Astrophysical Probes of Modified Gravity Theory

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Abstract

Review of geodesics of general relativity is undertaken to attain Newtonian approximations. Attempts are made to understand G^3–Galileon theory beyond the Horndeski class so as to how Vainshtein Mechanism is employed to non-relativistic sources. Consequently new scalar potential fields have been derived and are used for astrophysical probes of this theory. Rotation curves of Milky Way galaxy are calculated and predictions are compared to actual GR, similarly strong lensing potentials are quantified and analyzed with GR predictions.
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Introduction

The basic concept of General relativity is the geometrization of space-time. Using the geodesics that govern general relativity one deviates from the general convention of Euclidean geometry to a new design that alters the curvature of space with influence of gravity. We study the general curves of space and derive the necessary general geodesic equation that eventually leads to PPN.

Although the current theory of general relativity has a firm grasp on working of gravity however there seems to be breakdown of gravity at large distances, like galaxy rotation curves. Dark matter and Dark energy are so far the most exclusive reason for the problem, also the motivation lies in the fact that there must be some mechanism to explain the vacuum energy density (cosmological constant). We need to modify gravity in such a way that at long distances the cosmological constant does not vary and ruin the large scale structure [1]. Modified theories of gravity have been proposed as an alternative to Dark energy. These theories are described by Scalar-Tensor gravity, in which an additional degree of freedom is introduced to gravitational waves [2]. One of the many ideas is to have our brane in extra-dimensions and the vacuum energy density provides tension to the brane such that it gives rise to gravitation only in the bulk. The DGP model is useful for this reason as it cannot be associated to fluids and is formally represented by 5-dimensional spacetime, this provides the concept of diluting the cosmological constant into extra dimensions [3]. The Idea of DGP is to capture gravity on brane and make it 4-dimensional. DGP theory however involves ghost instabilities that do not provide self-accelerating solutions [1]. One of the basic advantages of DGP theory is it successfully implements the Vainshtein mechanism. On the basis of the DGP one gets a Galileon defined in the decoupling limit possessing shift symmetry. It is interesting to note that when one implements the Vainshtein mechanism, the symmetry leads to higher order derivatives in action but ensures the equation of motion are second order and hence ghost free [4]. In order to preserve this ghost free nature, one introduces non--minimal coupling of the scalar to the curvature tensor to the action. These are covariant Galileons. Horndeski theory [5] is one of these theories that have quadratic field equations. Further in the paper, attempts are made at using the modified theory of gravity – The Galilean G^3-theory.

We study the screening effects of Vainshtein mechanism, so as to how it operates outside sources where the fifth force is not completely suppressed due to time dependence of cosmological fields [6] [7]. Using various probes of astrophysics we can explore these theories in extended format. New equations for metric potentials inside the sources have been described and variables have been chosen to carefully work under Newtonian parameters. Thus the equations of ϕ and T have been algebraically derived. Using the equations of metric potentials, modified rotation curves and strong lensing potentials have been analyzed and comparison have been made with the actual GR predictions.

Curves and their Measurements

In order to measure distances along the curved surfaces, one finds out how the distances are measured in $\mathbf{R}_3$. By using Pythagoras rule:

$$s(r_1 - r_2) = |r_1 - r_2| = \sqrt{(r_1 - r_2)} = \sqrt{\delta_{ij}(x_1^i - x_2^i)(x_1^j - x_2^j)} \tag{1}$$

where $s(r_1, r_2)$ is the distance between any two points r_1 and r_2, Kronecker's δ symbol gives $\delta_{ij} = 1$ if $i = j$ else $\delta_{ij} = 0$.

$$ds = |r(u + du) - r(u)| = |\frac{dr}{du}|du = \sqrt{\frac{dr}{du}.\frac{dr}{du}}du = \sqrt{\frac{\partial_{ij}dx^i}{du}\frac{dx^j}{du}}du \tag{2}$$

Arc length is then obtained by integration:

$$s(u_1 - u_2) = \int_{u_1}^{u_2} du\sqrt{\frac{\partial_{ij}dx^i}{du}\frac{dx^i}{du}} \tag{3}$$

For a general curve, r_u is parameterized to u. A tangent vector to the curve is a unit vector and is calculated as:

$$\frac{dr}{ds} = \frac{dr}{du}.\frac{du}{ds} = \mathbf{e} \tag{4}$$

Inscribing a Curve

Consider a surface $r(u, v) = x^i(u, v)\mathbf{e}_i$ in $\mathbf{R}_3$, to inscribe a curve $x(w) = x^i(w)\mathbf{e}_i$ in $\mathbf{R}_3$ such that all the points lie within the mentioned surface, one parameterizes the curves $u(w), v(w)$

such that it traces out all the points along the curve expressed as $x(w) = r(u(w), v(w))$. The tangent to this inscribed curve can then be expressed as

$$t = \frac{dx}{dw} = \frac{d}{dw} r(u(w), v(w)) = \frac{\partial r}{\partial u}\frac{\partial u}{\partial w} + \frac{\partial r}{\partial v}\frac{\partial v}{\partial w} \tag{5}$$

Einstein summation convention combines the above expression in compact notation and is represented as

$$t = \frac{dx}{dw} = \frac{d}{dw} r(u(w), v(w)) = \frac{\partial r}{\partial u^a}\frac{\partial u^a}{\partial w} = \frac{\partial x^i}{\partial u^a}\frac{\partial u^a}{\partial w}\mathbf{e}_i \tag{6}$$

where $a = 1, 2$ and $u^a = u$ and $u^2 = v$, Similarly the tangent to this inscribed curve is then given as (for above notation $u^a = u^1, u^2 = u, v$)

$$t = \frac{dr}{du^a} = \frac{dx^i}{du^a}\mathbf{e}_i \tag{7}$$

Distances Along Surfaces

Distances along a surface are measured along the curve inscribed to the surface. The distances are measured between two points that connect the curve. Likewise consider two points $r(u, v)$ and $r(u + du, v + dv)$ that are infinitesimally separate on the surface. This is similar to an arc inscribed on the surface. Thus, the distance then becomes

$$ds = |r(u, v) - r(u + du, v + dv)| = |\frac{dr}{du^a} du^a| = \sqrt{\frac{\delta_{ij} dx^i}{du^a}\frac{dx^j}{du^b} du^a du^b} \tag{8}$$

$$ds^2 = \gamma_{ab}(u, v) du^a du^b \tag{9}$$

Equation (9) signifies the fact that for geometry of surfaces; its properties i.e. distances and angles associated with the curves on the surface can be expressed in terms of $\gamma_{ab}(u, v) =$ and its corresponding derivatives. Where $\gamma_{ab}(u, v) = \frac{\delta_{ij} dx^i}{du^a}\frac{dx^j}{du^b}$ Similarly in order to find the arc length along the inscribed curve running between the points A and B, one integrates equation (8).

$$s(A, B) = \int_{w_a}^{w_b} \frac{ds}{dw} dw = \int_{w_a}^{w_b} \sqrt{\gamma_{ab}(w)\frac{du^a}{dw}\frac{du^b}{dw}} \tag{10}$$

where $\gamma_{ab}(w) = \gamma_{ab}(u(w), v(w))$

Geodesics

Geodesics are curves that determine the geometry of spacetime in general relativity. Straight lines as we conventionally define on general surfaces in $\mathbf{R}_3$ are no longer possible, in fact the shortest possible line joining two points in $\mathbf{R}_3$ is the curve along which the distance between the two points is minimal.

Parametrization: contravariant vectors - While describing surface $r(u, v)$, the tangent $t_a = \frac{dr}{du^a}$ is based on the parameter $u^a = u, v$. Since this forms the basis it can define components of any vector tangent to the surface.

$$c = c^a t_a = c^u t_u + c^u t_v \tag{11}$$

Now if we define new parameters - $u^{a'} = u'(u,v), v'(u,v)$ for the surface $r(u,v) = r(u'(u,v), v'(u,v))$ then the tangent to this new basis is $t_{a'} = \dfrac{\partial r}{\partial u^{a'}}$ then

$$c = c^{a'} t_{a'} = c^{u'} t_{u'} + c^{u'} t_{v'} \tag{12}$$

By using chain rule

$$t_a = \frac{dr}{du^a} = \frac{\partial u^{a'}}{\partial u^a} \frac{\partial r}{\partial u^{a'}} = \frac{\partial u^{a'}}{\partial u^a} t'_a \tag{13}$$

so

$$\mathbf{c} = c^a t_a = c^a \frac{\partial u^{a'}}{\partial u^a} t'_a \tag{14}$$

$$c_{a'} = c^a \frac{\partial u^{a'}}{\partial u^a} \tag{15}$$

Components of c^a that transform in this way under a change of parameters are contravariant components, $\mathbf{c}$ is the contravariant vector. Since γ_{ab} forms the basis of determining the shape of the curve in general space, it must be noted that when parameters are changed it correspondingly transforms too. Referring to equation (9), if $(u,v) \to (u', v')$ then applying chain rule we get:

$$ds^2 = \gamma_{ab}(u,v) du^a du^b = \gamma_{ab} \frac{\partial u^a}{\partial u^{c'}} \frac{\partial u^b}{\partial u^{d'}} du^{c'} du^{d'} \tag{16}$$

this yeilds

$$\gamma_{c',d'}(u', v') = \gamma_{ab}(u(u', v'), v(u', v')) \frac{\partial u^a}{\partial u^{c'}} \frac{\partial u^b}{\partial u^{d'}} \tag{17}$$

Similarly for a tangent vector t_i, the tangent t, to any other curve that is defined by $x^i(w)$ will have components

$$t = \frac{dx^i}{dw} t_i \tag{18}$$

Components in equation (18) define the contravariant vector, now if we change the coordinates from x^i to $x^{i'}$ the component $t_{i'}$ are given by

$$\frac{dx^i}{dw} = \frac{\partial x^{i'}}{\partial x^j} \frac{dx^j}{dw} \tag{19}$$

Metrics

As discussed previously the geodesics of General relativity is determined by least possible distance between two points, this is so done by defining a 3×3 matrix. This is denoted by a so-called metrics $g_{ij}(x) = g_{ji}(x), g_{ij}(x)$ is defined to provide distance between the two infinitesimal points at x^i and $x^i + dx^i$ given as

$$ds^2 = g_{ij}(x) dx^i dx^j \tag{20}$$

Now we find out the geodesic equation for distance between two points A and B such that its minimum, the distance is found by integrating the infinitesimal length according to equation (20) along the curve $x^i(u)$.

$$S_{AB} = \int_{u_A}^{u_B} du \frac{ds}{du} = \int_{u_A}^{u_B} du \sqrt{g_{ij}(x(u)) \frac{dx^i}{dx^u} \frac{dx^j}{dx^u}} = \int_{u_A}^{u_B} du \sqrt{g_{ij}(x(u)) d\dot{x}^i d\dot{x}^j} \tag{21}$$

Now if we vary the above equation with respect to the curve $x^i(u)$ then the quantity S_{AB} is constant to small changes $x^i(u) \to x^i(u) + \delta x^i(u)$ to the first order of $\delta x^i(u)$. Thus applying the principle of least action. We get

$$\delta S = \int \delta du \sqrt{g_{ij}\dot{x}^i\dot{x}^j} = \int \delta \frac{(g_{ij}\dot{x}^i\dot{x}^j)du}{2\sqrt{g_{ij}\dot{x}^i\dot{x}^j}} = 0 \tag{22}$$

$\therefore$

$$\int \delta(g_{ij}\dot{x}^i\dot{x}^j) = 0 \tag{23}$$

$\implies$

$$\int (\delta g_{ij}\dot{x}^i\dot{x}^j + g_{ij}\delta\dot{x}^i\dot{x}^j + g_{ij}\dot{x}^i\delta\dot{x}^j)du = \int (\dot{x}^i\dot{x}^j\partial_k g_{ij}\delta x^k + 2g_{ij}\dot{x}^i\dot{x}^j)du = 0 \tag{24}$$

$\therefore$ integrating by parts

$$\int du(\dot{x}^i\dot{x}^j\partial_k g_{ij}\delta x^k - 2\delta x^i \frac{d}{du}(g_{ij}\dot{x}^j)) = \int du(\dot{x}^i\dot{x}^j\partial_k g_{ij}\delta x^k - 2\delta x^i\partial_k g_{ij}\dot{x}^k\dot{x}^j - 2\delta x^i g_{ij}\ddot{x}^j) = 0 \tag{25}$$

$\therefore$

$$\int du\delta x^i(-2g_{ij}\ddot{x}^j + \dot{x}^k\dot{x}^j\partial_i g_{kj} - 2\dot{x}^k\dot{x}^j\partial_k g_{ij}) = 0 \tag{26}$$

$\therefore$

$$\int du\delta x^i(-2g_{ij}\ddot{x}^j + \dot{x}^k\dot{x}^j\partial_i g_{kj} - \dot{x}^k\dot{x}^j\partial_k g_{ij} - \dot{x}^j\dot{x}^k\partial_j g_{ik}) = 0 \tag{27}$$

multiply equation (27) by $-\frac{1}{2}$

$$\int du\delta x^i(g_{ij}\ddot{x}^j + \frac{1}{2}\dot{x}^k\dot{x}^j(\partial_k g_{ij} + \partial_j g_{ik} - \partial_i g_{kj})) = 0 \tag{28}$$

Using Hamilton's principle yields

$$g_{ij}\ddot{x}^j + \frac{1}{2}\dot{x}^k\dot{x}^j(\partial_k g_{ij} + \partial_j g_{ik} - \partial_i g_{kj}) = 0 \tag{29}$$

Multiply (29) by inverse metric tensor g^{il} we get

$$\ddot{x}^l + \frac{1}{2}g^{il}(\partial_k g_{ij} + \partial_j g_{ik} - \partial_i g_{kj})\dot{x}^k\dot{x}^j = 0 \tag{30}$$

$\implies$

$$\ddot{x}^l + \Gamma^l_{jk}\dot{x}^k\dot{x}^j = 0 \tag{31}$$

Equation (31) represents the general geodesic equation and Γ^l_{jk} is the Christoffel symbol of the first kind. Similarly, the second kind of Christoffel symbol is given as $\Gamma^i_{jk} = \Gamma^i_{kj} = \frac{1}{2}g^{il}(\partial_k g_{ij} + \partial_j g_{ik} - \partial_i g_{kj})$

Stress-Energy Tensor

Stress energy tensor is a combination of energy density ρ, energy flux s^j, momentum density π^i, and stress t^{ij}, under Lorentz transformation given as:

$$T^{\mu\nu} = \begin{pmatrix} \rho & s^j \\ \pi^i & t^{ij} \end{pmatrix} \tag{32}$$

Also equivalence between mass and energy indicates that flow of energy carries momentum and is equal to the density of momentum. Thus the total energy tensor is symmetric $T^{\mu\nu} = T^{\nu\mu}$

Potential Fields

Gravitational potential field is defined as $\Phi(r,t)$ and the strength is determined by the equation:

$$\nabla^2\Phi = 4\pi G\mu \tag{33}$$

Geometry and PPN

Now since we have derived the basic geodesics of space, we need a certain idea about the behavior of the metric $g_{\mu\nu}(x)$ that describes the gravity, which affects the space-time time geometry. For spherically symmetric metric, there must exist symmetry in the three coordinates of space. $x^i \to M^i_j x^j$ for $\Lambda^\mu_\nu = \begin{pmatrix} 1 & \\ & M^i_j \end{pmatrix}$ where $i,j = 1,2,3$ and M^i_j is a 3×3 orthogonal matrix: $\delta M^i_k M^j_l = \delta_{kl}$ This is the condition for symmetry [8].

The general equation for a spherically symmetric metric is given as:

$$ds^2 = -e^{2a(r,t)}dt^2 + e^{2b(r,t)}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \tag{34}$$

Coordinates that form equation (34) are called the Schwarzschild coordinates. To describe a weak gravitational field outside a spherical source, it is assumed that the geodesics is of flat geometry such that $ds^2 \approx -dt^2 + dr^2 + r^2((d\theta^2 + sin^2\theta d\phi^2)$ i.e $e^{2a(r,t)} = 1 + 2\Phi(r,t)$ and $e^{2b(r,t)} = 1 + 2\Psi(r,t)$ [8]. This implies that the Newtonian limit must be that functions Φ and Ψ potentials. Where $\Phi \simeq \Psi << 1$. Also, Newtonian analysis requires that $v^2 \simeq \dfrac{GM}{r} << 1$, thus $\Phi \simeq \Psi \simeq v^2 \simeq \dfrac{GM}{r} << 1$.

For a time dependent metric, (34) can be written as:

$$ds^2 = -[1 + 2\Phi(r)]dt^2 + [1 + 2\Psi(r)]dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \tag{35}$$

Approximations for Φ and Ψ are given as:

$$\Phi(r) = -\frac{GM}{r} + (\beta - \gamma)\left(\frac{GM}{r}\right)^2 + ...,\ and\ \Psi(r) = \gamma\left(\frac{GM}{r}\right) + ... \tag{36}$$

Where β and γ are dimensionless quantities and differ with theories of gravity, the spherical solution to Einstein's equation gives

$$e^{2a(r)} = [1 + 2\Phi(r)] = e^{-2b(r)} = [1 + 2\Psi(r)]^{-1} = 1 - \frac{2GM}{r} \tag{37}$$

Thus equation (35)can be written as:

$$ds^2 = -\left[1 - \frac{2GM}{r}\right]dt^2 + \left[1 - \frac{2GM}{r}\right]^{-1}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \tag{38}$$

This is the general expression for gravitational field outside stationary, spherically symmetrical and non-rotating mass M . θ and ϕ are spherical coordinates in two orthogonal directions, the first term represents the time interval while the second term indicates the radial change, and the last represents the space interval relative to the mass.

Modifying Gravity

As mentioned in the introduction we use the covariant Galileons as the starting point to impose screening mechanisms; these are defined using the Lagrangian.

$$\mathcal{L} = M_{pl}^2 \sum_i \frac{\mathcal{L}_i}{\Lambda_i^{2(i-2)}} + \alpha\phi T + \frac{T^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}{M^4} \tag{39}$$

$T^{\mu\nu}$ is the energy momentum and T is its trace

$$\begin{aligned} \mathcal{L}_2\lambda &= X \\ \mathcal{L}_3 &= X\Box\phi - \phi_\mu\phi^{\mu\nu}\phi_\nu \\ \mathcal{L}_4 &= -X\left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right] - \left[\phi^\mu\phi^\nu\phi_{\mu\nu}\Box\phi - \phi^\mu\phi^{\mu\nu}\phi_\rho\phi^{\rho\nu}\right] \\ \mathcal{L}_5 &= -2X\left[(\Box\phi)^3 - 3\phi_{\mu\nu}\phi^{\mu\nu}\Box\phi + 2\phi_{\mu\nu}\phi^{\rho\nu}\phi^\mu_\rho\right] - \\ \frac{3}{2}&\left[(\Box\phi)^2\phi^\mu\phi^\nu\phi_{\mu\nu} - 2\phi_\mu\phi^{\mu\nu}\phi_{\rho\nu}\phi^\rho - \phi_{\mu\nu}\phi^{\mu\nu}\phi_\rho\phi^{\rho\sigma}\phi_\sigma + 2\phi_\mu\phi^{\mu\nu}\phi_{\rho\nu}\phi^{\rho\sigma}\phi_\sigma\right] \end{aligned} \tag{40}$$

Here $\phi_{\mu_1 \dots \mu_n} = \nabla_{\mu_1 \dots \mu_n}$ and $X = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi^3$, Planck's Mass $M_{pl} = (8\pi G)^{-1}$; for $\lambda = -1$ for stable solutions that include the higher order terms and $\lambda = 1$ is considered for screening procedures using α as the conformal decoupling limit. The study has been performed [4] wherein it is showed that the coupling of Galileons to the curvature tensor for the covariantised quartic term requires $\lambda = -1$.

In fact it is shown by [9] that Lagrangians with higher order equation have a stable solution with ghost free nature due to certain hidden constraints. Equations (40) are subset of general Lagrangians and in order to use the Vainshtein Mechanism, the Lagrangian used is

$$\frac{\mathcal{L}}{\sqrt{-g}} = M_{pl}^2\left[\frac{R}{2} + X + \frac{\mathcal{L}_4}{\Lambda^4}\right] \tag{41}$$

(41) represents equation beyond Horndeski theory [9] [10]. [4] has referred to this as G^3 Galileon. Now a review of screening of quartic Galileon with the decoupling limit α is shown. Equation of motion for static, spherically symmetric densities $T^{\mu\nu} = (\rho(r), 0, 0, 0)$ is given as [4]

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\phi' + \frac{2}{\lambda^4}\phi'^3\right] = 8\alpha\pi G\rho \tag{42}$$

The first term is $\nabla^2\phi$ and occurs due to the quadratic term in Lagrangian the second term is due to quartic Lagrangian. Fifth force $F_5 = -\alpha\phi$ occurs due to conformal coupling to matter and is analogous to the scalar tensor theories [6]. Integrating equation (42) we get

$$F_5 + \frac{2}{\alpha^2\lambda^4 r^2}F_5^3 = 2\alpha^2 F_N \tag{43}$$

Where F_N is the Newtonian force, at large distances $F_5 = 2\alpha^2 F_N$ and the force is unscreened at smaller distances, we get a relation

$$\frac{F_5}{F_N} = \left(\frac{r}{r_V}\right)^2 \tag{44}$$

where $r_V = (\alpha\lambda)^{\frac{-2}{3}}(GM)^{\frac{1}{3}}$.

Vainshtein radius r_V provides a transition between screened and unscreened regime. When $r >> r_v$ the force is unscreened and conversely $F_5 << F_N$ for screened force. In order to explore the non-relativistic limits of G^3-Galileon and how best the Vainshtein mechanism screens, we investigate the governing of Newtonian perturbations with specific profile $\phi(r,t) = \phi_0(t)+\pi(r,t)$ for the metric as:

$$ds^2 = -[1+2\Phi(r)]dt^2 + [1-2\Psi(r,t)]d_{ij}dx^i dx^j a^2(t) \tag{45}$$

dynamics of ϕ, Ψ and π have been solved by [2]. Variables used are given below, primes denote derivatives with respect to r and M is the mass within sphere r.

$$x = \frac{\pi'}{r},\ y = \frac{\Phi'}{r},\ z = \frac{\Psi'}{r},\ A = \frac{M(r)}{8\pi M_{pl}^2 r^3} \tag{46}$$

Using the above variables, equation of motion by parts are derived (see 1 appendix)

$$z + \frac{5\varepsilon}{2}x^2 = A \tag{47}$$

$$y - z - \frac{\varepsilon}{2}x^2 - \varepsilon x(rx' + x) = 0 \tag{48}$$

$$-x - \frac{4x^3}{\Lambda^4} + 10\varepsilon xy + 2\varepsilon x(rz' + 2z) = 0 \tag{49}$$

Where $\varepsilon = \frac{\dot{\phi}_0^2}{\Lambda^4}$ Using equation (47),(48) and (49), we find the algebraic expression for x:

$$\left(\frac{4}{\Lambda^4} + 20\varepsilon^2\right)x^2 - 2\epsilon[(7A + rA')] - 1 = 0 \tag{50}$$

Equation (50) is a cubic equation, and $x = 0$ is a constant solution to the above equation; so eliminating x from the above equation one obtains a quadratic equation. Also assuming that for any density profile $A >> 1$ and $rA' \sim A$, we get

$$x^2 = \varepsilon\Lambda^4\left(\frac{7A + rA'}{10\gamma + 2}\right) \tag{51}$$

where $\gamma = \varepsilon^2\Lambda^4 = \left(\frac{\dot{\phi}_0}{4}\right)^2$ Using the value of x^2 from (51) in equation

$$y = \frac{A}{1+5\gamma} + \frac{\gamma}{4(1+5\gamma)}\frac{(r^3A)''}{r},\ z = \frac{A}{1+5\gamma} - \frac{5\gamma}{4(1+5\gamma)}\frac{(r^3A)''}{r^2} \tag{52}$$

If we restrict ourselves to Newtonian parameters $ds^2 = -\left(1 + \frac{2GM(r)}{r}\right) + \left(1 - 2\gamma\frac{GM(r)}{r}\right)$ See [4], Then equation (52) can be written as:

$$\frac{d\Phi}{dr} = \frac{GM(r)}{r^2} + \frac{\gamma}{4}GM''(r) \tag{53}$$

$$\frac{d\Psi}{dr} = \frac{GM(r)}{r^2} - \frac{5\gamma}{4}GM'(r) \tag{54}$$

equation (53) and (54) are the initial equations required for the test of astrophysical probes, the first term of these two equations agree with GR, however the second terms the above two equations contain first and second derivatives of mass. These terms appear due time derivatives of cosmological profile that Vainshtein mechanism fails to screen. The perturbations occurring due to π' have been suppressed by Vainshtein as $x = 0$ is a constant solution. When using scalar tensor field (Jordan frame)[11], the energy momentum tensor is conserved ($\nabla_\mu T^{\mu\nu} = 0$), conservation of Mass equation remains unaltered although [4].

$$\frac{dM}{dr} = 4\pi r^2 \rho(r) \tag{55}$$

Astrophysical Probes

Certain cosmological objects like galaxies can reveal a great detail about the presence of dark matter in the cosmos. Rotation curves of various galaxies especially spiral galaxies that can be determined show that contribution of actual mass that aids in rotation is calculatively less for actual rotation curves, hence the there must me some matter are intrinsic and not detected, also the brightness extends to more massive stars and so lensing techniques can also be used to detect matter surrounding the object.

We analyze the breaking down of Vainshtein mechanism using rotation curves and lensing. For this we use NFW (Navarro-Frank-White) density profile that provides the distribution of dark matter as:

$$\rho(r) = \frac{\rho_s}{\frac{r}{r_s}\left(1 + \frac{r}{r_s}\right)^2} \tag{56}$$

For a spiral galaxy like Milky Way, $r_s = 20kPc$ and $\rho_s = 6.68 \mathrm{M}_\odot kPc^{-3}$ for a halo mass of $10^{12}\mathrm{M}_\odot$ [12].

Rotation curves of a galaxy can be determined by gravitational potential, assuming a circular motion velocity and radius are related as:

$$\frac{v^2}{r} = \frac{d\Phi}{dr} \tag{57}$$

Using equation (57) one gets

$$v^2 = \frac{4\pi G r_s^3 \rho_s}{r}\left[ln\left(1 + \frac{r}{r_s}\right) - \left(1 + \frac{r_s}{r}\right)^{-1} + \frac{\gamma\left(\frac{r_s}{r} - 1\right)}{4\left(\frac{r_s}{r} + 1\right)^3}\right] \tag{58}$$

The rotation for Milky Way galaxy is plotted in **Figure 1**, for various values of γ indicated in the plot, the plot illustrates that at larger radii the discrepancy between the two theories is more pronounced, it is over here the curves begin to flatten. Also at these larger radii G^3 suggests that for higher values of γ smaller velocities are obtained than predicted by GR.

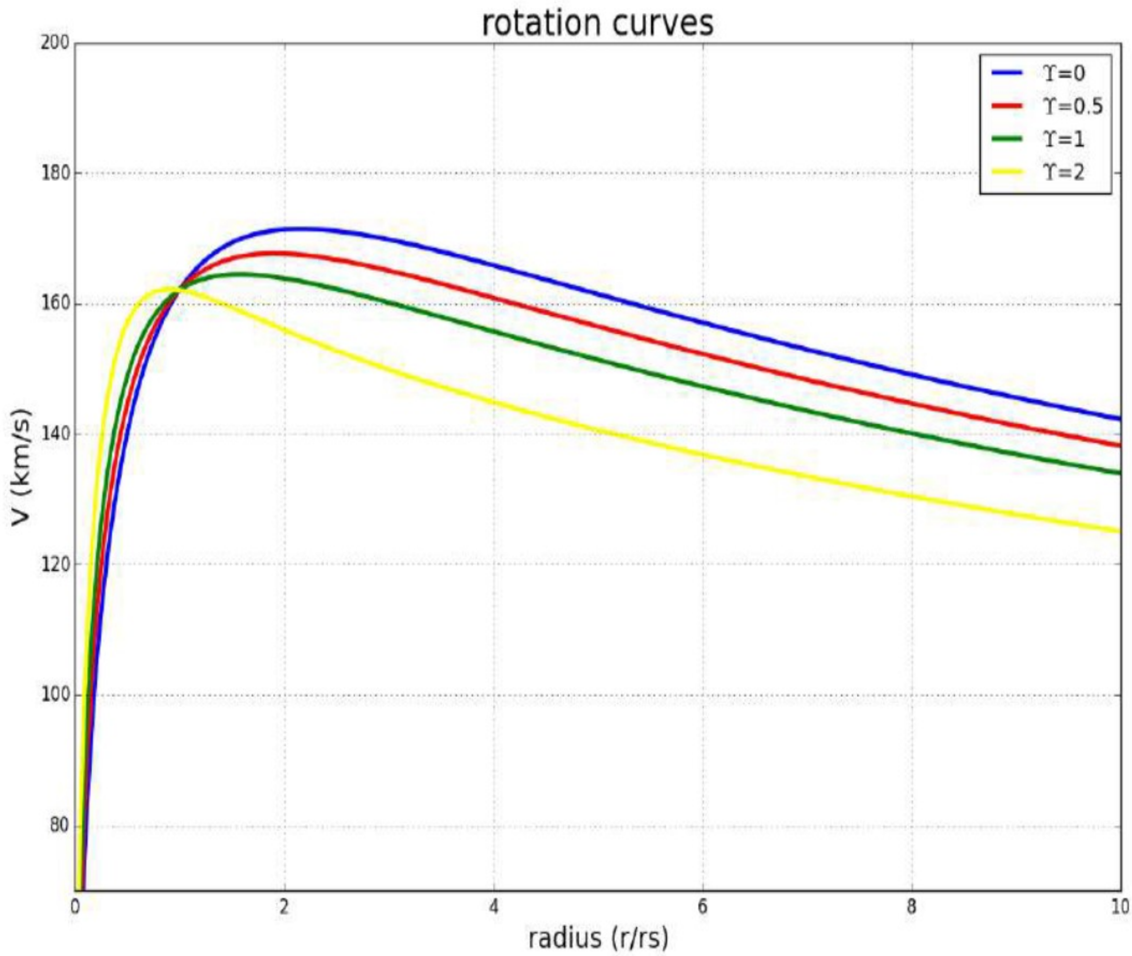

Figure 1: Rotation curve for NFW profile with $r_s = 20kPc$ and $\rho_s = 6.68 \mathrm{M}_{\odot} kPc^{-3}$.The GR curve is shown in blue while other curves correspond to various values of γ.

Bending of light depends upon the potential $\Phi + T$, using equation (53) and (54) one gets:

$$\Phi + T = -\frac{8\pi G r_s^3 \rho_s}{r} \left[\ln(1 + \frac{r}{r_s}) + \frac{\gamma \left(\frac{r_s}{r} + 6\right)}{8 \left(\frac{r_s}{r} + 1\right)^2} \right] \tag{59}$$

For GR predictions $\Phi = T$ **Figure 2** plots the Strong lensing potential with the ratio $\frac{\Phi+T}{2\Phi}$ that signifies the amount of light bent by gravity relative to the gravitational force felt by objects moving at non-relativistic velocities for various values of γ. We find that the ratio of the potential is larger than GR predictions and increases with higher values of γ. This indicates that the lensing mass is greater than the actual source mass and thus strong lensing can be used as probe. However one also needs to constrain the parameters of γ , which is not performed currently.

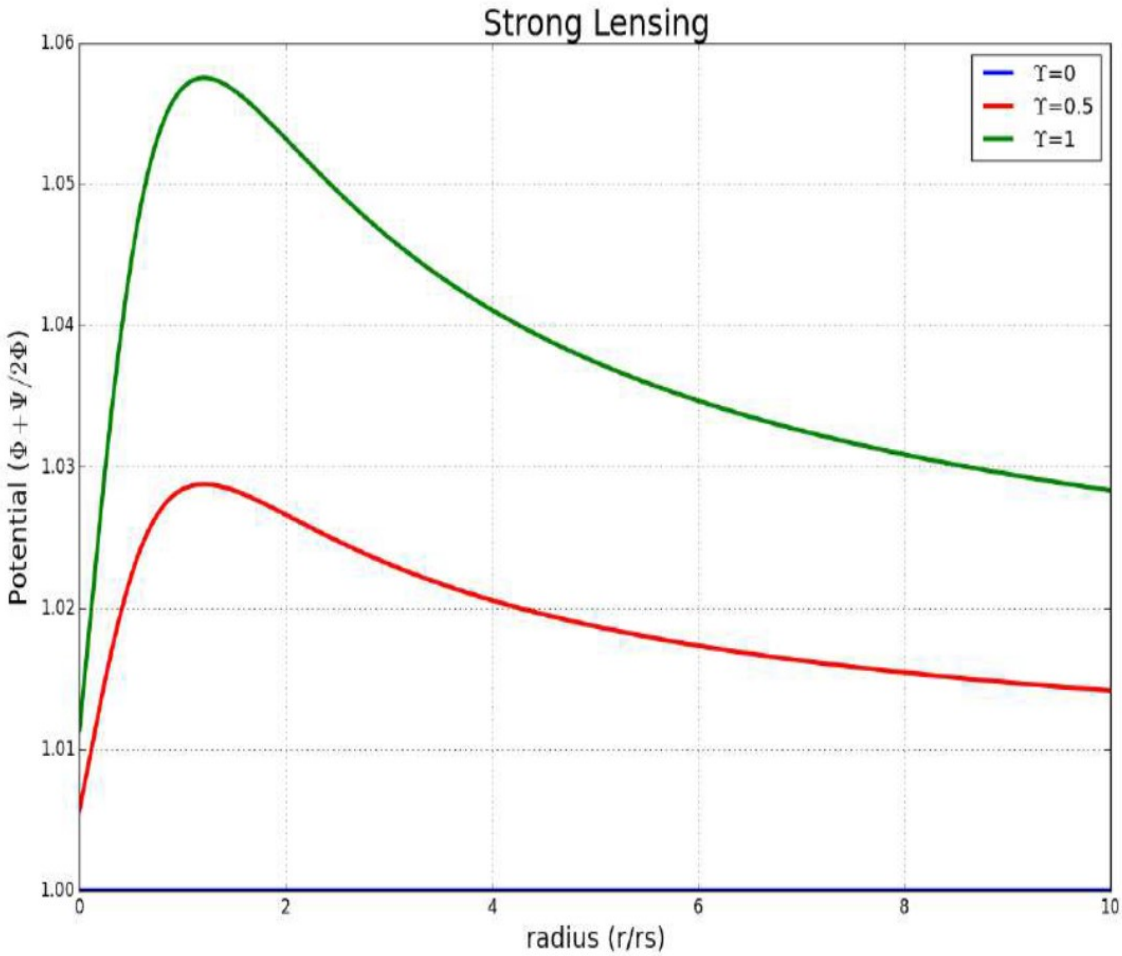

Figure 2: Strong lensing Potential for NFW profile with $r_s = 20kPc$ and $\rho_s = 6.68\mathrm{M}_\odot kPc^{-3}$.The GR curve is shown in blue while other curves correspond to various values of γ.

Discussion and Conclusion

In the current paper we have reviewed the basic or the starting point of general geodesics that govern the geometry in GR. We have derived the general geodesic equation to find Christoffel symbols that aid in determining the curvature of spacetime. Thus, we reviewed the equation for spherically symmetric metric for non-relativistic case given by Schwarzschild coordinates. In order to describe weak Gravitational field, assumption are made to attain the Parameterized Post Newtonian Approximation (PPN).

In the second part of the paper attempts are made to study the Lagrangians beyond the Hordenski theory known as G^3 - Galileons. Also, a brief screening mechanism of Galileon with decoupling limit α is shown wherein it is described how the fifth force that originates due to conformal coupling is successfully screened beyond the Vainshtein radius. Subsequently the non-relativistic limit of G^3 - Galileons is explored to derive the new scalar potential fields of ϕ and T. Is it found that Vainshtein mechanism successfully suppresses the gradients of scalar fields and that the equations obtained for it are different from Poisson's equation with an additional term can contain first and second derivatives of mass. They tend to weaken the strength of gravity.

Next, we have used galactic probes, rotation curves for the Milky Way have been calculated

using the NFW profile and it's shown through the plots that object with rotate with smaller circular velocities than predicted by GR with increasing radii. The lensing potential was also calculated and it is shown that lensing potential is higher than GR predictions.
Breaking of Vainshtein Mechanism only occurs when the cosmological scalar has non-vanishing time derivative [4] [6]; all of the effects presented here were non-negligible when $\gamma \gtrsim \mathcal{O}(1)$, further improvements can be made by providing specific constraints on γ. One can also determine the effects of using a different energy profile for astrophysical probes for instance, Einasto profile to investigate the rotation curves and their differences with GR, one can also use different classes of galaxies like the elliptical and dwarfs, however in order to do that one must find the central density and the scale radius using number of galaxies of the same class to determine the best possible fit. Any theory that predicts γ in range of cosmological parameters can be used as probes of astrophysics. Nonetheless G^3-Galileons serve as fine models for astrophysical probes.

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