

The United Nations publishes a Human Development Index (HDI) for each country, aggregating data regarding each country's achievement with respect to health, education and income.<sup>1</sup> HDI is calculated as the geometric mean of the following three criteria:

- *Health*, measured by life expectancy at birth
- *Education*, measured as the average of (a) mean years of schooling for adults aged 25 years and over, and (b) expected years for schooling for children of school entering age
- *Standard of living*, measured using gross national income per capita

HDI provides one way to combine these criteria into a ranking of countries. In this assignment, we will apply two alternative methods presented in today's lecture.

With direct comparisons of a subset of alternatives, we can infer criteria weights to generate a ranking. We will use comparisons provided by the World Bank's Human Capital Index<sup>2</sup> (HCI) to represent our decision maker. Based on HCI, we observe the following pairwise comparisons, where preference is defined according to the country with the greater HCI score:

- Oman is preferred to Brazil
- Ireland is preferred to Portugal
- Turkey is preferred to Ukraine
- Zimbabwe is preferred to Haiti
- Japan is preferred to Estonia
- Algeria is preferred to Panama
- Kenya is preferred to India
- Peru is preferred to Romania

Formulate a linear program (LP) to infer a set of weights for each of the three HDI criteria such that the inferred rankings applying these weights to the HDI criteria minimize the errors associated with the HCI pairwise comparisons. You should normalize each of the HDI criteria such that all criteria scores lie between zero and one.

1. What value(s) did you use for the constraint RHS  $\delta_{ik}$ ?
2. Did your optimal solution exhibit any inconsistencies with respect to the pairwise comparisons provided? If yes, which ones?
3. What were the optimal criteria weight values  $w_j$  that you obtained?
4. What is Canada's ranking out of all countries, applying your optimal weights? How does this compare with its HDI ranking?

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<sup>1</sup><http://hdr.undp.org/en/content/human-development-index-hdi>

<sup>2</sup><https://www.worldbank.org/en/publication/human-capital>

Another alternative is to apply Data Envelopment Analysis to identify those countries lying on the efficient frontier for the set of HDI criteria. Formulate an LP to identify an efficiency rating ( $\theta$ ) for each country using an output-oriented model assuming variable returns to scale.

**1. Which countries lie on this efficient frontier?**

**2. What is New Zealand's efficiency rating ( $\theta$ )? If New Zealand is not efficient, (a) what are its target values for each criteria? and (b) what is the convex combination of ERS members used to generate these target values?**