Optimal Lunar Flyby using Multiple Shooting

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Course : Lunar Mission Internship

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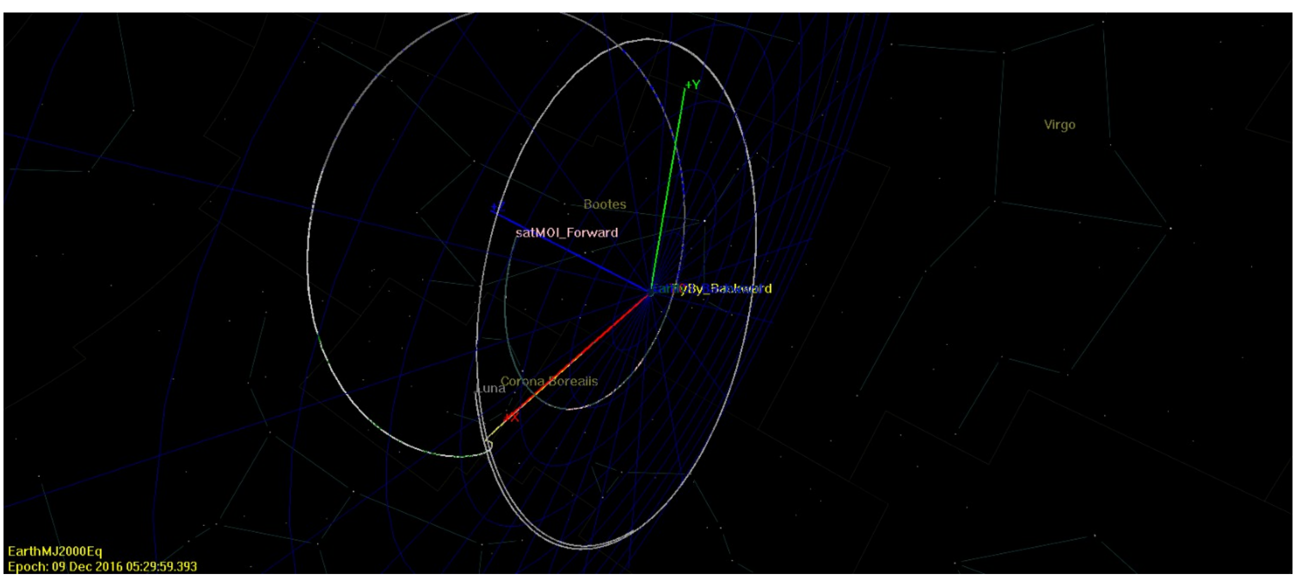

Acknowledgement

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I am also grateful to my only friend for her unwavering support and motivation throughout this process. Her encouragement was a constant source of strength. Finally, I would like to acknowledge the developers and community of the GMAT software for providing a powerful and accessible tool that made this analysis possible.

Abstract

This report details the design and optimization of a lunar flyby trajectory using the Multiple Shooting Method, as demonstrated in the "Optimal Lunar Flyby using Multiple Shooting (Advanced)" tutorial. The primary objective was to design a trajectory that utilizes the Moon's gravity to achieve a specific mission orbit periapsis and inclination, while minimizing fuel usage. The methodology employed three trajectory segments—Transfer Orbit Insertion (TOI), lunar periapsis, and Mission Orbit Insertion (MOI)—to break down the sensitive boundary value problem. Key findings include the successful design of a flyby that meets the mission constraints, with the final solution providing a C3 value of −0.09171 km/s. The report demonstrates the effectiveness of modern optimization techniques in solving complex astrodynamics problems.

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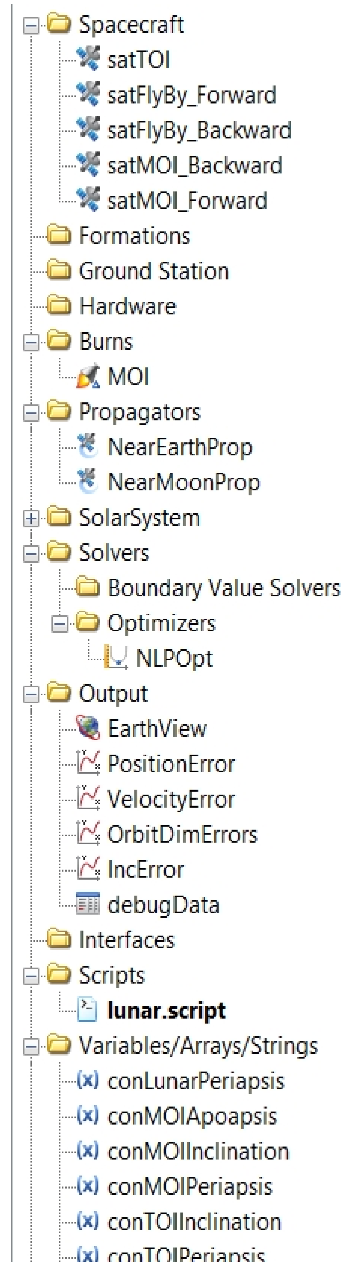


Spacecraft
satTOI
satFlyBy_Forward
satFlyBy_Backward
satMOI_Backward
satMOI_Forward
Formations
Ground Station
Hardware
Burns
MOI
Propagators
NearEarthProp
NearMoonProp
SolarSystem
Solvers
Boundary Value Solvers
Optimizers
NLPOpt
Output
EarthView
PositionError
VelocityError
OrbitDimErrors
IncError
debugData
Interfaces
Scripts
lunar.script
Variables/Arrays/Strings
conLunarPeriapsis
conMOIApoapsis
conMOIInclination
conMOIPeriapsis
conTOIInclination

1. Introduction

1.1 Background

Highly Elliptic Earth Orbits (HEO) are a common target for spacecraft, but achieving them efficiently can be a challenge. Using a celestial body's gravity, such as the Moon's, for a "gravity assist" or flyby is a highly effective method to change a spacecraft's trajectory without expending significant fuel. This is particularly useful for raising periapsis or performing plane changes. However, designing these flybys to meet specific mission constraints, like a target periapsis or inclination, is a non-trivial problem that requires modern optimization techniques.

1.2 Problem Statement

The goal of this assignment is to design an optimal lunar flyby trajectory. The trajectory must provide a mission orbit with a specific periapsis and inclination. The optimization must simultaneously minimize fuel usage, which is a key performance metric for any space mission.

Design a lunar flyby that:

- Inserts from a 285 km, 28.5° transfer orbit
- Executes a flyby at lunar altitude ≥ 100 km
- Inserts into a mission orbit with 15 Re perigee, 60 Re apogee, and 10° inclination

Minimize the total ΔV required for the Mission Orbit Insertion (MOI) burn.

1.3 Objectives

The objectives of this report are to:

1. Explain the application of the Multiple Shooting Method to this problem.
2. Detail the setup of the optimization, including the trajectory segments.

3. Present the key results, including the initial conditions and the final optimized solution.
4. Discuss the effectiveness of this method and the significance of the final trajectory.

2. Methodology: The Multiple Shooting Method

To efficiently solve this complex problem, the Multiple Shooting Method was employed. This technique breaks down a sensitive boundary value problem into smaller, more manageable segments. Instead of a single continuous trajectory, the mission is modelled with three distinct, but connected, segments.

2.1 Trajectory Segments

The GMAT script, lunar.script ([GMAT/Lunar Assignment a21/lunar.script at main · a21amit/GMAT]), defines three main trajectory segments, each with its own control point:

- Segment 1: Begins at the Transfer Orbit Insertion (TOI) and propagates forward in time.
- Segment 2: Is centred at the lunar periapsis and propagates both forwards and backwards.
- Segment 3: Is centred on the Mission Orbit Insertion (MOI) and also propagates both forwards and backwards.

These segments are linked by "control points" that are optimized to ensure the trajectory is smooth and continuous. The optimizer adjusts variables such as the spacecraft's state (position and velocity) at these points to satisfy the constraints.

2.2 Optimization Setup

The optimization process minimizes fuel usage by minimizing the ΔV (change in velocity) required for the Mission Orbit Insertion (MOI) maneuverer. The script defines a nonlinear optimization problem with multiple constraints, including:

- Matching the position and velocity states of the different trajectory segments at their connection points.

- Achieving a specific inclination for the final mission orbit.
- Satisfying boundary conditions for the periapsis and apoapsis of the final orbit.

The optimizer then finds the set of variables that satisfies all constraints while minimizing the objective function (fuel usage).

3. Results and Analysis

3.1 Initial Conditions

The initial guess for the optimization was crucial for a successful convergence. The report begins with a guess for the key epochs and the spacecraft's state at the TOI point. The GMAT script then uses the final solution from a previous run as a new initial guess, as detailed in the PDF tutorial. This iterative process helps in finding an optimal solution more efficiently.

Solver Window - Optimize 'Optimize Flyby' NLPOpt {SolveMode = Solve, ExitMode = Disca...

MOI.Element1	-0.09192574337852834	-0.09182574337852834	-0.0001000000000000029

Constraints	Desired	Achieved	Difference
(==) satTOI.INC	28.5	28.4999999999736	-2.639666263348772e-11
(==) satTOI.RadPer	6663	6662.999999967938	-3.206241672160104e-08
(==) satMOI_Backward.RadP	95672.0445	95672.04449936227	-6.377376848831773e-07
(==) launchRdotV	0	-2.492015482857824e-13	-2.492015482857824e-13
(==) satTOI.EarthMJ2000Eq.X	140804.4516059846	140804.4522913997	0.0006854150851722807
(==) satTOI.EarthMJ2000Eq.Y	-41212.38141202374	-41212.38141699491	-4.971174348611385e-06
(==) satTOI.EarthMJ2000Eq.Z	22191.27247769132	22191.27249591363	1.822230478865094e-05
(==) satTOI.EarthMJ2000Eq.V	1.790840047506368	1.790840040820796	-6.685572095932457e-09
(==) satTOI.EarthMJ2000Eq.V	-0.07353662586722309	-0.07353662399181789	1.875405206130942e-09
(==) satTOI.EarthMJ2000Eq.V	0.03733116886189006	0.03733116769966248	-1.162227578066499e-09
(==) satMOI_Backward.Earth	504202.6748992605	504202.6867126295	0.0118133689975366
(==) satMOI_Backward.Earth	353465.3224617756	353465.368277343	0.04581556736957282
(==) satMOI_Backward.Earth	23053.68987783697	23053.69629257981	0.006414742849301547
(==) satMOI_Backward.Earth	-0.1809556240646748	-0.1809556799780389	-5.591336404986436e-08
(==) satMOI_Backward.Earth	0.3898296276089206	0.3898295536091186	-7.399980195277323e-08
(==) satMOI_Backward.Earth	0.06341508356815177	0.06341507415412447	-9.414027307697026e-09
(==) satMOI_Forward.EarthM	10	9.999999999678604	-3.213962429526873e-10
(==) satMOI_Forward.RadAp	382688.178	382688.1785403523	0.0005403522518463433

Objective Function	Current Value	Last Value	Difference
Cost	0.09192574337852834	0.09192566482942076	7.854910757942868e-08

CONVERGED
Optimization Completed in 11 passes through the Solver Control Sequence

Solver Window - Optimize 'Optimize Flyby' NLPOpt {SolveMode = Solve, ExitMode = Disca...

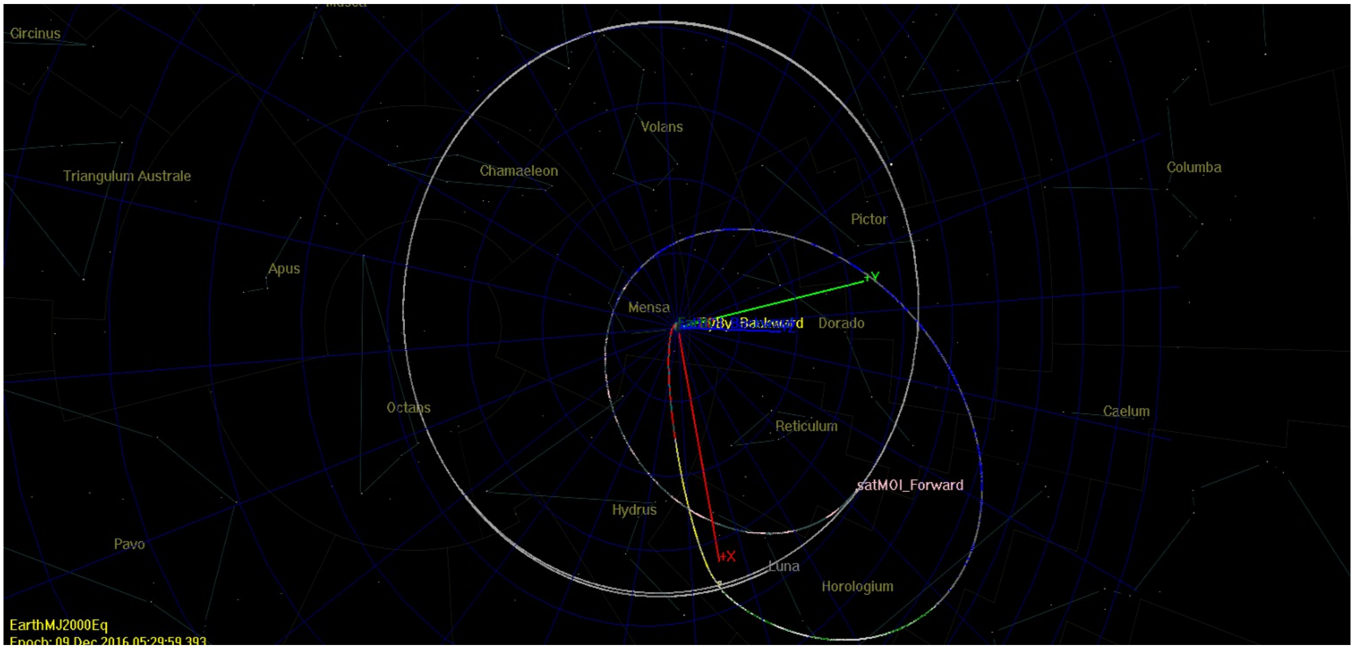
Control Variable	Current Value	Last Value	Difference
toiEpoch	27698.54756076427	27698.54756076427	3.637978807091713e-12
flybyEpoch	27703.57719146816	27703.57719146816	3.637978807091713e-12
moiEpoch	27725.23435494771	27725.23435494771	3.637978807091713e-12
satTOI.X	-6662.750580075303	-6662.750580075303	0
satTOI.Y	-48.96437827921044	-48.96437827921044	0
satTOI.Z	30.4334838418114	30.4334838418114	0
satTOI.VX	0.09364842531779116	0.09364842531779116	0
satTOI.VY	-9.52775946355343	-9.52775946355343	0
satTOI.VZ	5.173094293133505	5.173094293133505	0
satFlyBy_Forward.X	426.2493240129227	426.2493240129227	0
satFlyBy_Forward.Y	-6712.926059740425	-6712.926059740425	0
satFlyBy_Forward.Z	-3644.105663933052	-3644.105663933052	0
satFlyBy_Forward.VX	1.117421068628345	1.117421068628345	2.220446049250313e-16
satFlyBy_Forward.VY	-0.7568466648495614	-0.7568466648495614	0
satFlyBy_Forward.VZ	-0.5521139681591446	-0.5521139681591446	0
satMOI_Backward.X	-53045.68518589674	-53045.68518589674	0
satMOI_Backward.Y	-80663.5932896466	-80663.5932896466	0
satMOI_Backward.Z	-8422.053973216372	-8422.053973216372	0
satMOI_Backward.VX	2.05202330560308	2.05202330560308	4.440892098500626e-16
satMOI_Backward.VY	-1.63560879294263	-1.63560879294263	-4.440892098500626e-16
satMOI_Backward.VZ	-0.4212927359177253	-0.4212927359177253	5.551115123125783e-17
MOI.Element1	-0.09192574337852834	-0.09182574337852834	-0.0001000000000000029

CONVERGED
Optimization Completed in 11 passes through the Solver Control Sequence

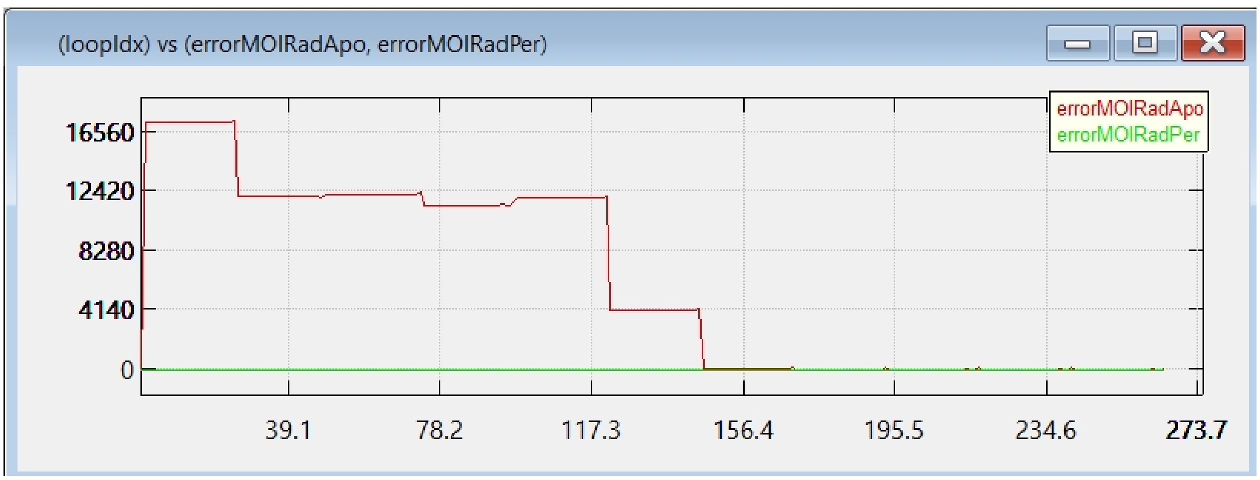
3.2 Final Solution and Maneuver Summary

The optimization successfully converged to a solution that meets all mission requirements. The key findings from the GMAT analysis are:

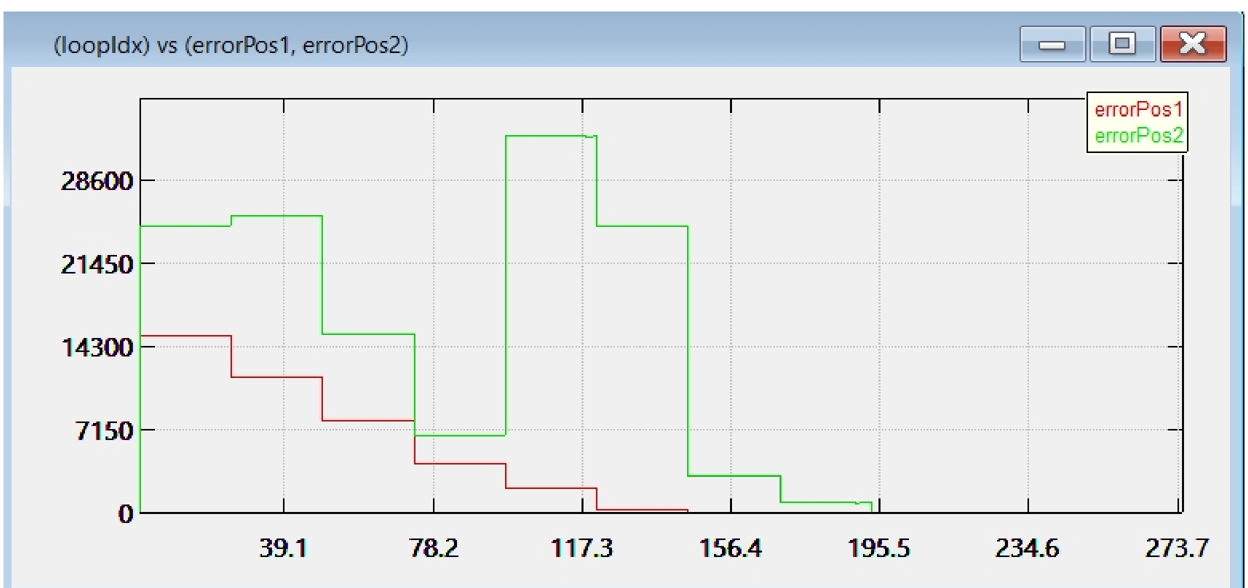
- MOI ΔV: The final MOI maneuver required a ΔV of −0.09171 km/s.
- Trajectory: The optimized trajectory successfully executes a lunar flyby, providing the necessary change in velocity to achieve the target mission orbit.
- Mission Constraints: The final orbit satisfies the specified periapsis and inclination constraints, as verified by the GMAT propagator.



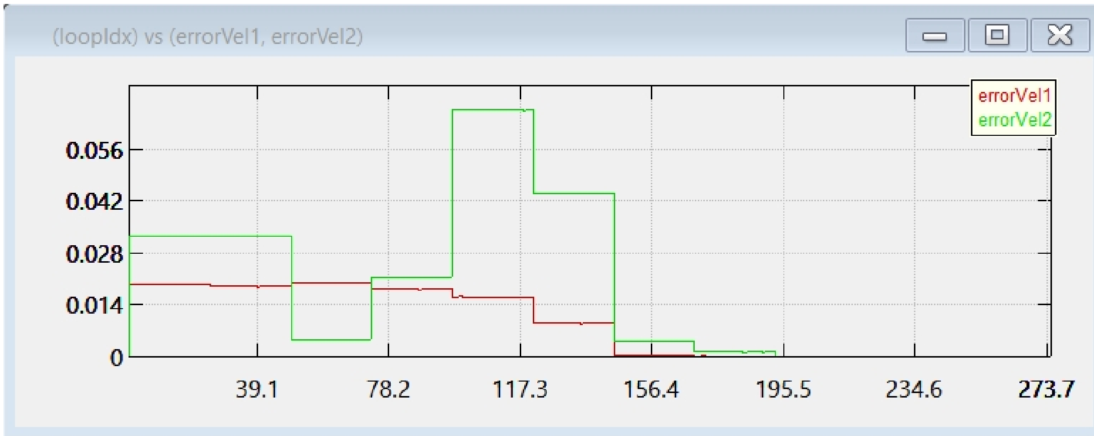
Circinus
Volans
Triangulum Australe
Chamaeleon
Columba
Pictor
Apus
Mensa
Dorado
Octans
Reticulum
Caelum
satMOI_Forward
Hydrus
Pavo
+X
Luna
Horologium
EarthMJ2000Eq



(loopIdx) vs (errorMOIRadApo, errorMOIRadPer)
errorMOIRadApo
errorMOIRadPer
16560
12420
8280
4140
0
39.1
78.2
117.3
156.4
195.5
234.6
273.7



(loopIdx) vs (errorPos1, errorPos2)
errorPos1
errorPos2
28600
21450
14300
7150
0
39.1
78.2
117.3
156.4
195.5
234.6
273.7



4. Discussion

4.1 Signifiicance ofi the Flyby

The successful optimization of this lunar flyby demonstrates the power of the Multiple Shooting Method for complex trajectory design. By leveraging the Moon's gravity, the spacecraft is able to achieve its desired mission orbit with a minimal ΔV burn at MOI. This method is far more fuel-efficient than a direct burn from a low-Earth orbit to the final mission orbit, highlighting its importance for long-duration and resource-constrained space missions.

4.2 Limitations and Future Work

While successful, this analysis has limitations. It assumes a simplified model of the solar system and does not account for all perturbations. Future work could include adding more complex gravitational models, considering other celestial bodies, and incorporating navigation and guidance errors to make the design more robust for a real-world mission.

5. Conclusion

In conclusion, this report demonstrates the successful application of the Multiple Shooting Method to design an optimal lunar flyby trajectory. By breaking down the problem into smaller segments and using a numerical optimizer, the process efficiently found a fuel-optimal solution that satisfies all mission constraints. The results confirm that a lunar gravity assist is a viable and highly efficient method for achieving a desired final orbit, validating the use of modern optimization techniques in astrodynamics.

```
Mission run completed.
===> Total Run Time: 70.179 seconds
```