

A Five-Factor Asset Pricing Model

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[Journal of Financial Economics 116 \(2015\) 1–22](#)

Introduction

$$\bullet \frac{M_t}{B_t} = \frac{\sum_{\tau} E(Y_{t+\tau} - dB_{t+\tau}) / (1+r)^{\tau}}{B_t}$$

- M_t : market cap at time t
 - B_t : book equity at time t
 - $Y_{t+\tau}$: total equity earnings for period $t + \tau$
 - $dB_{t+\tau}$: change in total book equity
 - r : long-term average expected stock return
- 只看分子 (股票市值) 的話，一家公司的現在價值等於所有的未來收益減去投資的折現值
 - 賺越多錢 & 投資越少 → 公司的現值越大

Definitions of Variables

Samples	從CRSP當中，選用在NYSE、AMEX、NASDAQ上市的股票 (with SHRCD 10 or 11 i.e., ordinary common shares)
	1963/07 – 2013/12, 606 months
	NYSE breakpoints，每年六月重新計算一次
Size	Market Capitalization
B/M	Book-to-Market ratio
OP	<u>O</u> perating <u>p</u> rofitability
	= $\text{revenue}_{t-1} - \text{COGS}_{t-1} - \text{interest}_{t-1} - \text{other operating expenses}_{t-1}$ (以上數字都有除以該公司的book equity進行標準化)
Inv	<u>I</u> nvestment
	= $(\text{Assets}_{t-1} - \text{Assets}_{t-2}) \div \text{Assets}_{t-1}$ (i.e., growth rate of total assets)

Explanation of Effects







Size	市值越大的股票，平均報酬率越低
Value	市值固定的情況下，B/M越大(i.e., book equity比較接近market equity)，平均報酬率越高
Profitability	賺錢能力越健全的股票，平均報酬率越高
Investment	投資力度越強的股票，平均報酬率越低

因為[Novy-Marx\(2013\)](#) & [Titman, Wei, and Xie\(2004\)](#)分別指出了三因子的不足之處，所以產生五因子模型。

Explanation of Effects (cont.)

Table 1

Average monthly percent excess returns for portfolios formed on *Size* and *B/M*, *Size* and *OP*, *Size* and *Inv*; July 1963–December 2013, 606 months.

	Low	2	3	4	High
<i>Panel A: Size-B/M portfolios</i>					
					
Small	0.26	0.81	0.85	1.01	1.15
2	0.48	0.72	0.94	0.94	1.02
3	0.50	0.78	0.79	0.88	1.07
4	0.60	0.57	0.71	0.85	0.86
Big	0.46	0.51	0.48	0.56	0.62
					
<i>Panel B: Size-OP portfolios</i>					
					
Small	0.56	0.94	0.90	0.95	0.88
2	0.59	0.78	0.84	0.81	0.98
3	0.53	0.77	0.72	0.78	0.94
4	0.57	0.65	0.63	0.70	0.82
Big	0.39	0.33	0.43	0.47	0.57
					
<i>Panel C: Size-Inv portfolios</i>					
					
Small	1.01	0.98	0.99	0.89	0.35
2	0.92	0.91	0.92	0.90	0.48
3	0.90	0.93	0.81	0.82	0.50
4	0.79	0.72	0.71	0.75	0.54
Big	0.71	0.52	0.49	0.48	0.42
					

Size Effect

Value Effect

Profitability Effect

Investment Effect

Concepts of Five Factors

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it}$$

$R_M - R_f$	return on value-weight market portfolio
SMB	(return on small stocks) – (return on big stocks)
HML	(return on high B/M stocks) – (return on low B/M stocks)
RMW	(return on high profitability stocks) – (return on low profit. stocks)
	high profitability = robust low profitability = weak
CMA	(return on low investment stocks) – (return on high invest. stocks)
	low investment = conservative high investment = aggressive

Concepts of Five Factors (cont.)

Table 3

Construction of *Size*, *B/M*, profitability, and investment factors.

We use independent sorts to assign stocks to two *Size* groups, and two or three *B/M*, operating profitability (*OP*), and investment (*Inv*) groups. The VW portfolios defined by the intersections of the groups are the building blocks for the factors. We label these portfolios with two or four letters. The first always describes the *Size* group, small (*S*) or big (*B*). In the 2×3 sorts and 2×2 sorts, the second describes the *B/M* group, high (*H*), neutral (*N*), or low (*L*), the *OP* group, robust (*R*), neutral (*N*), or weak (*W*), or the *Inv* group, conservative (*C*), neutral (*N*), or aggressive (*A*). In the $2 \times 2 \times 2 \times 2$ sorts, the second character is *B/M* group, the third is *OP* group, and the fourth is *Inv* group. The factors are *SMB* (small minus big), *HML* (high minus low *B/M*), *RMW* (robust minus weak *OP*), and *CMA* (conservative minus aggressive *Inv*).

Sort	Breakpoints	Factors and their components
2×3 sorts on <i>Size</i> and <i>B/M</i> , or <i>Size</i> and <i>OP</i> , or <i>Size</i> and <i>Inv</i>	<i>Size</i> : NYSE median <i>B/M</i> : 30th and 70th NYSE percentiles <i>OP</i> : 30th and 70th NYSE percentiles <i>Inv</i> : 30th and 70th NYSE percentiles	$SMB_{B/M} = (SH + \boxed{SN} + SL)/3 - (BH + BN + BL)/3$ $SMB_{OP} = (SR + SN + SW)/3 - (BR + BN + BW)/3$ $SMB_{Inv} = (SC + SN + SA)/3 - (BC + BN + BA)/3$ $SMB = (SMB_{B/M} + SMB_{OP} + SMB_{Inv})/3$ $HML = (SH + BH)/2 - (SL + BL)/2 = [(SH - SL) + (BH - BL)]/2$ $RMW = (SR + BR)/2 - (SW + BW)/2 = [(SR - SW) + (BR - BW)]/2$ $CMA = (SC + BC)/2 - (SA + BA)/2 = [(SC - SA) + (BC - BA)]/2$
2×2 sorts on <i>Size</i> and <i>B/M</i> , or <i>Size</i> and <i>OP</i> , or <i>Size</i> and <i>Inv</i>	<i>Size</i> : NYSE median <i>B/M</i> : NYSE median <i>OP</i> : NYSE median <i>Inv</i> : NYSE median	$SMB = (SH + SL + SR + SW + SC + SA)/6 - (BH + BL + BR + BW + BC + BA)/6$ $HML = (SH + BH)/2 - (SL + BL)/2 = [(SH - SL) + (BH - BL)]/2$ $RMW = (SR + BR)/2 - (SW + BW)/2 = [(SR - SW) + (BR - BW)]/2$ $CMA = (SC + BC)/2 - (SA + BA)/2 = [(SC - SA) + (BC - BA)]/2$
$2 \times 2 \times 2 \times 2$ sorts on <i>Size</i> , <i>B/M</i> , <i>OP</i> , and <i>Inv</i>	<i>Size</i> : NYSE median <i>B/M</i> : NYSE median <i>OP</i> : NYSE median <i>Inv</i> : NYSE median	$SMB = (SHRC + \boxed{SHRA} + SHWC + SHWA + SLRC + SLRA + SLWC + SLWA)/8$ $- (BHRC + BHRA + BHCW + BHWA + BLRC + BLRA + BLWC + BLWA)/8$ $HML = (SHRC + SHRA + SHWC + SHWA + BHRC + BHRA + BHCW + BHWA)/8$ $- (SLRC + SLRA + SLWC + SLWA + BLRC + BLRA + BLWC + BLWA)/8$ $RMW = (SHRC + SHRA + SLRC + SLRA + BHRC + BHRA + BLRC + BLRA)/8$ $- (SHWC + SHWA + SLWC + SLWA + BHCW + BHWA + BLWC + BLWA)/8$ $CMA = (SHRC + SHWC + SLRC + SLWC + BHRC + BHCW + BLRC + BLWC)/8$ $- (SHRA + SHWA + SLRA + SLWA + BHRA + BHWA + BLRA + BLWA)/8$

HML: a redundant factor

Table 6

Using four factors in regressions to explain average returns on the fifth: July 1963–December 2013, 606 months.

	<i>Int</i>	$R_M - R_F$	<i>SMB</i>	<i>HML</i>	<i>RMW</i>	<i>CMA</i>	R^2
<i>2 × 3 Factors</i>							
<i>R_M - R_F</i>							
Coef	0.82		0.25	0.03	-0.40	-0.91	0.24
<i>t</i> -Statistic	4.94		4.44	0.38	-4.84	-7.83	
<i>SMB</i>							
Coef	0.39	0.13		0.05	-0.48	-0.17	0.17
<i>t</i> -Statistic	3.23	4.44		0.81	-8.43	-1.92	
<i>HML</i>							
Coef	-0.04	0.01	0.02		0.23	1.04	0.51
<i>t</i> -Statistic	-0.47	0.38	0.81		5.36	23.03	
<i>RMW</i>							
Coef	0.43	-0.09	-0.22	0.20		-0.44	0.21
<i>t</i> -Statistic	5.45	-4.84	-8.43	5.36		-7.84	
<i>CMA</i>							
Coef	0.28	-0.10	-0.04	0.45	-0.21		0.57
<i>t</i> -Statistic	5.03	-7.83	-1.92	23.03	-7.84		

HML: a redundant factor (cont.)

- 改進的方法：將HML改成HMLO
- HMLO
 - Orthogonal HML
 - Sum of the intercept and residual from the regression of HML on $R_M - R_F$, SMB, RMW, and CMA
- 回歸式的 a_i 和 e_{it} 保持不變

$$R_{it} - R_{Ft} = \mathbf{a}_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_i\textcolor{red}{HML}_t + r_iRMW_t + c_iCMA_t + \mathbf{e}_{it}$$



$$R_{it} - R_{Ft} = \mathbf{a}_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_i\textcolor{red}{HMLO}_t + r_iRMW_t + c_iCMA_t + \mathbf{e}_{it}$$

Performance Comparisons

Table 5
Summary statistics for tests of three-, four-, and five-factor models; July 1963–December 2013, 606 months.

	2 × 3 Factors				2 × 2 Factors				2 × 2 × 2 × 2 Factors			
	GRS	$A a_i $	$\frac{A a_i }{A \bar{r}_i }$	$\frac{A(\hat{\alpha}_i^2)}{A(\hat{\mu}_i^2)}$	GRS	$A a_i $	$\frac{A a_i }{A \bar{r}_i }$	$\frac{A(\hat{\alpha}_i^2)}{A(\hat{\mu}_i^2)}$	GRS	$A a_i $	$\frac{A a_i }{A \bar{r}_i }$	$\frac{A(\hat{\alpha}_i^2)}{A(\hat{\mu}_i^2)}$
<i>Panel A: 25 Size-B/M portfolios</i>												
HML	3.62	0.102	0.54	0.38	3.54	0.101	0.53	0.36	3.40	0.096	0.51	0.36
HML RMW	3.13	0.095	0.50	0.24	3.11	0.096	0.51	0.26	3.29	0.089	0.47	0.24
HML CMA	3.52	0.101	0.53	0.39	3.46	0.100	0.53	0.37	3.18	0.096	0.51	0.35
RMW CMA	2.84	0.100	0.53	0.22	2.78	0.093	0.49	0.19	2.78	0.087	0.46	0.13
HML RMW CMA	2.84	0.094	0.50	0.23	2.80	0.093	0.49	0.23	2.82	0.088	0.46	0.18
<i>Panel B: 25 Size-OP portfolios</i>												
HML	2.31	0.108	0.68	0.51	2.31	0.109	0.68	0.51	1.91	0.089	0.56	0.37
RMW	1.71	0.067	0.42	0.12	1.82	0.078	0.49	0.16	1.73	0.059	0.37	0.05
HML RMW	1.64	0.062	0.39	0.16	1.74	0.058	0.36	0.03	1.62	0.064	0.40	0.06
HML CMA	3.02	0.137	0.86	0.90	2.85	0.135	0.85	0.86	2.06	0.102	0.64	0.49
RMW CMA	1.87	0.075	0.47	0.12	1.67	0.066	0.42	0.05	1.61	0.068	0.43	0.05
HML RMW CMA	1.87	0.073	0.46	0.12	1.73	0.066	0.42	0.06	1.60	0.069	0.43	0.07
<i>Panel C: 25 Size-Inv portfolios</i>												
HML	4.56	0.112	0.64	0.57	4.40	0.107	0.61	0.53	4.32	0.100	0.57	0.56
CMA	4.03	0.105	0.60	0.47	4.05	0.106	0.61	0.47	4.23	0.123	0.70	0.62
HML RMW	4.40	0.106	0.61	0.57	4.26	0.103	0.59	0.52	4.45	0.116	0.66	0.66
HML CMA	4.00	0.099	0.57	0.43	3.97	0.098	0.56	0.41	3.70	0.084	0.48	0.35
RMW CMA	3.33	0.085	0.49	0.29	3.28	0.082	0.47	0.26	3.50	0.082	0.47	0.27
HML RMW CMA	3.32	0.085	0.49	0.29	3.27	0.082	0.47	0.27	3.59	0.082	0.47	0.28

皆省略了 $R_M - R_F$ 、SMB兩個因子

Performance Comparisons (cont.)

Table 7

Regressions for 25 value-weight *Size-B/M* portfolios; July 1963 to December 2013, 606 months.

$$R(t) - R_F(t) = a + b[R_M(t) - R_F(t)] + sSMB(t) + hHML(t) + rRMW(t) + cCMA(t) + e(t).$$

<i>B/M</i> →	Low	2	3	4	High	Low	2	3	4	High
<i>Panel A: Three-factor intercepts: $R_M - R_F$, <i>SMB</i>, and <i>HML</i></i>										
	<i>a</i>					<i>t(a)</i>				
Small	-0.49	0.00	0.02	0.16	0.14	-5.18	0.07	0.40	2.88	2.37
2	-0.17	-0.04	0.12	0.07	-0.02	-2.75	-0.80	2.24	1.40	-0.38
3	-0.06	0.06	0.02	0.06	0.12	-0.98	0.92	0.33	0.96	1.66
4	0.14	-0.10	-0.04	0.07	-0.08	2.24	-1.46	-0.55	1.05	-0.94
Big	0.17	0.02	-0.07	-0.11	-0.18	3.53	0.40	-0.95	-1.86	-1.92
<i>Panel B: Five-factor coefficients: $R_M - R_F$, <i>SMB</i>, <i>HML</i>, <i>RMW</i>, and <i>CMA</i></i>										
	<i>a</i>					<i>t(a)</i>				
Small	-0.29	0.11	0.01	0.12	0.12	-3.31	1.61	0.17	2.12	1.99
2	-0.11	-0.10	0.05	-0.00	-0.04	-1.73	-1.88	0.95	-0.04	-0.64
3	0.02	-0.01	-0.07	-0.02	0.05	0.40	-0.10	-1.06	-0.25	0.60
4	0.18	-0.23	-0.13	0.05	-0.09	2.73	-3.29	-1.81	0.73	-1.09
Big	0.12	-0.11	-0.10	-0.15	-0.09	2.50	-1.82	-1.39	-2.33	-0.93
	<i>h</i>					<i>t(h)</i>				
Small	-0.43	-0.14	0.10	0.27	0.52	-10.11	-4.38	3.90	10.12	17.55
2	-0.46	-0.01	0.29	0.43	0.69	-15.22	-0.45	11.77	16.78	24.44
3	-0.43	0.12	0.37	0.52	0.67	-14.70	3.71	12.28	17.07	18.75
4	-0.46	0.09	0.38	0.52	0.80	-15.18	2.76	11.03	15.88	20.26
Big	-0.31	0.03	0.26	0.62	0.85	-14.12	1.09	7.54	21.05	18.74
	<i>r</i>					<i>t(r)</i>				
Small	-0.58	-0.34	0.01	0.11	0.12	-13.26	-10.56	0.31	3.89	3.95
2	-0.21	0.13	0.27	0.26	0.21	-6.75	4.89	10.35	9.86	7.04
3	-0.21	0.22	0.33	0.28	0.33	-6.99	6.77	10.36	8.98	8.88
4	-0.19	0.27	0.28	0.14	0.25	-6.06	7.75	7.99	4.16	6.14
Big	0.13	0.25	0.07	0.23	0.02	5.64	8.79	2.07	7.62	0.49
	<i>c</i>					<i>t(c)</i>				
Small	-0.57	-0.12	0.19	0.39	0.62	-12.27	-3.46	6.59	13.15	19.10
2	-0.59	0.06	0.31	0.55	0.72	-17.76	1.94	11.27	19.39	22.92
3	-0.67	0.13	0.42	0.64	0.78	-20.59	3.64	12.52	18.97	19.62
4	-0.51	0.31	0.51	0.60	0.79	-15.11	8.33	13.35	16.41	18.03
Big	-0.39	0.26	0.41	0.66	0.73	-16.08	8.38	10.80	19.88	14.54

Conclusions

- 五因子模型沒辦法達成截距項等於零的目標 (GRS test failed)
- 報酬率解釋力有比三因子模型來的好 (Table 7 & 9~11)
- 如果想要用最少因子的回歸式，HML可以考慮拿掉，或是改用HMLO去替代HML。