

# Developing Topological Representations for Robotic Exploration of Known Environments

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**Abstract—test**

## I. INTRODUCTION

In the context of mobile robotics, it is vital for a robot or a planner to understand the topological and metric connectivity of the environment it is operating in. In order to conduct higher level planning such as monitoring locations of interest or harvesting resources, a map of the environment is needed that is able to represent through which paths two areas in the environment are connected. There has been a lot of work in persistent coverage and autonomous surveillance which solve variations of the travelling salesman problem on graphical representations of an environment that assume knowledge of the paths between areas of interest in their search [1], [2], [3], [4] but do not provide a method of determining these paths. This work seeks to provide a simple graphical representation such that there are a low number of nodes in the representation in order for solutions to the travelling salesman problem and its variants are still feasible and a user of the presented algorithm will be able to simply look-up the path between areas of interest.

There has been a lot of work done in generating roadmaps of the environment (i.e. graphs that represent the connectivity of the environment). Kavraki et al developed the very popular probabilistic roadmap which randomly samples points in the configuration space of a robot and connects them in a graphical structure if it is feasible for the robot to move from one configuration to another avoiding collisions [5]. LaValle developed the rapidly-exploring random tree which incrementally develops a tree like structure of paths that would lead a robot from an initial to a goal configuration [6]. However, these approaches do not provide a concise graphical representation of the environment.

There has also been a lot of work in thinning free space to provide a more concise representation. Beenson et al developed a topological thinning algorithm using an Extended Voronoi Graph (EVG) that can show topologically distinct paths in free space [7]. However, the work lacks a concise graphical structure. Instead of providing a graph representing the connectivity, the work has provided a list of points which lie on the thinned representation of free space. Similarly, work has been done using morphology operations to produce a thin skeleton of the original volume [8], [9] but do not provide information about the connectivity of

regions. Portugal and Rocha were able to use the work done by Beenson et al to determine the location of critical points that can be used as nodes in a graph [10], however this work did not provide a means of determining edges between the discovered nodes.

This paper presents an algorithm that is able to provide a graphical structure that represents both the topological and metric connectivity of the environment with a smaller number of nodes than work done previously by Portugal and Rocha and a simple look-up procedure to determine the path between nodes.

## II. DEVELOPING THE INITIAL VORONOI DIAGRAM

We develop the initial graphical representation of the environment by determining points along the edge of all the obstacles in the environment. We use these points as centers of the Voronoi cells and provide them to the Voronoi diagram algorithm to determine the initial topological representation. This is the technique used by Choset and Burdick when developing generalized Voronoi graphs of occupancy grids [11]. Algo. 1 shows how these boundary points are chosen from the occupancy grid.

In order to extract a graphical structure from the Voronoi diagram, the points along the ridges are used as vertices. The vertices are connected to other points along the same ridge. Ridges that intersect obstacles and ridges totally contained by obstacles are discarded. This connected structure provides the first graphical representation of the environment.

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### Algorithm 1 BOUNDARYPOINTS( $\mathcal{O}$ )

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**Input:**

- $\mathcal{O}$ :  $M \times N$  binary matrix representing obstacles in the environment, where if  $\mathcal{O}_{i,j}$  is 1,  $(i,j)$  is inside an obstacle

**Output:**

- A set of pairs representing boundary points in the occupancy grid

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1:  $S \leftarrow \{\}$ 
2: for  $i = 1$  to  $M$  do
3:   for  $j = 1$  to  $N$  do
4:     if  $\mathcal{O}_{i,j} = 1$  then
5:       for  $k = -1$  to  $1$  do
6:         for  $l = -1$  to  $1$  do
7:           if  $\mathcal{O}_{i+k,j+l} == 0$  then
8:              $S \leftarrow S \cup \{(i,j)\}$ 
9: return  $S$ 
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### III. PRUNING REDUNDANT NODES

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**Algorithm 2** RTVD( $\mathcal{O}$ )

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**Input:**

- $\mathcal{O}$ :  $M \times N$  binary matrix representing obstacles in the environment

**Output:**

- A graph representing the topological connectivity of the environment,  $\mathcal{O}$ , with redundant nodes

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1:  $P \leftarrow \text{BOUNDARYPOINTS}(\mathcal{O})$ 
2:  $\mathcal{V} \leftarrow \text{VORONOI}(P)$ 
3:  $G = (V, E) \leftarrow \text{DIGRAPH}()$ 
4: for  $i \in \text{CRITICALNODES}(\mathcal{V})$  do
5:   for  $j \in \text{NEIGHBOURS}(i)$  do
6:      $\Pi \leftarrow \{i\}$ 
7:      $S \leftarrow \{j\}$ 
8:     while  $|S| > 0$  do
9:        $n \leftarrow \text{POP}(S)$ 
10:       $\Pi \leftarrow \Pi \cup \{n\}$ 
11:      if  $n \in \text{CRITICALNODES}(\mathcal{V})$  then
12:         $V \leftarrow V \cup i$ 
13:         $E \leftarrow E \cup (i, n, \Pi)$ 
14:        break
15:      else
16:        for  $m \in \text{NEIGHBOURS}(n)$  do
17:          if  $m \notin \Pi$  then
18:             $S \leftarrow S \cup \{m\}$ 
19: return  $H$ 

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**Algorithm 3** TVD( $\mathcal{O}$ )

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**Input:**

- $\mathcal{O}$ :  $M \times N$  binary matrix representing obstacles in the environment

**Output:**

- A graph representing the topological connectivity of the environment,  $\mathcal{O}$

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1:  $G = (V, E) \leftarrow \text{MULTIDIGRAPH}()$ 
2:  $H \leftarrow \text{RTVD}(\mathcal{O})$ 
3:  $H' \leftarrow \text{REMOVECLOSENODES}(H, \mathcal{O})$ 
4: for  $i \in \text{CRITICALNODES}(H')$  do
5:   for  $j \in \text{NEIGHBOURS}(i)$  do
6:      $\Pi \leftarrow \{i\}$ 
7:      $S \leftarrow \{(i, j)\}$ 
8:     while  $|S| > 0$  do
9:        $(n, m) \leftarrow \text{POP}(S)$ 
10:       $\Pi \leftarrow \Pi \cup \Pi_{H'}(n, m)$ 
11:      if  $n \in \text{CRITICALNODES}(H')$  then
12:         $V \leftarrow V \cup i$ 
13:         $E \leftarrow E \cup (i, n, \Pi)$ 
14:        break
15:      else
16:        for  $k \in \text{NEIGHBOURS}(n)$  do
17:          if  $k \notin \Pi$  then
18:             $S \leftarrow S \cup \{(n, k)\}$ 
19: return  $G$ 

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### IV. DEVELOPING THE FINAL TOPOLOGICAL REPRESENTATION

### V. EXPERIMENTAL RESULTS

### VI. CONCLUSION

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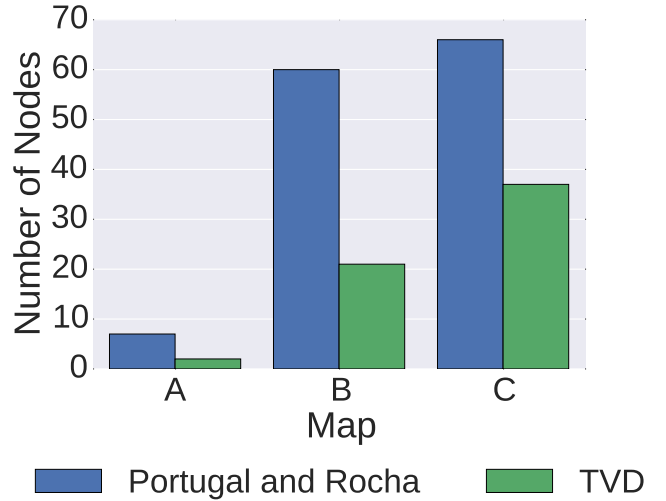


Fig. 3. Plot comparing the number of nodes generated using the proposed TVD algorithm and the algorithm developed in [10]

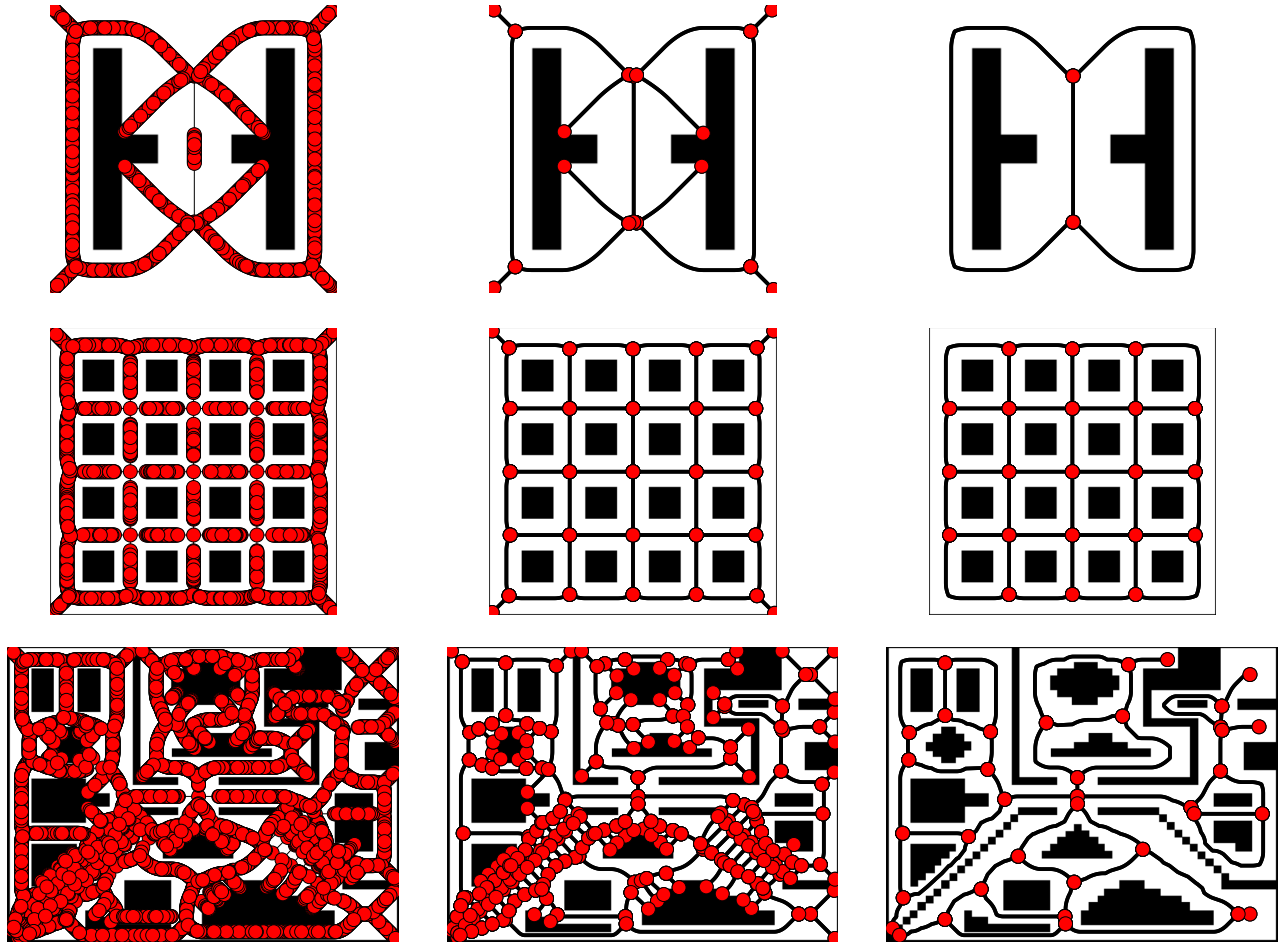


Fig. 1. The topological extraction algorithm running on each of the three scenes presented. The first column represents the graph generated from the initial Voronoi decomposition. The second column is a result of the first node reduction using RTVD and the last column is the final graph representing the connectivity of the environment generated by the second node reduction using TVD.

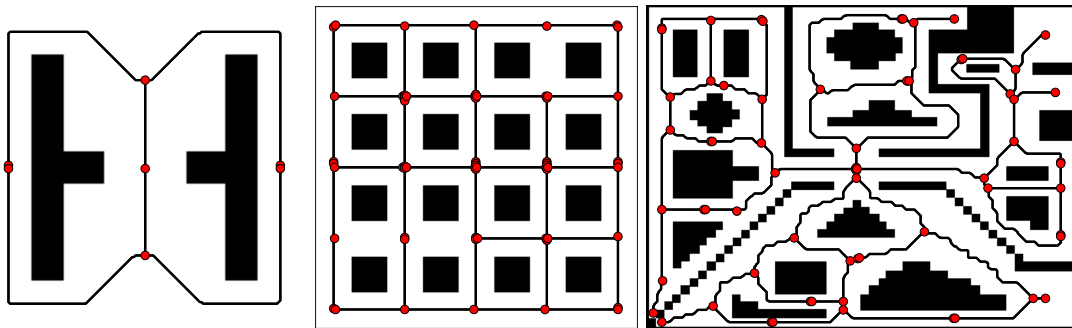


Fig. 2. The topological graph extracted using the algorithm by Portugal and Rocha shown in [10]

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