Homework 1

Asset Allocation with Stock Portfolios and Bills

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第一題

1. 已知：

為了簡化和的結果，所以在此定義了幾個矩陣表示法：

，因此；；

特別注意在這裡，。

1. 已知是equally weighted portfolio，因此
2. 按照Homewok 1 spec所使用的符號，我們可以獲得以下推導過程：

第二題

* 已知、：

第三題

* 已知條件如下：
* 為了方便計算，我們額外定義以下兩個符號：
* 再次，為了方便計算，我們定義以下一個符號：
* 因為我們的目標是，所以將可以透過來求之。
* 接著，將代入上式，並且經過一些項次的整理、整併，即可得到以下結果：
* 特別可以注意到：為了讓最後結果具有一致的符號，因此將原本改成。然而，這並不影響結果，受益於共變異數不因平移而改變大小，也就是。

第四題

1. 詳細數值請參閱表格一

Table 1 Descriptive Statistics of Average Equal Weighted Portfolios

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| (Year: 1999~2018) | Average Return | Standard Deviation | Skewness | Kurtosis |
| Small / Growth | 10.4233% | 35.0582% | 0.5229 | 0.1462 |
| Small / Value | 17.3648% | 31.4953% | 0.4508 | 1.1832 |

1. 詳細數值請參閱表格二

Table 2 Covariance Matrix and Correlation Coefficient

|  |  |  |
| --- | --- | --- |
|  | Small / Growth | Small / Value |
| Small / Growth | 1229.0785 | 982.4749 |
| Small / Value | 982.4749 | 991.9553 |
| Correlation Coef. | 0.8898 | |

1. 詳細數值請參閱圖一和表格三

Figure 1 Portfolio Opportunity Set of “Small/Value”and“Small/Growth”Portfolio

Table 3 Detailed Proportions for Minimum-Variance Portfolio of Figure 1

|  |  |  |  |
| --- | --- | --- | --- |
| Minimum-Variance Portfolio | | | |
| Small / Growth % | Small / Value % | Expected Return | Standard Deviation |
| 3.7021% | 96.2979% | 17.1078% | 31.4897% |

1. 詳細數值請參閱圖二和表格四

Figure 2 Portfolio Opportunity Set with Specific Correlation Coefficient

Table 4 Detailed Proportions for Minimum-Variance Portfolio of Figure 2

|  |  |  |  |
| --- | --- | --- | --- |
| Minimum-Variance Portfolio (ρ=-1) | | | |
| Small / Growth % | Small / Value % | Expected Return | Standard Deviation |
| 47.3233% | 52.6767% | 14.0798% | 0% |
| Minimum-Variance Portfolio (ρ=+1)  Table 3 Detailed Proportions for Minimum-Variance Portfolio of Figure 1 | | | |
| Small / Growth % | Small / Value % | Expected Return | Standard Deviation |
| -883.9816% | 983.9816% | 78.7268% | 0% |

1. 詳細數值請參閱(3)的表格三
2. 已知risk-free rate=1.76%，即可推得以下兩個比率：

接著，我們將下列表格的數值代入第三題的公式當中，去求Small/Growth和Small/Value在optimal risky portfolio P之中的占比：

|  |  |  |  |
| --- | --- | --- | --- |
|  | 8.6633% |  | 991.9553 |
|  | 15.6048% |  | 1229.0785 |
|  | | | 982.4749 |

因此，我們可以得知portfolio P具有以下的特性：

|  |  |  |  |
| --- | --- | --- | --- |
| Optimal Risky Portfolio | | | |
| Small / Growth % | Small / Value % | Expected Return | Standard Deviation |
| -171.4300% | 271.4300% | 29.2647% | 41.1550% |

1. 根據定義，CAL的斜率就等於Sharpe ratio，因此帶入前幾個小題所得到的數值就可以得到，如圖三所示。

Figure 3 Capital Allocation Line and Portfolio Opportunity Set

1. 詳細數值請參閱圖四

Figure 4 Indifference Curve and Capital Allocation Line (risk aversion = 4)

特別注意到：根據圖四可知，indifference curve和CAL的交點就是optimal complete portfolio，因此我們可以配合(6)的結果，推算出表格五。

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Optimal Complete Portfolio | | | | |
| Risk-Free % | Small / Growth % | Small / Value % | Expected Return | Standard Deviation |
| 61.3056%  Table 5 Detailed Proportions for Optimal Complete Portfolio of Figure 4 | -66.3388% | 105.0282% | 12.4028% | 16.3116% |

1. 詳細數值請參閱圖五

Figure 5 Indifference Curve and Capital Allocation Line (risk aversion = 1)

按照如同(8)的方法，我們也可以得到以下的表格六。

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Optimal Complete Portfolio | | | | |
| Risk-Free % | Small / Growth % | Small / Value % | Expected Return | Standard Deviation |
| -54.7776%  Table 6 Detailed Proportions for Optimal Complete Portfolio of Figure 5 | -265.3352% | 420.1129% | 44.3311% | 65.2465% |