

# Title

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### 1 Question 15

Let  $p$  be a prime number. Let  $A$  be a  $p \times p$  matrix over a field  $F$  with 1 in all entries except 0 on the main diagonal.

Determine the Jordan canonical form (JCF) of  $A$

- (a) When  $F = \mathbb{Q}$ ,
- (b) When  $F = \mathbb{F}_p$ .

Hint: In both cases, all eigenvalues lie in the ground field. In each case find a matrix  $P$  such that  $P^{-1}AP$  is in JCF.

### 2 Question 16

Let  $\zeta = e^{2\pi i/8}$ .

- (a) What is the degree of  $\mathbb{Q}(\zeta)/\mathbb{Q}$ ?
- (b) How many quadratic subfields of  $\mathbb{Q}(\zeta)$  are there?
- (c) What is the degree of  $\mathbb{Q}(\zeta, \sqrt[4]{2})$  over  $\mathbb{Q}$ ?