Centro de Enseñanza Técnica Industrial

Desarrollo de Software

Actividad Para Evaluación Parcial –
100 ejercicios resueltos

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E.D. Homogéneas

1) $y'' - y' - 12y = 0$

$m^2 - m - 12 = 0$

$(m-4)(m+3)$

$m_1 = 4$

$m_2 = -3$

$y = C_1 e^{4x} + C_2 e^{-3x}$

2) $y'' - 4y = 0$

$m^2 - 4 = 0$

$m^2 = 4$

$m_1 = 2$

$m_2 = -2$

$y = C_1 e^{2x} + C_2 e^{-2x}$

3) $y'' - 2y' + 5y = 0$

$m^2 - 2m + 5 = 0$

$y = C_1 e^{x} \cos(2x) + C_2 e^{x} \operatorname{sen}(2x)$

$a = 1 \quad b = -2 \quad c = 5$

$\frac{2 \pm \sqrt{4 - 4(1)(5)}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i = m_{1,2}$

4) $4y'' - 4y' + y = 0$

$4m^2 - 4m + 1 = 0$

$(2m-1)^2 = 0 \quad m_{1,2} = \frac{1}{2}$

$y = C_1 e^{\frac{1}{2}} + C_2 x e^{\frac{1}{2}}$

5) $y'' + 16y = 0, \; y(0) = 2, \; y'(0) = -2$

$m^2 + 16 = 0$

$m^2 = -16$

$m_{1,2} = \pm 4i$

$y = C1 \cos(4x) + C_2 \operatorname{sen}(4x)$

$y' = -C1 \operatorname{sen}(4x) + C2 \cos(4x)$

$y(0) = 2 = C1 \cos(4(0)) + \cancel{C_2 \operatorname{sen}(4(0))}^{0} \quad C_1 = 2$

$y'(0) = -2 = \cancel{-C1 \operatorname{sen}(4(0))} + C2 \cos(4(0)) \quad C2 = -2$

$y_p = 2\cos(4x) - 2\operatorname{sen}(4x)$

6) $y'' + y = 0,\ y(\frac{\pi}{3}) = 0,\ y'(\frac{\pi}{3}) = 2$

$m^2 + 1 = 0 \qquad y = C_1 \cos(x) + C_2 \operatorname{sen}(x)$

$m^2 = -1 \quad m_{1,2} = \pm i \quad y' = -C_1 \operatorname{sen}(x) + C_2 \cos(x)$

$y(\frac{\pi}{3}) = 0 = C_1 \cos(\frac{\pi}{3}) + C_2 \operatorname{sen}(\frac{\pi}{3})$

$y'(\frac{\pi}{3}) = 2 = -C_1 \operatorname{sen}(\frac{\pi}{3}) + C_2 \cos(\frac{\pi}{3})$

7) $y'' - 4y' - 5y = 0,\ y(1) = 0,\ y'(1) = 2$

$m^2 - 4m - 5 = 0$

$(m-5)(m+1) \qquad m_1 = 5 \qquad y = C_1 e^{5x} + C_2 e^{-x}$

$m_2 = -1 \qquad y' = 5C_1 e^{5x} - C_2 e^{-x}$

$y(1) = 0 = C_1 e^{5(1)} + C_2 e^{-1} \qquad C_1 = \frac{2}{5e^5} \qquad C_2 = -\frac{2e}{5}$

$y'(1) = 2 = 5C_1 e^{5(1)} - C_2 e^{-1}$

$y_p = \frac{2}{5e^5} e^{5x} - \frac{2e}{5e^x}$

8) $4y'' - 4y' - 3y = 0,\ y(0) = 1,\ y'(0) = 5$

$4m^2 - 4m - 3 = 0$

$a = 4 \quad b = -4 \quad c = -3$

$\frac{4 \pm \sqrt{16 - 4(4)(-3)}}{2(4)} = \frac{4 \pm \sqrt{16 + 48}}{8} = \frac{4 \pm \sqrt{64}}{8} = \frac{4 \pm 8}{8} =$

$\Rightarrow m_1 = \frac{4+8}{8} = \frac{12}{8} = \frac{3}{4} \qquad y = C_1 e^{\frac{3}{4}x} + C_2 e^{-\frac{1}{2}x}$

$y' = \frac{3}{4} C_1 e^{\frac{3}{4}x} - \frac{1}{2} C_2 e^{-\frac{1}{2}x}$

$m_2 = \frac{4-8}{8} = -\frac{4}{8} = -\frac{1}{2}$

$y(0) = 1 = C_1 e^{\frac{3}{4}(0)} + C_2 e^{-\frac{1}{2}(0)} = C_1 + C_2 \qquad C_1 = \frac{22}{5}$

$y'(0) = 5 = \frac{3}{4} C_1 e^{\frac{3}{4}(0)} - \frac{1}{2} C_2 e^{-\frac{1}{2}(0)} = \frac{3}{4} C_1 - \frac{1}{2} C_2 \qquad C_2 = \frac{18}{5}$

$y = \frac{22}{5} e^{\frac{3}{4}x} + \frac{18}{5} e^{-\frac{1}{2}x}$

9) $y''' - 3y'' - y' + 3y = 0$

$m^3 - 3m^2 - m + 3 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & -1 & 3 \\ & & 1 & -2 & -3 \\ \hline & 1 & -2 & -3 & 0 \end{array} \qquad m_1 = 1$$

$m^2 - 2m - 3 = 0$

$(m-3)(m+1)$ $\qquad m_2 = 3 \qquad m_3 = -1$

$$y = C_1 e^{x} + C_2 e^{3x} + C_3 e^{-x}$$

10) $y''' + 2y'' - 19y' - 20y = 0$

$m^3 + 2m^2 - 19m - 20 = 0$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -19 & -20 \\ & & -1 & -1 & 20 \\ \hline & 1 & 1 & -20 & 0 \end{array} \qquad m_1 = -1$$

$m^2 + m - 20 = 0$

$(m+5)(m-4)$ $\qquad m_2 = -5 \qquad m_3 = 4$

$$y = C_1 e^{-x} + C_2 e^{-5x} + C_3 e^{4x}$$

11) $y''' - y'' + 2y = 0$

$m^3 - m^2 + 2 = 0$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 0 & 2 \\ & & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & 0 \end{array} \qquad m_1 = -1$$

$$y = C_1 e^{-x} + C_2 e^{x}\cos(x) + C_3 e^{x}\operatorname{sen}(x)$$

$m^2 - 2m + 2 = 0$

$a = 1 \quad b = -2 \quad c = 2$

$$\frac{2 \pm \sqrt{4 - 4(1)(2)}}{2} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i = m_{2,3}$$

12) $y^{IV} - y = 0 \qquad y = C_1 e^{x} + C_2 e^{-x} + C_3 \cos(x) + C_4 \operatorname{sen}(x)$

$m^4 - 1 = 0$

$m^4 = 1 \qquad u = m^2 \qquad m_1 = 1$

$u^2 = 1 \qquad m^2 = 1 \qquad m_2 = -1$

$u = -1 \qquad m^2 = -1 \qquad m_3 = i$

$m_4 = -i$

13) $y^{IV} + 4y''' + 6y'' + 4y' + y = 0$

$m^4 + 4m^3 + 6m^2 + 4m + 1 = 0$

$(m^2 + 2m + 1)^2 = 0$

$(m+1)^4 = 0 \qquad y = C_1 e^{x} + C_2 x e^{x} + C_3 x^2 e^{x} + C_4 x^3 e^{x}$

$m_{1,2,3,4} = 1$

14) $y^{IV} + 2y''' + 10y'' + 18y' + 9y = 0$

$m^4 + 2m^3 + 10m^2 + 18m + 9 = 0$

$(m+1)^2 (m^2 + 9) \qquad m_{1,2} = -1$

$m_{3,4} = \pm 3i$

$y = C_1 e^{-x} + C_2 x e^{-x} + C_3 \cos(3x) + C_4 \operatorname{sen}(3x)$

15) $y''' - y'' - 4y' + 4y = 0, \; y(0) = -4, \; y'(0) = -1, \; y''(0) = -19$

$m^3 - m^2 - 4m + 4 = 0 \qquad y = C_1 e^{x} + C_2 e^{2x} + C_3 x e^{2x}$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -4 & 4 \\ & & 1 & 0 & -4 \\ \hline & 1 & 0 & -4 & 0 \end{array} \qquad m_1 = 1$$

$y' = C_1 e^{x} + 2C_2 e^{2x} + 2C_3 e^{2x}$

$y'' = C_1 e^{x} + 4C_2 e^{2x} + 4C_3 e^{2x}$

$m^2 - 4 = 0 \qquad m_{2,3} = 2$

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$y(0) = -4 = c1 + c2 + c3$

$y'(0) = -1 = c1 + 2c2 + 2c3$

$y''(0) = -19 = c1 + 4c2 + 4c3$

16) $y''' + 7y'' + 14y' + 8y = 0,\ y(0) = 1,\ y'(0) = -3,\ y''(0) = 13$

$m^3 + 7m + 14m + 8 = 0$

$$\begin{array}{r|rrrr} -1 & 1 & 7 & 14 & 8 \\ & & -1 & -6 & -8 \\ \hline & 1 & 6 & 8 & 0 \end{array}$$

$m_1 = -1$

$y = C_1 e^{-x} + C_2 e^{-4x} + C_3 e^{-2x}$

$m^2 + 6m + 8 = 0$

$y' = -C1 e^{-x} - 4C2 e^{-4x} - 2C3 e^{-2x}$

$(m+4)(m+2)$

$y'' = C1 e^{-x} + 16C2 e^{-4x} + 4C3 e^{-2x}$

$m_2 = -4 \quad m_3 = -2$

$y(0) = C1 + C2 + C3$

$y'(0) = -C1 - 4C2 - 2C3$

$y''(0) = C1 + 16C2 + 4C3$

$C1 = -\frac{2}{3}$

$C2 = \frac{1}{6}$

$C3 = \frac{3}{2}$

$y = -\frac{2}{3}e^{-x} + \frac{1}{6}e^{-4x} + \frac{3}{2}e^{-2x}$

17) $y'' + 9y = 0$

$m^2 + 9 = 0$

$m_{1,2} = \pm 3i$

$y = C1\cos(3x) + C2\,\mathrm{sen}(3x)$

18) $y'' + y = 0$

$m^2 + 1 = 0$

$m_{1,2} = \pm i$

$y = C1\cos(x) + C2\,\mathrm{sen}(x)$

19) $z'' - 6z' + 10z = 0$ $\qquad y = C_1 e^{3x}\cos(x) + C_2 e^{3x} sen(x)$

$a = 1 \quad b = -6 \quad c = 10$

$$\frac{6 \pm \sqrt{36 - 4(1)(10)}}{2} = \frac{6 \pm \sqrt{36 - 40}}{2} = \frac{6 \pm 2i}{2} = 3 \pm i$$

20) $y'' - 10y' + 26y = 0$ $\qquad y = C_1 e^{5x}\cos(x) + C_2 e^{5x} sen(x)$

$a = 1 \quad b = -10 \quad c = 26$

$$\frac{10 \pm \sqrt{100 - 4(1)(26)}}{2} = \frac{10 \pm \sqrt{100 - 104}}{2} = 5 \pm i$$

21) $w'' + 4w + 6w = 0$ $\qquad y = C_1 e^{-2x}\cos(\sqrt{2}x) + C_2 e^{-2x} sen(\sqrt{2}x)$

$a = 1 \quad b = 4 \quad c = 6$

$$\frac{-4 \pm \sqrt{16 - 4(1)(6)}}{2} = \frac{-4 \pm \sqrt{16 - 24}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}i$$

22) $y'' - 4y' + 7y = 0$ $\qquad y = C_1 e^{2x}\cos(\sqrt{3}x) + C_2 e^{2x} sen(\sqrt{3}x)$

$a = 1 \quad b = -4 \quad c = 7$

$$\frac{4 \pm \sqrt{16 - 4(1)(7)}}{2} = \frac{4 \pm \sqrt{16 - 28}}{2} = \frac{4 \pm 2\sqrt{3}i}{2} = 2 \pm \sqrt{3}i$$

23) $4y'' - 4y' + 26y = 0$ $\qquad y = C_1 e^{\frac{1}{2}x}\cos(\frac{5}{2}x) + C_2 e^{\frac{1}{2}x} sen(\frac{5}{2}x)$

$a = 4 \quad b = -4 \quad c = 26$

$$\frac{4 \pm \sqrt{16 - 4(4)(26)}}{2(4)} = \frac{4 \pm \sqrt{16 - 416}}{8} = \frac{4 \pm \sqrt{400}i}{8} = \frac{1 \pm 5i}{2}$$

24) $4y'' + 4y' + 6y = 0$ $\qquad y = C_1 e^{-\frac{1}{2}x}\cos(\frac{\sqrt{5}}{2}x) + C_2 e^{-\frac{1}{2}x} sen(\frac{\sqrt{5}}{2}x)$

$a = 4 \quad b = 4 \quad c = 6$

$$\frac{-4 \pm \sqrt{16 - 4(4)(6)}}{2(4)} = \frac{-4 \pm \sqrt{16 - 96}}{8} = \frac{-4 \pm \sqrt{80}}{8} = \frac{-1 \pm \sqrt{5}}{2}$$

25) $y'' - 8y' + 7y = 0$ $\qquad y = C_1 e^{7x} + C_2 e^{x}$

$m^2 - 8m + 7 = 0$

$(m-7)(m-1) = 0$ $\qquad m_1 = 7$

$m_2 = 1$

26) $y'' + 4y' + 8y = 0$ $\qquad y = C_1 e^{-2x}\cos(2x) + C_2 e^{-2x} sen(2x)$

$a = 1 \quad b = 4 \quad c = 8$

$$\frac{-4 \pm \sqrt{16 - 4(1)(8)}}{2} = \frac{-4 \pm \sqrt{16-32}}{2} = \frac{-4 \pm \sqrt{-16}}{2} = -2 \pm 2i$$

27) $z'' + 10z' + 25z = 0$ $\qquad y = C_1 e^{-5x} + C_2 x e^{-5x}$

$m^2 + 10m + 25 = 0$

$(m+5)^2 = 0 \quad m_{1,2} = -5$

28) $u'' + 7u = 0$ $\qquad y = C_1 \cos(\sqrt{7}x) + C_2 sen(\sqrt{7}x)$

$m^2 + 7 = 0$

$m_1 = \pm\sqrt{7}i$

29) $y'' + 2y' + 5y = 0$ $\qquad y = C_1 e^{-x}\cos(2x) + C_2 e^{-x} sen(2x)$

$a = 1 \quad b = 2 \quad c = 5$

$$\frac{-2 \pm \sqrt{4 - 4(1)(5)}}{2} = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm 4i}{2} = -1 \pm 2i$$

30) $y'' - 2y' + 26y = 0$ $\qquad y = C_1 e^{x}\cos(5x) + C_2 e^{x} sen(5x)$

$a = 1 \quad b = -2 \quad c = 26$

$$\frac{2 \pm \sqrt{4 - 4(1)(26)}}{2} = \frac{2 \pm \sqrt{4-104}}{2} = \frac{2 \pm \sqrt{100}\,i}{2} = 1 \pm 5i$$

31) $y''+10y'+41y=0$ $\quad Y=C_1e^{-5x}\cos(8x)+C_2e^{-5x}sen(5x)$

$a=1 \quad b=10 \quad c=41$

$$\frac{-10\pm\sqrt{100-4(1)(41)}}{2}=\frac{-10\pm\sqrt{100-164}}{2}=-5\pm8i$$

32) $y''-3y'-11y=0$ $\quad Y=C_1e^{4x}+C_2e^{-x}$

$a=1 \quad b=-3 \quad c=-11$

$$\frac{3\pm\sqrt{9-4(1)(-11)}}{2}=\frac{3\pm\sqrt{9+44}}{2}=\frac{3\pm5}{2}\rightarrow m_1=4 \quad m_2=-1$$

33) $y''-y'+7y=0$ $\quad Y=C_1e^{x}\cos\left(\frac{3\sqrt{3}}{2}x\right)+C_2e^{x}sen\left(\frac{3\sqrt{3}}{2}x\right)$

$a=1 \quad b=-1 \quad c=7$

$$\frac{1\pm\sqrt{1-4(1)(7)}}{2}=\frac{1\pm\sqrt{1-28}}{2}=\frac{1\pm3\sqrt{3}}{2}$$

34) $2y''+13y'-7y=0$ $\quad y=C_1e^{\frac{1}{2}x}+C_2e^{-7x}$

$a=2 \quad b=13 \quad c=-7$

$$\frac{-13\pm\sqrt{169-4(2)(-7)}}{2(2)}=\frac{-13\pm\sqrt{169+56}}{4}=\frac{-13\pm15}{4}$$

$m_1=\frac{1}{2} \quad m_2=-7$

35) $y'''+y''+3y'-5y=0$ $\quad y=C_1e^{x}+C_2e^{-\frac{1}{2}x}\cos\left(\frac{\sqrt{19}}{2}x\right)+C_3e^{-\frac{1}{2}x}sen\left(\frac{\sqrt{19}}{2}x\right)$

$m^3+m^2+3m-5=0$

1	1	1	3	-5	$m_1=1$
		1	2	5	
	1	2	5	0	

$m^2+m+5=0$

$a=1 \quad b=1 \quad c=5$

$$\frac{-1\pm\sqrt{1-4(1)(5)}}{2}=\frac{-1\pm\sqrt{1-20}}{2}=\frac{-1\pm\sqrt{19}i}{2}=m_{2,3}$$

36) $y''' - y'' + 2y = 0 \qquad y = C_1 e^{-x} + C_2 e^{x} \cos(x) + C_3 e^{x} \operatorname{sen}(x)$

$m^3 - m^2 + 2 = 0$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 0 & 2 \\ & & -1 & -2 & -2 \\ \hline & 1 & -2 & 2 & 0 \end{array} \qquad m_1 = -1$$

$m^2 - 2m + 2 = 0$

$a = 1 \quad b = -2 \quad c = 2$

$$\frac{2 \pm \sqrt{4 - 4(1)(2)}}{2} = \frac{2 \pm \sqrt{4 - 8}}{2} = 1 \pm i = m_{2,3}$$

37) $y'' + 2y' + 2y = 0, \quad y(0) = 2, \quad y'(0) = 1$

$m^2 + 2m + 2 = 0 \qquad y = C_1 e^{-x} \cos(x) + C_2 e^{-x} \operatorname{sen}(x)$

$a = 1 \quad b = 2 \quad c = 2$

$$\frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2} = \frac{-2 \pm \sqrt{4 - 8}}{2} = -1 \pm i = m_{1,2}$$

$y(0) = 2 = C_1 e^{0} \cos(0) + \cancel{C_2 e^{0} \operatorname{sen}(0)} \qquad C_1 = 2$

$y'(0) = 1 = -C_1 (e^{0} \cos(0) + \cancel{e^{0} \operatorname{sen}(0)}) + C_2 (\cancel{-e^{0} \operatorname{sen}(0)} + e^{0} \cos(0)) =$

$= -C_1 + C_2 = -2 + C_2 \qquad C_2 = 3$

$y = 2 e^{-x} \cos(x) + 3 e^{-x} \operatorname{sen}(x)$

38) $y'' + 2y' + 17y = 0, \quad y(0) = 1, \quad y'(0) = -1$

$a = 1 \quad b = 2 \quad c = 17 \qquad y = C_1 e^{-x} \cos(4x) + C_2 e^{-x} \operatorname{sen}(4x)$

$$\frac{-2 \pm \sqrt{4 - 4(1)(17)}}{2} = \frac{-2 \pm \sqrt{4 - 68}}{2} = \frac{-2 \pm 8i}{2} = -1 \pm 4i$$

$y(0) = 1 = C_1 e^{0} \cos(4(0)) + \cancel{C_2 e^{0} \operatorname{sen}(4(0))} \qquad C_1 = 1$

$y'(0) = -1 \Rightarrow C_2 = 1$

$y = \cos(4x) + \operatorname{sen}(4x)$

39) $w'' - 4w' + 2w = 0$, $w(0) = 0$, $w'(0) = 1$

$a = 1 \quad b = -4 \quad c = 2$

$$\frac{4 \pm \sqrt{16 - 4(1)(2)}}{2} = \frac{4 \pm \sqrt{16 - 8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

$W = C_1 e^{2+\sqrt{2}x} + C_2 e^{2-\sqrt{2}x}$

$w(0) = 0 = C_1 e^0 + C_2 e^0 = C_1 + C_2$

$w'(0) = 1 = (2+\sqrt{2}) C_1 e^0 + (2-\sqrt{2}) C_2 e^0$

40) $y'' + 9y = 0$, $y(0) = 1$, $y'(0) = 1$

$m^2 + 9 = 0 \qquad y = C_1 \cos(3x) + C_2 \operatorname{sen}(3x)$

$m_{1,2} = \pm 3i \qquad y' = -3C_1 \operatorname{sen}(3x) + 3C_2 \cos(3x)$

$y(0) = 1 = C_1 \cos(3(0)) + \cancel{C_2 \operatorname{sen}(3(0))} \xrightarrow{} 0 \qquad C_1 = 1$

$y'(0) = 1 = \cancel{-3C_1 \operatorname{sen}(3(0))} + 3C_2 \cos(3(0)) \qquad C_2 = \frac{1}{3}$

$y = \cos(3x) + \frac{1}{3} \operatorname{sen}(3x)$

41) $y'' - 2y' + 2y = 0$, $y(\pi) = e^{\pi}$, $y'(\pi) = 0$

$a = 1 \quad b = -2 \quad c = 2$

$$\frac{2 \pm \sqrt{4 - 4(1)(2)}}{2} = \frac{2 \pm \sqrt{4 - 8}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

$y = C_1 e^x \cos(x) + C_2 e^x \operatorname{sen}(x)$

$y' = -C_1 e^x \operatorname{sen}(x) + C_2 e^x \cos(x)$

$y(\pi) = e^{\pi} = \cancel{C_1 e^{\pi} \cos(\pi)} + C_2 e^{\pi} \operatorname{sen}(\pi) \qquad C_2 = 1$

$y'(\pi) = 0 = -C_1 e^{\pi} \operatorname{sen}(\pi) + \cancel{C_2 e^{\pi} \cos(\pi)} \qquad C_1 = -\frac{1}{e^{\pi}}$

$y = e^x \operatorname{sen}(x) - \frac{1}{e^{\pi}} \cos(x)$

42) $y'' - 2y' + y = 0, \quad y(0) = 1, \quad y'(0) = -2$

$m^2 - 2m + 1 = 0 \qquad y = C1\,e^{x} + C2\,x\,e^{x}$

$(m-1)^2 = 0 \qquad m_{1,2} = 1$

$y(0) = 1 = C1\,e^{0} + \cancel{C2(0)e^{0}} \qquad C1 = 1$

$y'(0) = -2 = C1\,e^{0} + C2\left(\cancel{(0)e^{0}} + e^{0}\right) \qquad C2 = -3$

$y = e^{x} - 3x\,e^{x}$

43) $y''' - 4y'' + 7y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 0$

$m^3 - 4m^2 + 7m - 6 = 0 \qquad y = C1\,e^{-2x} + C2\,e^{x}\cos(x) + C3\,e^{x}\,\text{sen}(x)$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 7 & -6 \\ & & 2 & -4 & 6 \\ \hline & 1 & -2 & 3 & 0 \end{array} \qquad m_1 = -2$$

$m^2 - 2m + 3 = 0$

$a = 1 \quad b = -2 \quad c = 3$

$$\frac{2 \pm \sqrt{4 - 4(1)(3)}}{2} = \frac{2 \pm \sqrt{4 - 12}}{2} = \frac{2 \pm 2\sqrt{2}\,i}{2} = 1 \pm \sqrt{2}\,i$$

$y(0) = 1 = C1\,e^{0} + C2\,e^{0}\cos(0) + \cancel{C3\,e^{0}\,\text{sen}(0)} = C1 + C2$

$y'(0) = 0 = -2\,C1\,e^{0} + C2\left(\cancel{-e^{0}\,\text{sen}(0)} + e^{0}\cos(0)\right) + C3\left(e^{0}\cos(0) + \cancel{e^{0}\,\text{sen}(0)}\right) = -2C1 + C2 + C3$

$y''(0) = 0 = 4\,C1\,e^{0} + C2\left(\left(-e^{0}\cos(0) + \cancel{e^{0}\,\text{sen}(0)}\right) + \left(\cancel{-e^{0}\,\text{sen}(0)} - e^{0}\cos(0)\right)\right) + C3\left(\left(\cancel{-e^{0}\,\text{sen}(0)} - e^{0}\cos(0)\right) + \left(e^{0}\cos(0) - \cancel{e^{0}\,\text{sen}(0)}\right)\right) =$

$= 4C1 - 2C2$

$C1 = \frac{1}{3} \qquad C2 = \frac{2}{3} \qquad C3 = 0$

$y = \frac{1}{3}e^{-2x} + \frac{2}{3}e^{x}\cos(x)$

TAREA

Metodo variacion de parametros

1; $y'' + y = \tan x \qquad = y'' + y = 0$

$r^2 + 1 = 0$

$r^2 = -1 \qquad r = \pm i$

$y_c = C_1 \cos x + C_2 \sin x$

$y_p = A \tan x + B$

$y'_p = A \sec^2 x$

$y''_p = 2A \sec^2 x \tan x$

$2A \sec^2 x \tan x + A \tan x + B = \tan x$

$2A \sec^2 x = 1 \qquad A = \dfrac{1}{2\cos^2 x}$

$y_p = \dfrac{1}{2\cos^2 x} \tan x + B$

$y = y_c + y_p = C_1 \cos x + C_2 \sin x + \dfrac{1}{2\cos^2 x} \tan x + B$

$y = C_1 \cos x + C_2 \sin x + \dfrac{1}{2\cos^2 x} \tan x + B$

2; $y'' + y = \sec x \tan x$

$y'' + y = 0$

$r^2 \, 2 + 1 = 0 \qquad r = \pm i$

$y = c_1^{\alpha} \cos x + c_2^{\alpha} \sin x$

$y'' + y = \sec x \tan x$

$y_p = A\sec x + B \tan x$

$y_p' = -A^x \sec x \tan x + B \sec^2 x$

$y_p'' = -A \sec x \tan x - 2B \sec^2 x \tan x$

$(-A\sec x \tan x - 2B \sec^2 x \tan x) + (A \sec x + B \tan x = \sec x \tan x$

Simplificando $\rightarrow = -A \sec x \tan x - 2B \sec^2 x \tan x + A \sec x + B \tan x = \sec x \tan x$

$-A - 2B \sec x = 1 \qquad A = 0 \qquad -A + B = 2$

$B = A = 0$

$y_g = y = c_1 \cos x + c_2 \sin x$

$3.- y'' + y = \sec^2 x \Rightarrow y'' + y = 0$

$r^2 + 1 = 0 \qquad r = \pm i$

$y_h = c_1 \cos x + c_2 \sin x$

$y_p = A \sec^2 x$

$y_p' = -2A \sec x \tan x$

$y_p'' = -2A(\sec^2 x - \tan^2 x) = -2A$

$A = -2$

$y_p = -2\sec^2 x$

$y = y_h + y_p = \cos x + c_2 \sin x - 2\sec^2 x$

$y = c_1 \cos x + c_2 \sin x - 2\sec^2 x.$ //

1. $y'' + 2y' - y = t^{-1}e^{t}$

$y = e^{rx}$
$y' = re^{rx}$
$y'' = r^2e^{rx}$

$r^2e^{rx} + 2(re^{rx}) - e^{rx} = 0$
$e^{rx}(r^2 + 2r - 1) = 0$

$r^2 + 2r - 1 = 0$ $\quad \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad \dfrac{-2 \pm \sqrt{4+4}}{2}$
$(r+1)(r+1)$

$m_1 = -2 + \sqrt{8} \quad -1 + \frac{\sqrt{8}}{2}$

$y_h = c_1e^{-1+\sqrt{8}x} + c_2e^{-1-\sqrt{8}x}$

$m_2 = -2 - \sqrt{8}$
$-1 - \frac{\sqrt{8}}{2}$

$y_p = (Ax^{-1} + Bx + C)$
$y_p' = (-Ax^{-2} + B)e^{x} + (Ax^{-1} + Bx + C)xe^{x}$
$y''_p = xe^{x}(-Ax^{-1} + Bx + Ax + Bx^2 + Cx + 2Ax^3 + 2Bx + C$

$y = 2A + 2B - 2Ax^{-1} + 2Bx - 2Ax^{-2} + 2Bx^2 + 3Cx + Ax +$
$A2x^{-3} + Bx^3 = x^{-1}$

$y = x^3B + x^2\,2B + x(A + 2B + 3C) + x^{-1}(-2A) + x^{-2}(-2C)$
$x^{-3}(2A) + (2A + 2B) = x^{-1}$

$B = 0 \qquad -2A = 1 \qquad C = \frac{1}{6}$
$A = -\frac{1}{2}$

2

$5y'' - 3y' + 2y = e^3 \cos 4t$

$r_1 = \frac{2}{3}$ $\quad r_2 = 1$

$y_h = C_1 e^{\frac{2}{3}x} + C_2 e^{1x}$

$y_p = At^3 \cos 4t + Bt^3 \operatorname{sen} 4t$

$y'_p = 3At^2 \cos 4t - 4At^3 \operatorname{sen} 4t + 4Bt^3 \operatorname{sen} 4t + Bt^3 \cos 4t$

$y''_p = 6At \cos 4t - 24At^2 \operatorname{sen} 4t - 8At^2 \operatorname{sen} 4t + 8B \operatorname{sen} 4t\ 5B$
$t^2 \cos 4t^3\ 4B \operatorname{sen} 4t$

$(30At - 9At^2 + 2At^3) \cos 4t + (-120At^2 - 40At^2 - 40B + 12$
$B \operatorname{sen} 4 + (40Bt^2 - 20Bt^3) \cos 4t + (40Bt - 2t^3) \operatorname{sen} 4t =$
$t \cos 4$

$23A = 1 \qquad A = \frac{1}{23}$

$-160A + 28B = 0 \qquad B = 0$

$20B = 0$
$38B = 0$

$y = C_1 e^{2/3 t} + C_2 e^{t} + \frac{1}{23} t^3 \cos 4t$

3

$x'' + 5x' - 3x = 3'$

$r^2 + 5r - 3 = 0$

$r = -\frac{5}{2} \pm \frac{\sqrt{37}}{2}$

$y_h = C_1 e^{(-\frac{5}{2} + \frac{\sqrt{37}}{2})t} + C_2 e^{(-\frac{5}{2} + \frac{\sqrt{37}}{2})t}$

$y_e = A$

$0 + 0 - 3A = 3 \qquad x_e = -1$

$A = -1$

$y_g = C_1 e^{(-\frac{5}{2} + \frac{\sqrt{37}}{2})t} + C_2 e^{(-\frac{5}{2} + \frac{\sqrt{37}}{2})t} - 1$ ///

④

$$2y''x - 6y'x + yx = (\operatorname{sen} x)/e^{4x}$$

$$2r^2 - 6r + 1 = 0 \qquad r = \frac{3 \pm \sqrt{7}}{2}$$

$$y_h = C_1 e^{\frac{3+\sqrt{7}}{2}x} + C_2 e^{\frac{3-\sqrt{7}}{2}x}$$

$$y_p = A\operatorname{sen}(x)e^{-4x}$$
$$y_p' = A\cos x e^{-4x} - 4A\operatorname{sen}(x)e^{-4x}$$
$$y_p'' = -2A\operatorname{sen}(x)e^{-4x} - 8A\cos(x)e^{-4x} - 4A\cos(x)e^{-4x} + 16A\operatorname{sen}(x)e^{-4x}$$

$$(-4Ax - 8Ax - 6Ax + Ax)\operatorname{sen} x e^{-4x} + (-16Ax + 32Ax - 24Ax)\cos x e^{-4x} = \frac{\operatorname{sen} x}{e^{-4x}}$$

$$(17Ax = 1 \qquad A = -\frac{1}{17}$$

$$y_p = -\frac{1}{17}\operatorname{sen}(x)e^{-4x}$$

$$y_g = C_1 e^{\frac{3+\sqrt{7}}{2}x} + C_2 e^{\frac{3-\sqrt{2}}{2}x} - \frac{1}{17}\operatorname{sen}(x)e^{-4x} ///$$

5

$y''(\theta)+3y'(\theta)-y(\theta)=\sec\theta$

$x=\dfrac{-3\pm\sqrt{3}}{2}$

$y_h=C_1e^{\frac{-3+\sqrt{13}}{2}\theta}+C_2e^{\frac{-3-\sqrt{13}}{2}}$

$y_p(\theta)=A\sec\theta+B\cos\theta+C\operatorname{sen}\theta$

$y'_p=-A\sec\theta\tan\theta-B\operatorname{sen}\theta\ A\ C\cos\theta$

$y''_p=0\quad -A\sec\theta\tan^2\theta-A\sec\theta+B\cos\theta-C\operatorname{sen}\theta$

$-A\sec\theta\tan^2\theta-4a\sec\theta-4D\operatorname{sen}\theta+4c\cos\theta=0$

$-A\tan^2\theta-4A=0$ $\qquad A=0$

$-A(1+\tan^2\theta)=0$ $\qquad B=0$

$-A\sec^2\theta=0$ $\qquad C=0$

$y_p=0$

$y_g=C_1e^{\frac{-3+\sqrt{13}}{2}\theta}+C_2e^{\frac{-3-\sqrt{13}}{2}\theta}$ //

(6)

$2w''(x) - 3w(x) = 4x\,sen^2 x + 4x\cos^2 x$

$x = \pm\sqrt{\frac{3}{2}}$

$w_h = C_1 e^{\sqrt{\frac{3}{2}}x} + C_2 e^{-\sqrt{\frac{3}{2}}x}$

$w_p = Ax^2 + Bx + C$
$w'_p = 2Ax + B$
$w''_p = 2A$

$2(2A) - 3(Ax^2 + Bx + C) = 4x\,sen^2 x + 4x\cos^2 x$

$4A - 3Ax^2 - 3Bx - 3C = 4x\,sen^2 x + 4x\cos^2 x$

$4A - 3C = 0$ $\quad A = -\frac{4}{3}$ $\quad B = 0$ $\quad C = -\frac{16}{9}$
$-3A = 4$
$-3B = 0$

$w_p = -\frac{4}{3}x^2 - \frac{16}{4}$

$w_s = C_1 e^{\sqrt{3/2}\,x} + C_2 e^{-\sqrt{3/2}} - \frac{4}{3}x^2 - \frac{16}{4}$

7

$+y'' - y' + 2y = \text{Sen } 3t$

$y'' - y' + 2y = 0$

$r\ \frac{1}{2} \pm \frac{\sqrt{7}}{2} i$

$y_h = e^{\frac{1}{2}t}\left(A\cos\left(\frac{\sqrt{7}}{2}t\right) + B\,\text{sen}\left(\frac{\sqrt{7}}{2}t\right)\right)$

$y_p = C\,\text{sen}(3t) + D\cos(3t)$

$y_p' = C\,\text{sen}(3t)(3) + D - \text{sen}(3t)\,3$

$y_p'' = 9C t - \text{sen}(3t)\,t$

$9C - \text{sen}(3t) = 1 \qquad 9D(-\cos(3t)) = 0$

$= \frac{1}{9} - \text{sen}(3t) = 1 \qquad 9DC - \cos(3t) = 0$

$C = \frac{1}{9}(-\text{sen}(3t)) \qquad D = \frac{0}{9}(-\cos(3t)) \qquad D = 0$

$\left(\frac{1}{9(-\text{sen}(3t))}\right)\text{sen } 3t = -\frac{1}{9} = y_p$

$y_G = e^{\frac{1}{2}t}\left(A\cos\left(\frac{\sqrt{7}}{2}t\right) + B\,\text{sen}\left(\frac{\sqrt{7}}{2}t\right)\right) - \frac{1}{9}$ //

8-) $8z'(x) - 2z(x) = 3x^{100} e^{4x} \cos 25x$

$e^{\int -2dx} = e^{-2x}$

$8e^{-2x} z'(x) - 2e^{2x} z(x) = 3x^{100} e^{2x} \cos(25x)$

$8e^{-2x} z(x) = \int 3x^{100} e^{2x} \cos(25x)\, dx$

$dy = e^{2x} \cos(25x)\, dx \quad du = 100x\, dx$

$v = \frac{1}{24}\left(e^{2x} \operatorname{sen}(25x) + 2\cos(25x)\right)$

$\int 3x^{100} e^{2x} \cos(25x)\, dx = \frac{1}{24}\left(x^{100}\left(e^{2x} \operatorname{sen}(25 + 2\right.\right.$
$\cos(25)$

$-\frac{100}{24} \int x^{99} \left(e^{2x} \operatorname{sen}(25x) + 2\cos(25)\right) dx$

9

$y'' + 3y = -9$

$y' = re^{rt}$

$y'' = r^2 e^{rt}$

$r^2 e^{rt} + 3e^{rt} = -9$

$e^{rt}(r^2+3) = -9$

$r = \pm 2i\sqrt{3}$

$yG = C_1 \cos(2\sqrt{3}\,t) + C_2 \operatorname{sen}(2\sqrt{3}t) - 3$

10

$y''(x) + y(x) = 2^x$

$A2^x = 1$

$A = \frac{1}{2}$ $\quad y_p = \frac{1}{2} 2^x$ //

11

$\theta''(t) - \theta(t) = t \operatorname{sen} t$

$y''(x) - y(x) = x \operatorname{sen} x$

$y_p = (Ax+B)\cos x + (Cx+D)\operatorname{sen} x$

$y'_p = (A\cos x + (Ax+B)(-\operatorname{sen} x)) + (C\operatorname{sen} x + (Cx+D)\cos x$

$y''_p = -\operatorname{sen} x + A(-\operatorname{sen} x) + (Ax+D) - \cos x + C\cos x +$
$\cos x + (Cx+D)\cos x$

$y''_p = -\operatorname{sen} x(1+A) + -\cos x(Ax+B + \cos x(2C + Cx + D)$

$1 + A = 0 \quad Ax + B = 0 \quad 2C + Cx + D = 0$

$\boxed{y_p = 0}$

$y''' - 3y'' + 4y = e^{2t}$

$y''' - 3y'' + 4y = 0$

$r^3 - 3r^2 + 4 = 0$

$(r-2)(r^2 - r - 2) = 0$

$r = 2, \frac{1}{2} \pm \frac{\sqrt{7}}{2}$

$y_c = C_1 e^{2t} + C_2 e^{\frac{1}{2}t} \cos\left(\frac{\sqrt{7}}{2}t\right) + C_3 e^{\frac{1}{2}t} \operatorname{sen}\left(\frac{\sqrt{7}}{2}\right)t$

$y_p = Ae^{2t}$

$y'_p = 2Ae^{2t}$

$y''_p = 4Ae^{2t}$

$y''' = 8Ae^{2t}$

$8Ae^{2t} - 3(4Ae^{2t}) + 4(Ae^{2t}) = e^{2t}$

$8Ae^{2t} - 12Ae^{2t} + 4Ae^{2t} = e^{2t}$

$0 = e^{2t}$

$y_G = C_1 e^{2t} + C_2 e^{\frac{1}{2}t} \cos\left(\frac{\sqrt{7}}{2}t\right) + C_3 e^{\frac{1}{2}t} \operatorname{sen}\left(\frac{\sqrt{7}}{2}t\right) + \frac{1}{16}$

te^{2t} //

$y'' + y = \tan^2 t$

$y'' + y = 0$

$r^2 + 1 = 0$

$r = \pm i$

$y_c = c_1 \cos t + c_2 \sin t$

$y_p(t)\ A \tan^2 t$

$y_p = A \tan^2 t$

$y'_p = 2A \tan t \sec^2 t$

$y''_p = 2A \sec^4 t + 8A \tan^2 t \sec^2 t$

$9A = 1$
$A = 1/9$

$y_p = \frac{1}{9} \tan^2 t$

$y_G = c_1 \cos t + c_2 \sin t + \frac{1}{9} \tan^2 t$

$y'' + y = \sec t$

$y'' + y = 0$

$r^2 + 1 = 0$

$r = \pm i$

$y_c = C_1 \cos t + c_2 \sin t$

$y_p = A \sec t + B \tan t$

$y'_p = A \sec t \tan t + B \sec^2 t$

$y''_p = A \sec t \tan^2 t + 2B \sec t \tan t$

$A = 0 \qquad 2B = 1 \qquad B = \frac{1}{2}$

$y_p = \frac{1}{2} \tan t$

$y(t) = C_1 \cos t + c_2 \operatorname{sen} t + \frac{1}{2} \tan t$ ///

$y'' - y = 2t + 4$

$y'' + y = 0$

$r^2 - 1 = 0$

$(r-1)(r+1) = 0$

$r = 1 \quad r = -1$

$y_h = C_1 e^{+x} + C_2 e^{-+x}$

$y_p = At + B$

$y'_p = A$

$y''_p = 0$

$0 - (At + B) = 2t + 4$

$-A = 2 \quad -B = 4$

$A = -2 \quad B = -4$

1. $y_G = C_1 e^{1x} + C_2 e^{-1x} - 2t - 4.$

$y'' + 2y' + 2y = 0$ $\quad \frac{-2\sqrt{4-4(2)}}{2}$ $\quad \frac{-2 \pm 2i}{2}$ $\quad m_1 = -1$

$m^2 + 2m + 2 = 0$ $\quad m_2 = 1$

$y_h = C_1 \cos -x + C_2 \operatorname{Sen} x$

$y'' + 2y' + 17y = 0$ $\quad -2\sqrt{4-4(17)} \quad -7 \quad \frac{-2 \pm 8i}{2}$ $\quad m_1 = -4$

$m^2 + 2m + 17y$ $\quad m_2 = 4$

$y_h = C_1 \cos -4x + C_2 \operatorname{Sen} 4x$

$y'' - 9y = 0$

$m^2 - 9 = 0$ $\quad y_h = C_1 e^{3x} + C_2 e^{-3x}$

$(m-3)(m+3)$ $\quad m_1 = 3 \quad m_2 = -3$

$y'' - 4y' + 2y = 0$

$m^2 - 4m + 2$ $\quad (m-2)(m-2) = 0$

$m_1 = m_2 = 2$

$y_h = C_1 e^{2x} + C_2 e^{2x}$

$y'' - 2y' + 2y = 0$ $\quad \frac{+2 \pm 2i}{2}$ $\quad m_1 = 1$

$m^2 - 2m + 2 = 0$ $\quad m_2 = -1$

$y_h = C_1 \cos x + C_2 \operatorname{sen} -x$

$y'' - 2y' + y = 0$
$m^2 - 2m + 1 = 0$

$(m-1)(m-1)$ $m_1 = m_2 = 1$

$y_h = C_1 e^x + C_2 e^x$

$y''' - 4y'' + 7y' - 6y = 0$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & +7 & -6 \\ & & 2 & -4 & 6 \\ \hline & 1 & 2 & 3 & 0 \end{array}$$

$m^2 - 2m + 3$

$\frac{\pm\sqrt{8i}}{2}$

$y_h = C_1 \cos \frac{\sqrt{8i}}{2} x + C_2 \sin -\frac{\sqrt{8i}}{2} x$

1: $y'' + 4y = \tan 2t$

$m^2 + 4 = \phi$

$m^2 = -4$

$m = \sqrt{-4}$

$m_{1,2} = \pm 2i$

$y_h = c_1 \cos(2x) + c_2 \operatorname{sen}(2x)$

$$w = \begin{vmatrix} \cos(2x) & \operatorname{sen}(2x) \\ 2\operatorname{sen}(2x) & -2\cos(2x) \end{vmatrix} = -2\cos(2x)^2 - 2\operatorname{sen}(2x)^2$$

$2\cos(2x)^2 + 2\operatorname{sen}(2x)^2$

$2(\underbrace{\cos(2x^2) + \operatorname{sen}(2x)^2}_{1})$

$2(1) = |2|$

$$y_1 = -\int \frac{\overset{y_2}{\operatorname{sen}(2x)} \overset{f(x)}{\tan 2x}}{w=2}\,dx$$

$$U_1 = -1/2 \int \frac{\operatorname{sen}(2x)\operatorname{sen}(2x)}{\cos(2x)}$$

$$-\frac{1}{2}\int \frac{\operatorname{sen}^2(2x)}{\cos(2x)}$$

$$\frac{1}{2}\int \frac{(1-\cos^2(2x))}{\cos(2x)}$$

$$-\frac{1}{2}\int \frac{1}{\cos 2x} - \frac{\cos^2 2x}{\cos 2x}$$

$$-\frac{1}{2}\left(\int \sec 2x - \int \cos 2x\right)$$

$$-\frac{1}{2}\left(\frac{1}{2}\int \sec u - \frac{1}{2}\int \cos u\right)$$

$$-\frac{1}{2}\left(\frac{\ln[\sec 2x + \tan 2x]}{2} - \frac{\operatorname{sen} 2x}{2}\right)$$

$$-\frac{\ln[\sec 2x + \tan 2x]}{4} - \frac{\operatorname{sen} 2x}{4}$$

$$U_2 = \int \frac{\cos(2x)\tan 2x}{2} = \frac{1}{2}\int \operatorname{sen} 2x\,dx$$

$$U_2 = \frac{1}{2} \cdot -\frac{\cos 2x}{2}$$

$$y_g = c_1\cos(2x) + c_2\operatorname{sen}(2x) - \frac{1}{4}\ln[\sec 2x + \tan 2x] - \frac{\operatorname{sen} 2x}{4} - \frac{\cos 2x}{4}$$

$2 - y'' + y = \sec t$

$y_h = C_1 \cos x + C_2 \operatorname{sen} x$

$m^2 + 1 = \emptyset$

$m^2 = -1$

$m = \pm\sqrt{1}\, i$

$m = i$

$$W = \begin{vmatrix} \cos x & \operatorname{sen} x \\ -\operatorname{sen} x & \cos x \end{vmatrix} = \cos^2 x + \operatorname{sen}^2 x \qquad W = 1$$

$$U_1 = -\int \operatorname{sen} x \cdot \sec x = -\int \frac{\operatorname{sen} x}{\cos x} = \ln(\cos x)$$

$$U_2 = \int \cos x \sec x = \int \frac{\cos x}{\cos x} dx = \int dx = x$$

$y_p = \ln(\cos x)\cos x + x \operatorname{sen} x$

$$y = C_1 \cos x + C_2 \operatorname{sen} x + \ln(\cos x)\cos x + x \operatorname{sen} x$$

3- $2x'' - 2x' - 4x = 2e^{3x}$

$2m^2 - 2m - 4 = \emptyset$ $\qquad y_h = C_1 e^{2x} + C_2 e^{-x}$

$m_1 = 2$

$m_2 = -1$

$$W = \begin{vmatrix} e^{2x} & e^{-x} \\ 2e^{2x} & -e^{-x} \end{vmatrix} = -e^{x} - 2e^{x} = 3e^{x}$$

$$U_1 = -\int \frac{e^{-x}\, 2e^{3x}}{3e^{x}} \Rightarrow -\frac{3}{2}\int \frac{e^{2x}}{e^{x}} dx \Rightarrow -\frac{3}{2} e^{x}$$

$$U_2 = \int \frac{e^{2x}\, 2e^{3x}}{3e^{x}} \Rightarrow \frac{2}{3}\int \frac{e^{5x}}{e^{x}} dx \Rightarrow \int e^{4x} dx = \frac{2e^{4x}}{12}$$

$$y_g = -\frac{3}{2} e^{3x} + \frac{e^{5x}}{5} + C_1 e^{2x} + C_2 e^{-x}$$

4- $y'' - y = 2x + 4$

$y_h = C_1 e^{x} + C_2 e^{-x}$

$m^2 - 1 = \emptyset \quad m = \pm\sqrt{1}$

$m_1 = 1 \quad m_2 = -1$

$$W = \begin{vmatrix} e^{x} & e^{-x} \\ e^{x} & -e^{-x} \end{vmatrix} = -1 - 1 = |-2| \qquad W = 2$$

$$U_1 = -\int \frac{e^{-x}\, 2x + 4}{2} = \int \frac{2xe^{-x}}{2} + \int \frac{4e^{-x}}{2} = \int xe^{-x} d + 2\int e^{-x} dx$$

$u = x \quad du = dx$

$v = e^{-x} \quad dv = e^{-x} dx$

$= xe^{-x} + e^{-x}$

$$U_2 = \int \frac{e^{x}\, 2x + 4}{2} = \int xe^{x} + 2\int e^{x} = xe^{x} - e^{x}$$

$u = x \quad du = dx$

$v = e^{x} \quad dv = e^{x} dx$

$y_p = x + 1 + 1 = 2x + 2$

$$y = C_1 e^{x} + C_2 e^{-x} + 2x + 2$$

5.- $y'' - 2y' + y = x^{-1} e^{x}$ $\quad y_h = c_1 e^{x} + c_2 x e^{x}$

$m^2 - 2m + 1 = 0$

$m_1 = 1$ $\quad W = \begin{vmatrix} e^{x} & xe^{x} \\ e^{x} & e^{x} + xe^{x} \end{vmatrix} = e^{2x}$

$$U_1 = -\int \frac{xe^{x}\, x^{-1} e^{x}}{e^{2x}}\, dx = \int \frac{e^{2x}}{e^{2x}}\, dx = \int dx = x$$

$$U_2 = \int \frac{e^{x}\, x^{-1} e^{x}}{e^{2x}}\, dx = \int x^{-1}\, dx = \ln |x|$$

$y = c_1 e^{x} + c_2 x e^{x} + x e^{x} + \ln x$

6.- $y'' + 9y = \sec^2(3x)$ $\quad y_h = c_1 \cos 3x + c_2 \operatorname{sen} 3x$

$m^2 + 9 = 0 \quad m^2 = -9$

$m = \pm 3i$ $\quad W = \begin{vmatrix} \cos 3x & \operatorname{sen} 3x \\ -3\operatorname{sen} 3x & 3\cos 3x \end{vmatrix} = 3\cos^2 3x + 3\operatorname{sen}^2 3x = 3$

$$U_1 = -\int \frac{\operatorname{sen} 3x \sec^2(3x)}{3} = \int \frac{\operatorname{sen} 3x}{\cos^2 3x}\, dx \qquad u = \cos 3x \quad du = -3\operatorname{sen} 3x$$

$$= -1/3 \int u^{-2}\, du = \frac{1}{3\cos(3x)}$$

$$U_2 = \int \cos 3x \sec^2 dx = \frac{\cos 3x}{\cos 3x} = \int \frac{1}{\cos 3x}\, dx$$

$$\int \sec(3x)\, dx = 1/3 \ln \tan\left(\frac{3x}{2} + \frac{\pi}{4}\right)$$

$$y = c_1 \cos 3x + c_2 \operatorname{sen} 3x + \frac{\cos 3x}{3\cos 3x} + \frac{1}{3} \ln \tan\left(\frac{3x}{2} + \frac{\pi}{4}\right)$$

7.- $y'' + 4y = \csc^2(2x)$ $\quad y_h = c_1 \cos(2x) + c_2 \operatorname{sen}(2x)$

$m^2 + 4 = 0$

$m = \pm 2i$ $\quad W = \begin{vmatrix} \cos 2x & \operatorname{sen} 2x \\ -2\operatorname{sen} 2x & 2\cos 2x \end{vmatrix} = 2\cos^2 + 2\operatorname{sen}^2 = 2$

$$U_1 = -\int \frac{\operatorname{sen} 2x \csc^2 2x}{2} = \frac{1}{2} \int \csc 2x\, dx = \frac{-1}{4} \ln [\tan x]$$

$$U_2 = \int \frac{\cos 2x \csc^2 2x}{2} = \frac{1}{2} \int \frac{\cos 2x}{\operatorname{sen}^2 2x} = -\frac{\csc 2x}{2}$$

$$y = c_1 \cos(2x) + c_2 \operatorname{sen}(2x) - \frac{1}{4} \ln(\tan x) \cos 2x - \frac{\csc 2x}{2}$$

8.- $y'' + y = \tan^2 x$ $\quad y_h = c_1 \cos x + c_2 \operatorname{sen} x$

$m^2 + 1 = 0$

$m = \pm i$ $\quad W = 1$

$U_1 = -\int \cos x \tan^2 x\, dx = -\ln(\tan x + \sec x) + \sin x + C$

$U_2 = \int \operatorname{sen} x \tan^2 x\, dx = \sec x + \cos x$ $\quad y_p = U_1 + U_2$

$Y = y_h + y_p$

9.- $y'' + 4y = \sec^4(2x)$ $\quad y_h = c_1\cos 2x + c_2 \operatorname{sen} 2x$

$m^2 + 4$ $\quad w = 2$

$m = \pm 2i$

$U_1 = -\int \frac{\operatorname{sen}2x \sec^4 2x}{2} dx = \frac{1}{12\cos^3(2x)}$

$U_2 = \int \frac{\cos 2x \sec^4 2x}{2} dx = \frac{\ln(\tan(2x) + \sec(2x)) + \sec 2x \tan 2x}{8}$

$y = c_1\cos 2x + c_2\operatorname{sen}2x + \frac{1}{12\cos^3(2x)} + \frac{\ln(\tan(2x) + \sec(2x)) + \sec 2x \tan^2 x}{8}$

10.- $y'' + y = \sec x$ $\quad y_h = c_1\cos x + c_2 \operatorname{sen} x$

$m^2 + 1 = \emptyset$ $\quad w = 1$

$m = \pm i$

$U_1 = -\int \operatorname{sen} x \sec x = \int \frac{\operatorname{sen} x}{\cos x} dx \rightarrow \int \tan x = -\ln \operatorname{sen} x$

$U_2 = \int \cos x \sec x\, dx = \int dx = x$

11.- $y'' + y = \operatorname{sen} x$ $\quad y_h = c_1\cos x + c_2\operatorname{sen} x$

$m^2 + 1 = \emptyset$ $\quad w = 1$

$m = \pm i$

$U_1 = -\int \operatorname{sen} x \operatorname{sen} x = \int \operatorname{sen}^2 x = -\frac{\operatorname{sen}(2x) - 2x}{4}$

$U_2 = \int \cos x \operatorname{sen} x = \frac{\operatorname{sen}^2 x}{2}$

12.- $y'' + y = \tan x$ $\quad y_h = c_1\cos x + c_2\operatorname{sen} x$

$m^2 + 1 = \emptyset$ $\quad w = 1$

$m = \pm i$

$U_1 = -\int \operatorname{sen} x \tan x\, dx = \int \frac{\operatorname{sen}^2 x}{\cos x} = \ln\left(\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right)$

$U_2 = \int \frac{\cos x}{\cos x} \operatorname{sen} x = \int \operatorname{sen} x\, dx = -\cos x$

13.- $y'' + y = \cos x$ $\quad y_h = c_1\cos x + c_2\operatorname{sen} x$

$m^2 + 1$ $\quad w = 1$

$m = \pm i$

$U_1 = -\int \operatorname{sen} x \cos x\, dx = \frac{-\operatorname{sen}^2 x}{2}$ $\quad y_p = U_1 + U_2$

$U_2 = \int \cos^2 x\, dx = 1/2 + \frac{\operatorname{sen}(2x)}{4}$

$y = c_1\cos x + c_2\operatorname{sen} x - \frac{\operatorname{sen}^2 x}{2} + \frac{\operatorname{sen}(2x)}{2}$

14.- $y'' - y' - 12y = \emptyset$

$m^2 - m - 12 = \emptyset$

$(m-4)(m+3) = \emptyset$ $\quad y_h = c_1 e^{4x} + c_2 e^{-3x}$

$m_1 = 4 \quad m_2 = -3$

15.- $y''' - 4y = \emptyset$

$m^2 - 4 = \emptyset$

$(m-2)(m+2) = \emptyset$ $\quad y_h = c_1 e^{2x} + c_2 e^{-2x}$

$m_1 = 2 \quad m_2 = -2$

16.- $y'' - 2y' + 5y = \emptyset$

$m^2 - 2m + 5 = \emptyset$ $\quad \dfrac{2 \pm \sqrt{4 - 4 \cdot 5}}{2} = \dfrac{\pm 4i}{2} = 2i$

$a = 1 \quad b = -2 \quad c = 5$

$y_h = c_1 \cos 2x + c_2 \operatorname{sen} 2x$

17.- $4y'' - 4y' + y = \emptyset$

$4m^2 - 4m + 1 = \emptyset$

$(2m-1)(2m-1) = \emptyset$ $\quad y_h = c_1 e^{1/2x} + c_2 e^{1/2x}$

$m_{1,2} = 1/2$

18.- $y'' + 2y' - y = 10$

$m^2 + 2m - 1 = \emptyset$ $\quad y = A$

$m_1 = -1 + \sqrt{2} \quad m_2 = -1 - \sqrt{2}$ $\quad y' = \emptyset$ $\quad 0 + 2(\emptyset) - A = 10 \quad A = -10$

$y'' = \emptyset$

$y_p = -10$ $\quad y = c_1 e^{(-1+\sqrt{2})} + c_2 e^{(-1-\sqrt{2})x} - 10$

19.- $2y'' + y = 9e^{2x}$

$2m^2 + 1 = \emptyset$ $\quad y_h = c_1 e^{\sqrt{-1/2}x} + c_2 e^{\sqrt{-1/2}x}$

$m = \pm\sqrt{-1/2}$

$y = 9e^{2x} = Ae^{2x}$ $\quad 2(2A) + Ae^{2x} = 9e^{2x}$

$y' = 2A$ $\quad A(4 + e^{2x}) = 9e^{2x}$

$A = \dfrac{9e^{2x}}{4 + e^{2x}}$

$y = c_1 e^{\sqrt{-1/2}x} + c_2 e^{-\sqrt{-1/2}x} + \dfrac{9e^{2x}}{4 + e^{2x}}$

21.- $y'' - y' + 9y = 3\,\text{sen}\,3x$

$m^2 - m + 9 = \emptyset$

$m_1 = \dfrac{-1 + \sqrt{35}\,i}{2} \quad m_2 = \dfrac{-1 - \sqrt{35}\,i}{2}$

$y_h = C_1 \cos\left(\dfrac{-1 + \sqrt{35}\,i}{2}\right)x + C_2\,\text{sen}\left(\dfrac{-1 - \sqrt{35}\,i}{2}\right)$

$3\,\text{sen}\,3x = A\,\text{sen}(3x) + B\cos(3x)$

$y' = 3A\cos(3x) - 3B\,\text{sen}(3x)$

$y'' = -9A\,\text{sen}(3x) - 9B\cos(3x)$

$\cancel{-3A\,\text{sen}(3x)} - \cancel{3B\cos(3x)} - A\cos(3x) + B\,\text{sen}\,3x + \cancel{3A\,\text{sen}(3x)}$
$+ \cancel{3B\cos(3x)} = \text{sen}\,3x$

$-A\cos(3x) + B\,\text{sen}\,3x = \text{sen}\,3x$

$-A\cos(3x) = \text{sen}\,3x - B\,\text{sen}\,3x = \text{sen}\,3x(1 + B)$

$\therefore \dfrac{-A}{\tan 3x} = 1 + B$

$-A\cos(3x) + A\dfrac{\cos^2 3x}{\text{sen}\,3x} - \cos 3x = \text{sen}\,3x + \cos 3x$

$A\left(-\cos 3x + \dfrac{\cos^2 3x}{\text{sen}\,3x}\right) = \left(\dfrac{\text{sen}\,3x + \cos 3x}{-\cos 3x + \dfrac{\cos^2 3x}{\text{sen}\,3x}}\right) = A$

$y_g = C_1 \cos \dfrac{-1 + \sqrt{35}\,i}{2}x + C_2\,\text{sen}\dfrac{-1 - \sqrt{35}\,i}{2} + \dfrac{\text{sen}\,3x + \cos x}{-\cos 3x\ \dfrac{\cos^2 3x}{\text{sen}\,3x}}$

$+ \dfrac{\dfrac{\text{sen}\,3x + \cos 3x}{-\cos 3x\ \dfrac{\cos^2 3x}{\text{sen}\,3x}} - 1}{\tan 3x}$