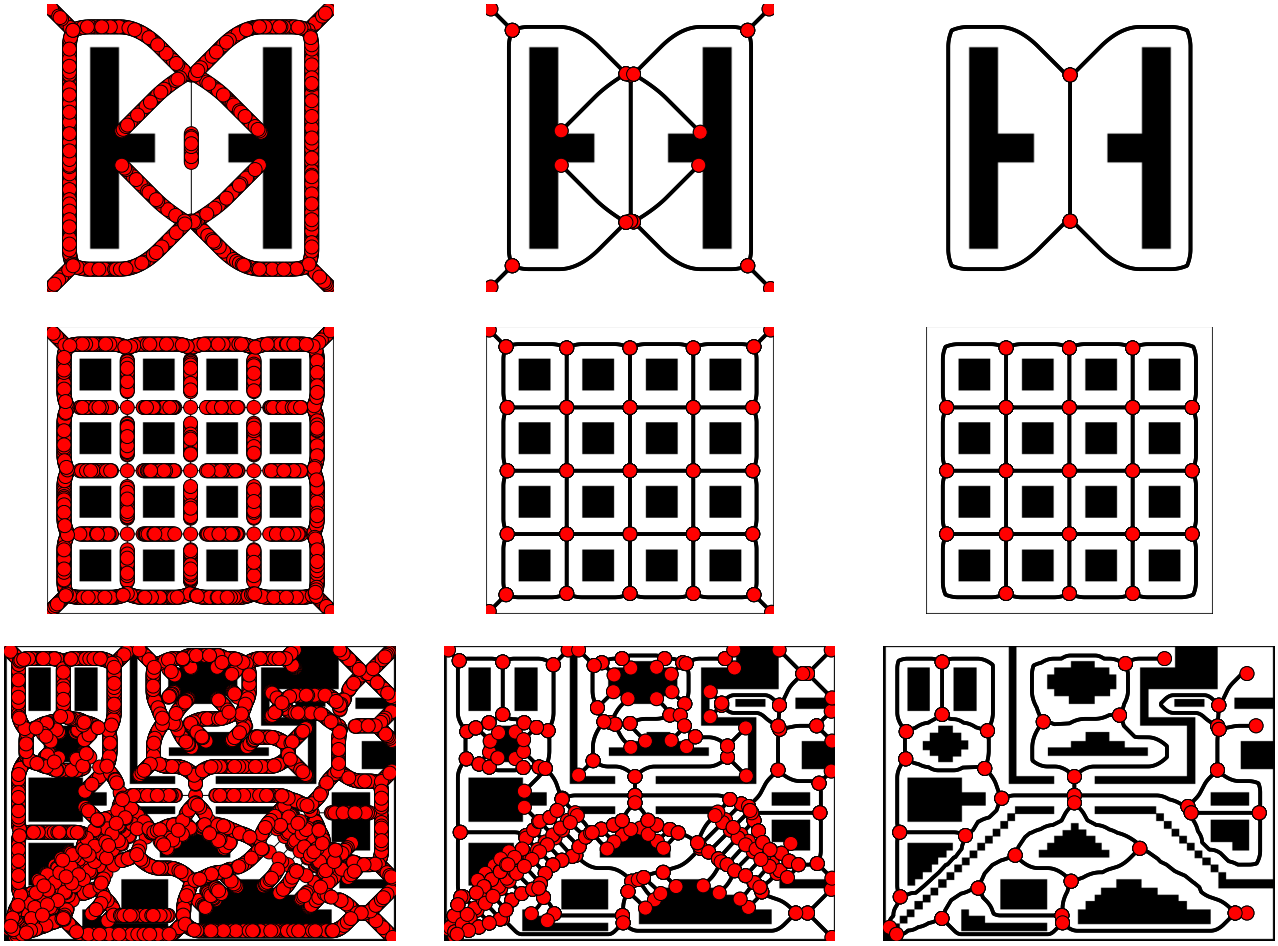

Fig. 2. The topological extraction algorithm running on each of the three scenes presented. The first column represents the graph generated from the initial Voronoi decomposition. The second column is a result of the first node reduction using RTVD and the last column is the final graph representing the connectivity of the environment generated by the second node reduction using TVD.

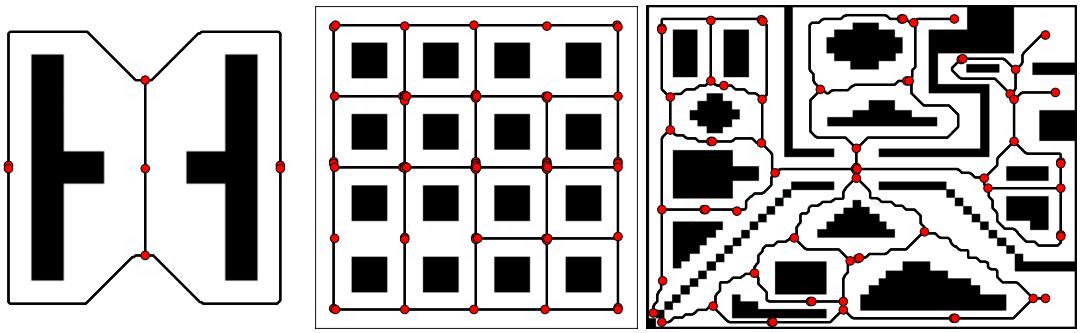

Fig. 3. The topological graph extracted using the algorithm by Portugal and Rocha shown in [10]

Algorithm 3 TVD($\mathcal{O}$)

Input:

- $\mathcal{O}$: $M \times N$ binary matrix representing obstacles in the environment

Output:

- A graph representing the topological connectivity of the environment, $\mathcal{O}$

$G = (V, E) \leftarrow$ MULTIDIGRAPH()
$H \leftarrow$ RTVD($\mathcal{O}$)
$H' \leftarrow$ REMOVECLOSENODES($H, \mathcal{O}$)
for $i \in$ CRITICALNODES(H') **do**
  for $j \in$ NEIGHBOURS(i) **do**
    $\Pi \leftarrow \{i\}$
    $S \leftarrow \{(i, j)\}$
    while $|S| > 0$ **do**
      $(n, m) \leftarrow$ POP(S)
      $\Pi \leftarrow \Pi \cup \Pi_{H'}(n, m)$
      if $n \in$ CRITICALNODES(H') **then**
        $V \leftarrow V \cup i$
        $E \leftarrow E \cup (i, n, \Pi)$
        break
      else
        for $k \in$ NEIGHBOURS(n) **do**
          if $k \notin \Pi$ **then**
            $S \leftarrow S \cup \{(n, k)\}$
return G

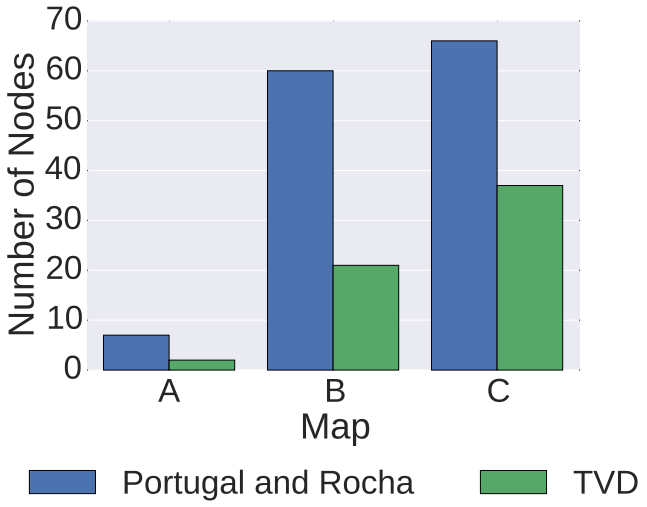

Fig. 4. Plot comparing the number of nodes generated using the proposed TVD algorithm and the algorithm developed in [10]

V. Experimental Results

VI. Conclusion

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