Wireless Systems

Final Exam Report

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1.0 INTRODUCTION

1.1 Description of the problem

In this project, links to 3 mobile stations at a time need to be established at the same frequency and maintain a significant quality of connection by measure of SINR. The cut-off is 24 dB and applies to all 15 base stations, and 5 timeslots. However the greatest challenge to getting 24 dB SINR is that the radiation from other links could reach any given link and reduce the quality of connection through interference. The interference happens depending on the radiation footprint of the base station as shown in Figure 1 as an example, also known as polar gain plots. This radiation signature is unique to specific base-station configuration, namely scan angle, antenna-count and antenna-spacing. The interference can also be changed among the mobile stations by moving them to different locations and changing the orienation with respect to the mobile stations.

The notion behind countering interference in this project is that the base stations need to be communicating with the MS (mobile station) that are far apart and preferably within the nulls of each other. It is difficult to communicate with two or MSs that are adjacent to each other from multiple BS (base-station) due to overlap of signal and high gain leaking into the MSs that a given BS does intend on communicating with.

Looking at the directivity pattern, unlike the previous assignment where there was one peak and direction the antenna was pointing at, this time the pattern looks somewhat like a cloverleaf where there are multiple peaks and multiple nulls. The plan is to have a BS direct the highest peak towards the MS that it needs to prioritize at the minimum required power level. This seems the most logical approach instead of keeping the angle constant (alpha) and hoping to communicate with another MS that falls under a beam different from the strongest beam in a different timeslot. In the figure notice the strongest beam is pointing at 0^o or East, and a second MS receives radition from the BS when talking to the MS under the east lob. It is imperative that the northern MS is either distant enough to not be affected and receive poor gain or the antenna configuration be changed to create a null there. There are numerous options available in this project and they will be explored in the upcoming sections. The following is the seating arrangement and base station locations found by trial and error:

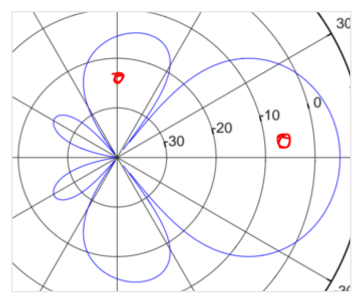

Figure 1: Single element azimuth pattern

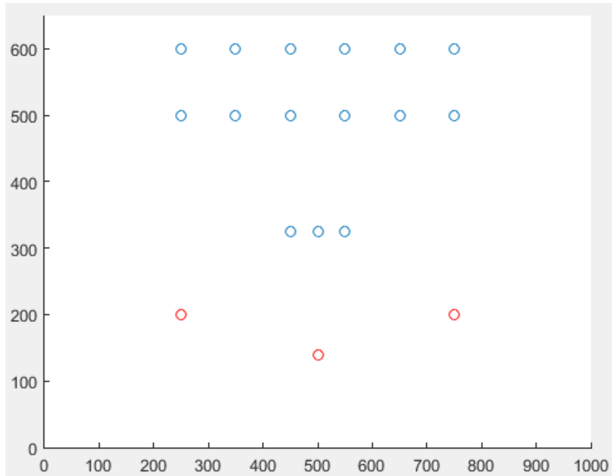

Figure 2: Arrangement pattern.
Scale is in meters. Red is BS, blue is MS.
Bottom three MS are VIP zones

As shown in Figure 2, the bottom 3 MS (VIPs) will be given priority. Because there will be 3 out of 5 timeslots pertaining to the service of VIPs, the base station that communicates with MS13,14,15 will be operating at a minimum required power and the rest of the base stations will be close to "blasting" the furthest MS in the public area at around +30 dBm i.e. if needed. This will ensure that the interference is limited. A case where all three base stations transmit at the higher end of the -30 to +30 dBm range is undesirable and will cause large drop in SINR. In order to attain a fair SINR for each MS there needs to be a scheduler that needs to choose the MSs that are most distant from each other in a given timeslot. The following is an example of how the algorithm could be designed such that the SINR is maximized in each timeslot (Figure 3). This sequence of MS can be tweaked after experimenting with the direction of gains and nulls from each of the base-stations, but this symmetry across timeslots in Figure 3 will be maintained.

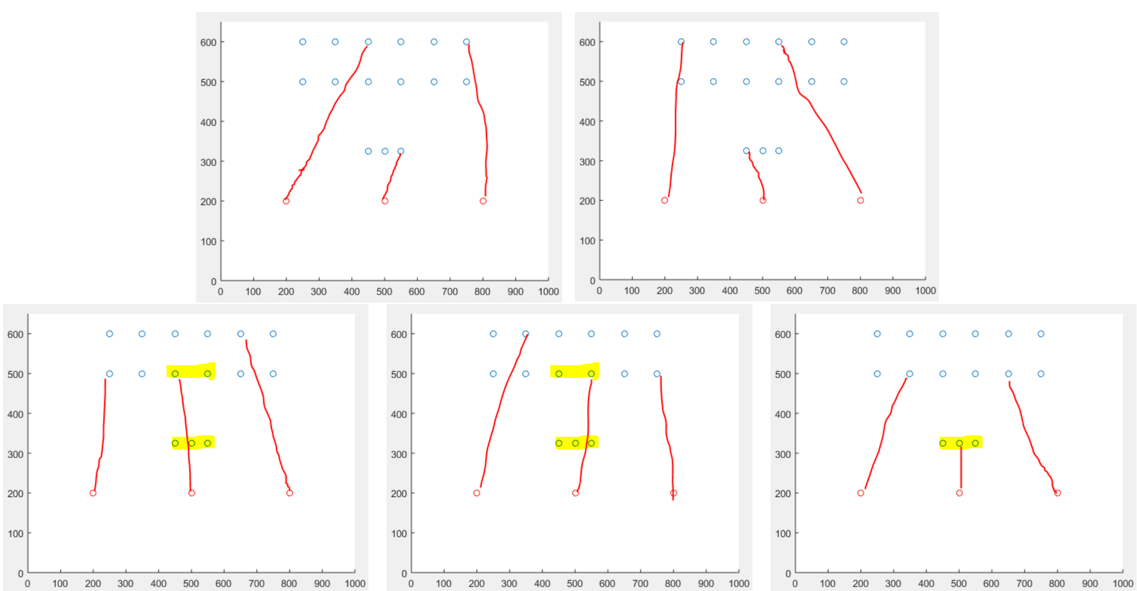

Figure 3: Scheduler priorities for each MS based on the maximum distance to each

1.2 Complexity of the challenge

Since it is unclear at this point whether the secondary lobes of a BS interfers with the peak lob of another base-station at another MS, this is when the scan angle (alpha), base-station orienation, antenna-count, antenna-spacing, and BS location play a role in the optimization. Appendix A shows the various gain plots that were used as reference when making an approximate guess about the gains and nulls at the MSs. For instance, the first image in Appendix which is also Figure 4, the combinatation of both the heat plot and the polar plot suggests that if MS8 is linked to BS1, then MS5, 13, 14 and probably 15 can be linked to other base stations, because of the darker blue-green lines in the heat map suggesting an absence of gain.

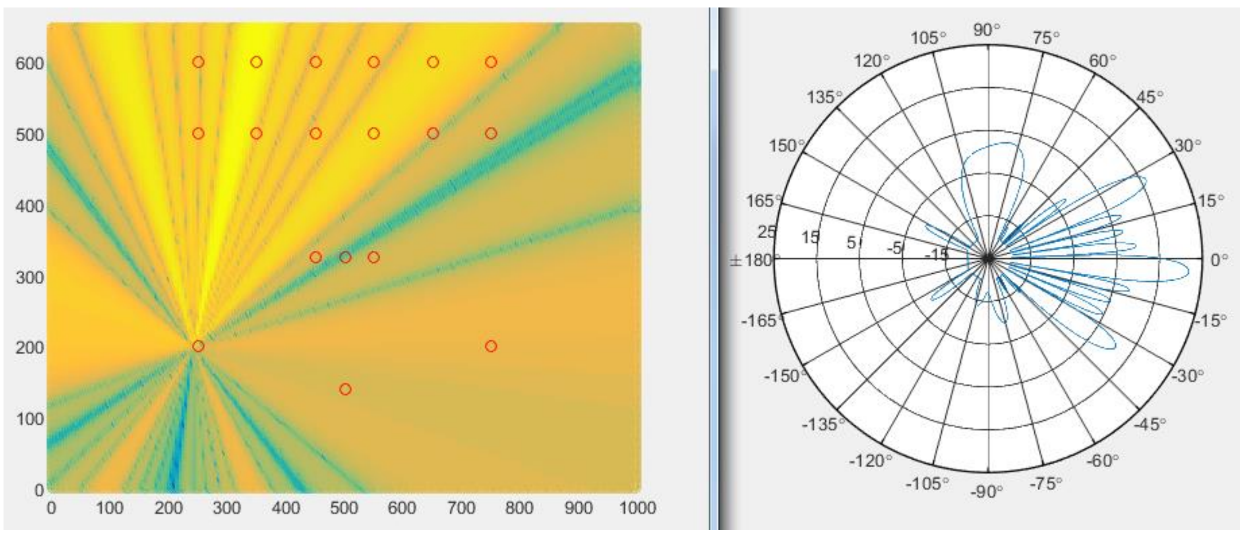

Figure 4: BS1, Antenna Count 5 Spacing 30cm, Alpha 45 degrees

Moreover, being in the direction of the highest directivity, MS8 should be at 20 dBi. In order to confirm the above speculation, another output after the calculation is produced, except this time, instead of using 40,000 or so points to show the gain heat, only gain in 15 MSs are calculated. The following table shows the output from the function for a given BS. In this case, the output corresponds to the above figure, for BS1 with 5 antennas, spacing 30 cm, alpha 45 degrees:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
7.611	9.3262	1.7204	8.663	-7.7902	-25.9502	7.611	14.4569	4.0021	7.6408	8.663	-7.159	-31.9498	-29.968	-64.2893

As speculated before, the gain for MS5 is indeed much lower than MS8, which are -7.7 dBi and 14.4 dBi respectively. Additionally, the gain for the VIP zones are favourable in reducing interference, i.e. ranging between -64 dBi to -30 dBi for MS13 to MS15. This confirms the initial speculation based on the visual data that if BS1 is linked to 8, then BS3 on the other side could be linked to MS 5 or 6, whereas BS3 could be used for MS 15. The table shown above is used to provide data that were missed from the heat map such as the fact that the gain at MS6 is lower than MS 5. Recall from Figure 3 with visualization of 5 timeslots, if BS 3 is linked to MS6, it will correspond to timeslot 4. The difference would be that instead BS2 being linked with MS4, it will be linked to MS15. Unfortunately, the complexity of comparing the gains and nulls emmitted from each BS is $O(n^2)$, which suggestes, that if we choose an MS to transmit to, the other N-1 BSs will need to be checked for interference at that MS. In this case, a similar comparison needs to be made for BS2 and then BS3. In Figure 5, note that BS2 has a considerable gain (directivity) in the direction of MS14, 15, and MS 4 from the regular audience. In terms of null, MS6 seems on the path of a null which is favourable when selecting MS 6 by BS3, but MS8 seems like it will be receiving much more interference. Looking at the polar plot, it seems that the gain on MS6 will be less than 0 dBi whereas the gain on MS8 will be around 10 dBi.

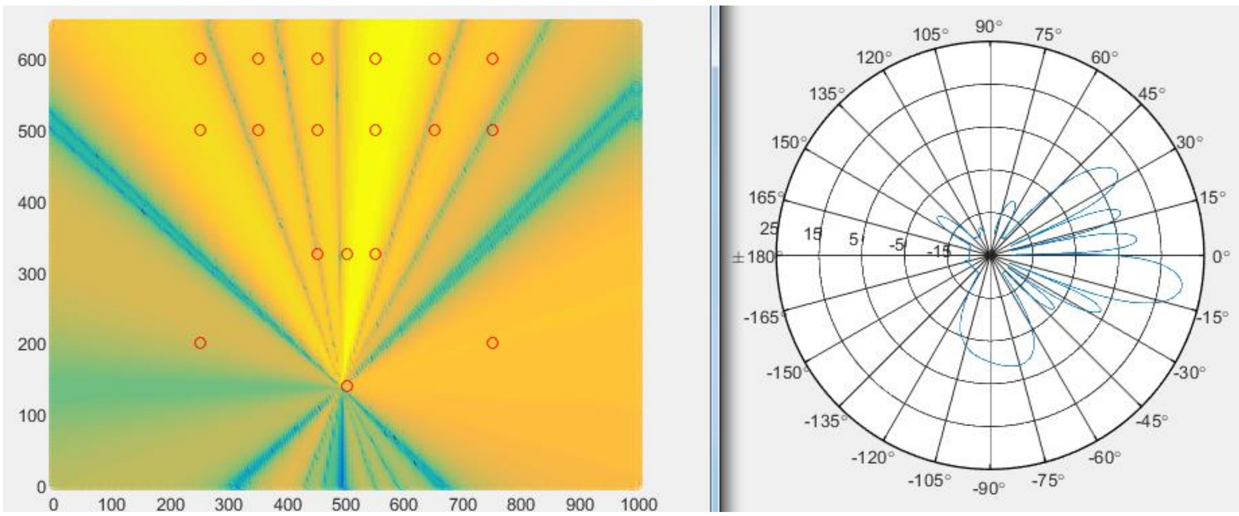

Figure 5: BS2, Antenna Count 4, Spacing 20.1cm, Alpha 80 degrees

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
10.25	0.3647	8.676	20.12	-5.7183	-27.4267	6.8495	7.0529	9.4598	19.187	10.61	2.9381	3.4011	3.5626	16.5226

Using the gain table for all 15 MSs, BS3-MS6 pair looks good, but BS1-MS8 pair does not. However, from the previous analysis, there is 14 dBi of gain from BS1, so it is upto the channel matrix calculation to decide whether 7.1 dBi of gain from BS2 is considerable. Moreover, BS2 is positioned 60 meters behind the other BSs, and therefore in overall h-values for other MSs from an interferer like BS2 should be smaller as a result of more free-space path loss. Recall that the h-values are representations of the terms in the equation used in the calculation of RSL without the transmit power (Tx) term. There is no breakpoint considered in this project.

Now consider the last BS in the current timeslot. BS3 is much further away from the MS8 in consideration which suggests that there is a degree of flexibility in case the gain is high in the direction of MS8, which at the time BS1 will be transmitting to. After speculation from Figure 6, it seems that MS1, 7, 13, 14 and 15 are in the direction of nulls, but not MS8. To get an idea about the values, either the polar or the gain table can be used to confirm the speculation.

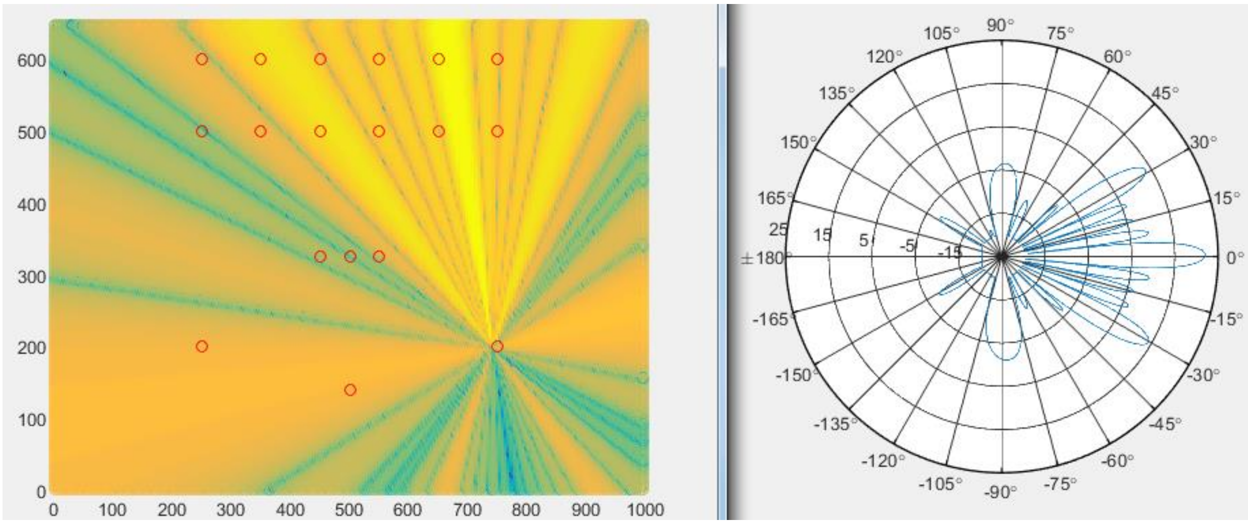

Figure 6: BS3, Antenna Count 5, Spacing 30cm, Alpha -5 degrees

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
-30	-7.74	9.5641	3.673	8.7519	6.1512	-8.8066	9.5641	5.0811	5.1179	16.21	6.1512	-31.482	-29.402	-47.0994

MS8 seems to be in the path of 9.5 dBi of gain whereas it is looking good for BS2-MS15 pair and their -47 dBi of interference coming from BS3. Now for all the interference in the unwanted, the calculation of the channel matrix is going to be reveal if there is a way compensate for the interference from other BSs. The algorithm used to calculate the SINR for 3 MSs for a given timeslot will be covered in a later section, but for now the SINR for MS8, 14, 6 are 18 dB, 26 dB and 12.5 dB respectively. This does not meet the requirement for 24 dB in all MSs but it does provide the 24+dB to MS15 which is one of the VIPs. The challenge in this project, is after the n^2 comparisons for the 3 base stations, the SINR requirement was not met, and now the experiment has to be repeated with a different set of scan angles and/or different physical parameters for the antennas at the base stations. This project is very long, time-consuming and requires using brute-force technique on certain occassions such as when looking for the best angles so that BS1 produces at least 1 good gain MSX, and 2 null MSs, MSY and MSZ; BS2 produces, one good gain MSY, but nulls on MSX and MSZ; BS3 produces a good gain MSZ, but nulls on MSX and MSY. Because the calculation of SINR involves a 3-dimentional brute-force approach, the number of computataions that Matlab is performing is already 61*61*61 or 226,981 permutitons for *BS1.power*, *BS2.power*, and *BS3.power* between -30 dBm to +30 dBm. Because of the time-constraint associated to the project, the locations of BSs, number of antenna, antenna spacing, scan angles and so on could not be accodmodated in the brute algorithm. This is the reason for performing hand calculations and making smart speculations.

2.0 TRIAL AND ERROR PHASE

2.1 Effect of changing antenna configuration

It has been determined that increasing antenna spacing reduces the difference between the primary and secondary peak gains. Using 20.1 cm and 30 cm spacing from Figure 7, the primary/secondary lobes had 22.05/8.47 dBi and 22.05/12.03 dBi gains respectively. Since in the first case, the secondary lobes are non-dominant, it is easier to focus on one MS as a time withouth interfering with another MS. On the other hand, with increasing antenna spacing, there is greater flexilibility to choose between MSs for an angle and link with the MS with great SINR.

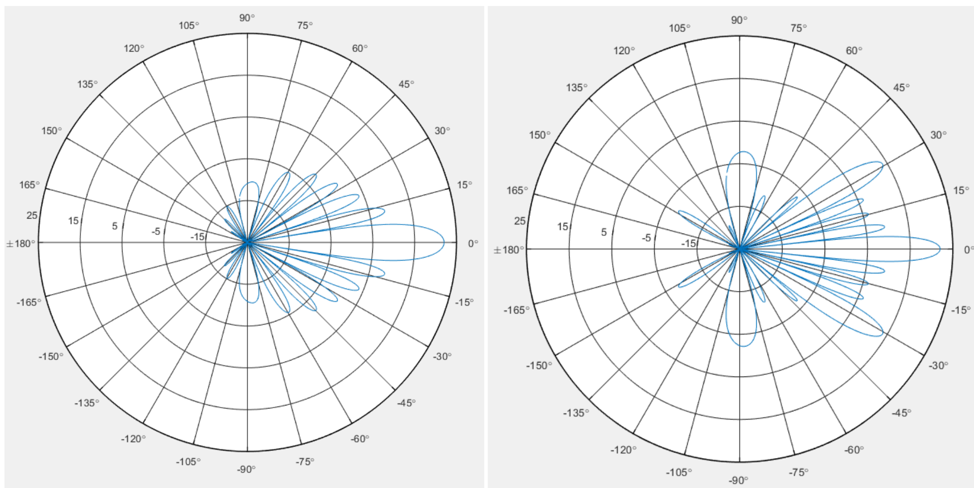

Figure 7: Left: 20.1 cm spacing, right: 30 cm spacing. 5 Antennas. Scan alpha is 0 degrees

Meanwhile, reducing the number of antennas in a linear array reduces the gain. In Figure 8, left plot has primary/secondary ratio of 22.05/12.03 dBi with 5 antennas, and right has 21.08/11.41 dBi with 4 antennas. Both scan angels are 0 degrees. Essentially, adding more antennnas gives more focus to the stations that are further away, but also increases the gain in the secondary lobs at ± 30 degrees and potentially causing more interference.

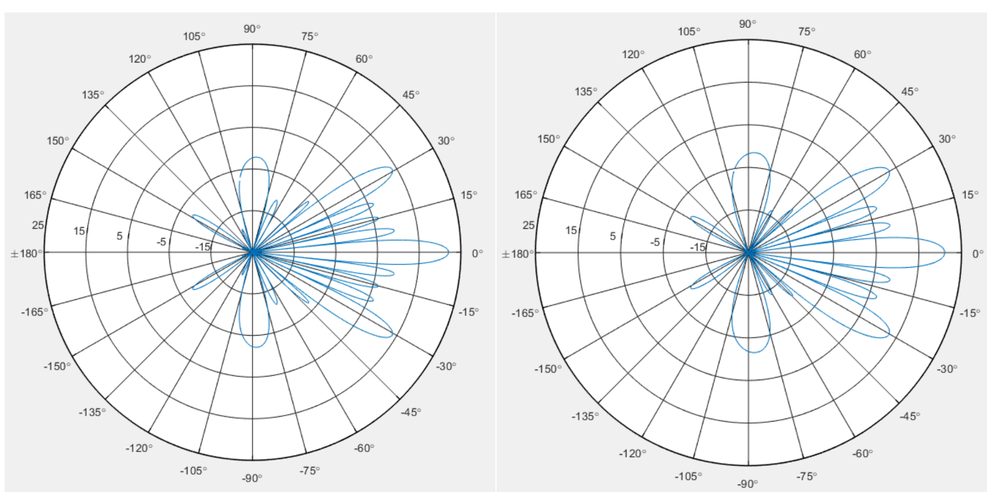

Figure 8: Left: 5 antenna, right: 4 antennas. 30 cm antenna spacing. Scan alpha is 0 degrees

As per trial and error the spacing decided for the project and the plots in Figure 7 and Figure 8 are 3.00, 2.01, 3.00 cms and antennas counts are 5, 4, 5 for BS1, BS2, BS3 respectively.

When finding the 3 best pairs in a given timeslot, these secondary lobes sometimes are a blessing and a curse. Based on the chosen physical parameters of base-stations, when the scan was varied, sidelobes provided considerable gain in one or more of the strongly undesirable MSs (7-17 dBi), and similar situation was observed with nulls (< 0 dBi) in a given timeslot. There is no simple way to determine through pattern-analysis or brute-force which physical paramater needs to be changed wihout performing a full analysis of the 5 different time-slots per change. If one of the timeslots has significant non-mutually exclusive radiation between the MSs, then it causes the developer of the algorithm to start from the beginning. This is because paramters like BS location, array orientation, antenna count and spacing has to be fixed throughout the 5 timeslots.

Meanwhile, the only parameter that can be changed between timeslots is the scan angle. Hence the pattern-analysis tables were developed to check for interference patterns between BSs. One set of tables are shown in Appendix B to provide an example of gain vs. scan information. Each BS has 2 tables, one for 0-90 degrees and another from -90-(-5) degrees both with 5 degree increments. The following table shows the predetermined binding between BSs and MSs from Figure 3:

Table 1: BS-MS Bindings

BS	MS
1	1,2,7,8,9
2	5,6,10,11,12
3	3,4,13,14,15

Based on this binding, in Appendix B, BS1 for example has rows for MS 1, 2, 7, 8 and 9 highlighted in blue to indicate that in order to form a link, any considerable gain value needs to correspond to that MS (row) for a given scan, say MSX. And then looking at the tables for BS2 and BS3, the gain values for MSX needs to be very low (~-20 to -70 dBi found empirically) for their respective scan columns at BS2 and BS3. Similar procedure applies to BS2 against BS1 and BS3 as well as BS3 against BS1 and BS2. This was done through observations, and highlighting good gains with green and bad gains with yellow as a means to quickly find a triplet of MSs. Figure 9 is a graphical representation from BS1 standpoint.

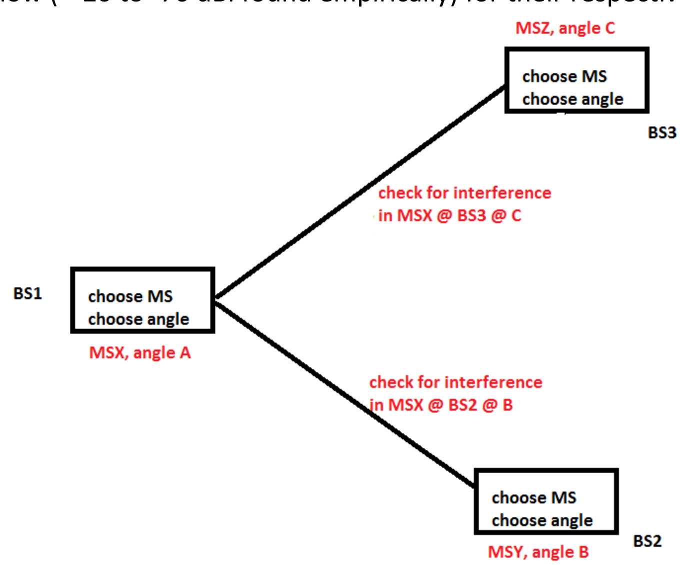

Figure 9: Inter-BS interaction for a potential interference-searching algorithm

A similar representation can be made for BS2 and BS3. The time-consuming part of the project after finding a triplet of mobile-stations in one time-slot is, if it turns out that there are no other compatible scan angles (among the BSs) that yeild another triplets in a different time-slot, a physical BS parameter needs to be changed, and the entire process has to be repeated – and that being trial and error in adjusting any one of base-station orienation, antenna-count, antenna-spacing, base station location and generating simlar gain tables as Appendix B.

2.2 Final design parameters

At the time of carrying out this procedure the triplet stations for all 5 time-slots are hard-coded as per Figure 3 and an extensive number of trial-and-error were done to get 24 dBs of SINR in all 15 MSs in all 5 time-slots. This requirement was not fulfilled due to time-constraints but the SINR numbers were descent in most cases. As mentioned earlier, the best polar plots were achieved by grouping 5, 4 and 5 antennas together for BS1, BS2 and BS3. The following is the outcome of the selected physical and dynamic (scan) parameters:

Table 2: SINR Calculations

Timeslot#	MS			Pxi chosen			Alpha			SINR		
1	2	13	11	30	30	30	10	65	35	25.2238	29.2679	24.2071
2	8	15	5	30	30	30	-10	-65	-35	25.7188	29.2679	27.8512
3	1	14	6	30	30	30	-75	-40	75	14.6615	31.1177	14.6616
4	7	4	10	-30	30	-30	40	65	-40	10.467	18.931	21.2402
5	9	3	12	-30	30	-30	40	-65	-40	21.2402	18.931	10.467

Note that the VIP locations are highlighted in green. Besides the locations of the MSs which were part of the problem statement, the physical parameters of the design project are as follows:

Table 3: Physical parameters for each BS

Parameters	Location (m)		Antenna Config		Orientation
	X	Y	Count	Spacing (cm)	(degrees)
BS1	250	200	5	30	80
BS2	500	140	4	20.1	90
BS3	750	200	5	30	100

The scan used during the 5 timeslots are as follows:

Table 4: Physical parameters for each BS

Timeslot	1	2	3	4	5
BS1	0	5	45	-40	40
BS2	90	-80	0	65	-65
BS3	-5	0	-45	40	-40

As required by the assignment, the scan angles from -90 to 90 in increments of 45 degrees is given in Figure 10 and Figure 11. If time was not a constraint, two things that require attention are: 1). More BS optimization such that the 24 dB requirment stated in the beginning of this section is fulfilled 2). The power of the BSs is adjusted by 1 dB for adjacent timeslots as required in the project.

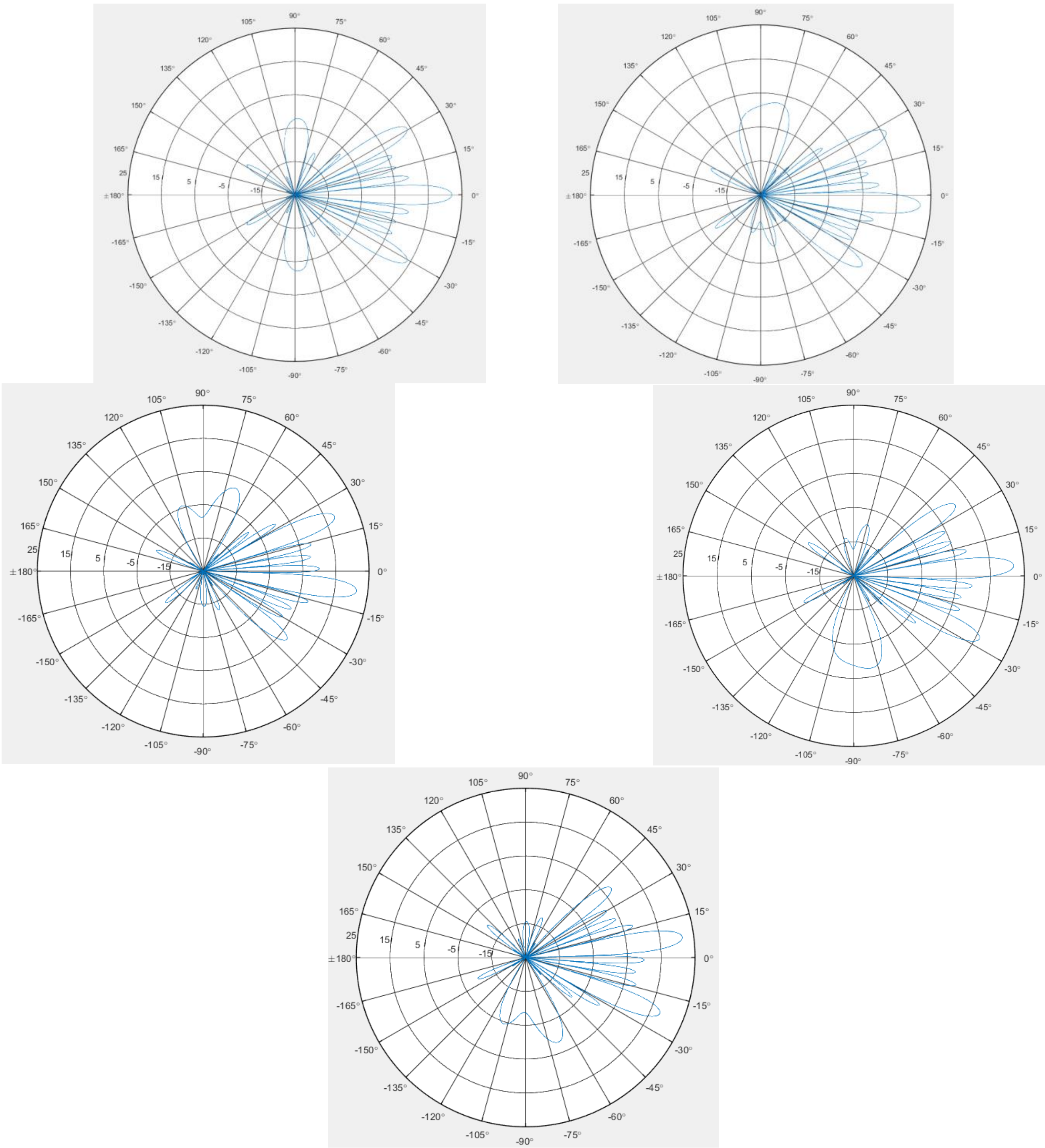

Figure 10: BS1 antenna count: 5. Starting from top-left corner, going clockwise, scan angle: 0, 45, -45, -90, 90. BS3 scan angle is the same because of same antenna count.

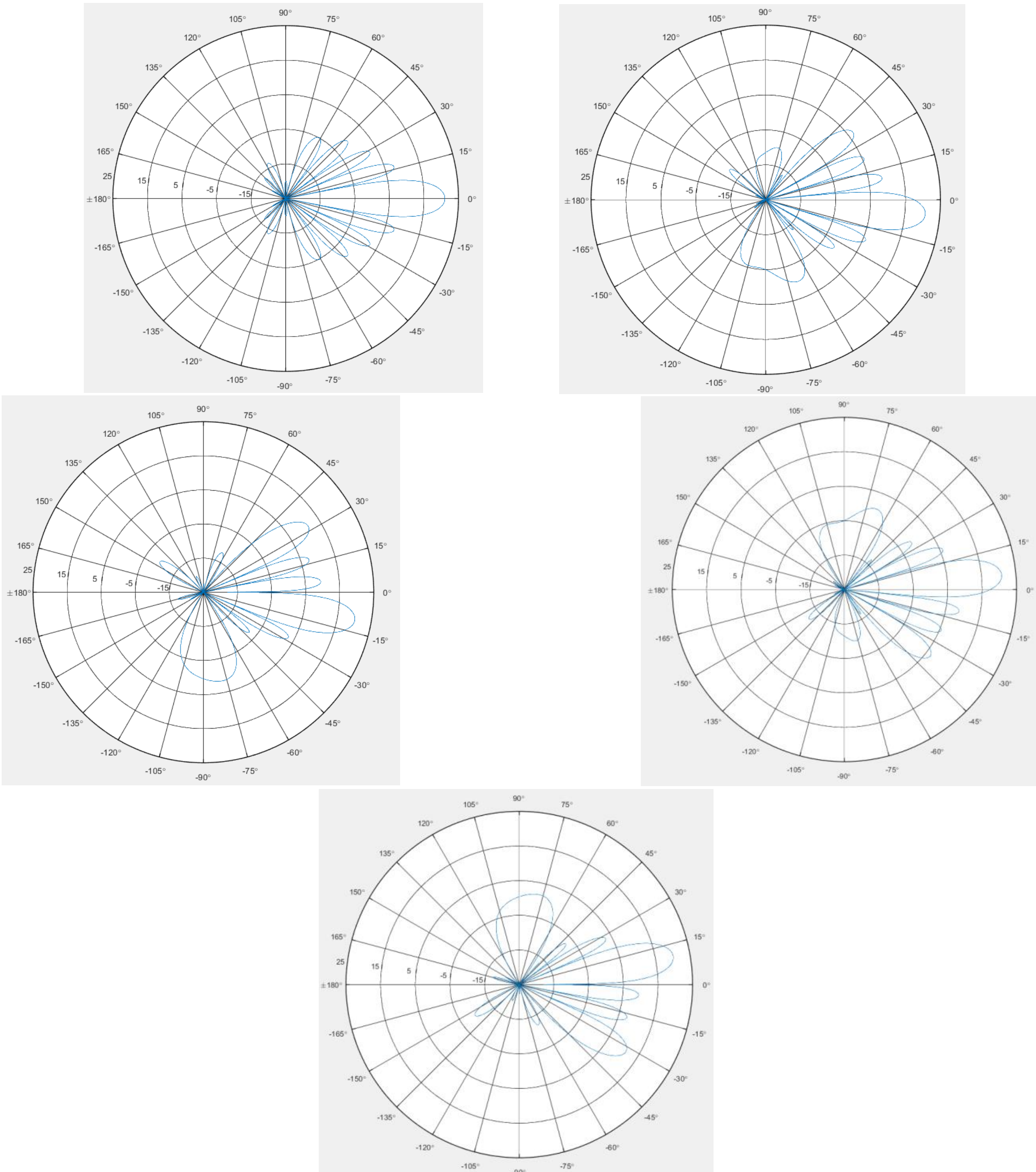

Figure 11: BS2 antenna count: 4. Starting from top-left corner, going clockwise, scan angle: 0, 45, -45, 90, -90

3.0 POWER CALCULATION

3.1 Channel Matrix

In the channel matrix script, the process is much simpler than the other stages. Since the breakpoint distance is not considered in this project, the Friis's Free Space Path Loss equation is used for the furthest MS in the map. Equations used to calculate the h-values in dBs is:

pathloss = 20log($\frac{4pi}{\lambda}$) + 20log(distance to MS in meters); (1)

h = Gtx + Grx – pathloss (2)

Where λ is speed of light (m/s) / frequency (Hz) or 3e8/1900e6, Gtx is the transmit gain that the previous sections discussed about and Grx is the receive gain which is -2 dBi. After attaining the h-values from equation (2), they were converted to linear scale for ease of math operations in later stages.

Because the equations for Gtx are part of the problem statement, they are not included in the report. A caution followed when using the single and multiple element gain equations were, in an event of indeterminate forms the value of the sin terms is set to 1. As for the single element polar gain equation, the equation becomes:

Gtx_single = $\frac{10*Lx*Ly}{\lambda^2}(1 + \frac{\cos(\,angles)}{2})^2$ (3)

And in a similar event, the gain of the linear array becomes:

Gtx = Gtx_single * # antennas in the BS (4)

3.2 Signal-to-Interference-Noise-Ratio calculation algorithm

In this stage of the SINR calculations, the equations used in the previous design project will also be used here to determine if the SINR is above 24 dB or ~251 in linear. In order to simplify calculations, the h-values were converted to linear scale so that it is easier to add the interference and noise in RMS. In order to use a particular power value for BS1, BS2 and BS3, all 61*61*61 or 226,981 permutations were iterated over by a 3-way brute force approach. In each iteration, a matrix of 9 x 9 dimensions was analyzed because of the requirement for 3 links at a given time. The matrix should look like a identity or diagonal matrix but not necessarily with 1's:

x	0	0
0	y	0
0	0	z

Figure 12: Ideal matrix for selected rows of channel matrix.

The values in the matrix other than the x, y and z components shoud be much smaller such that when multiplied with the power in linear scale, the overall SINR will be at or above the cut-off. The three rows chosen for a given time depends on the hard-coded

MS numbers as discussed in the bindings table, e.g. for TS1 the bindings are 1, 15 and 11. Hence, in a given TS, a smaller table of 9 x 9 is created out of the larger 15 x 3 table which stores the h-values of all 3 BSs for all 15 MSs. The following is the logic for computing the SINR:

h11	h12	h13
h21	h22	h23
h31	h32	h33

If $\frac{p1*h11}{p2*h12+p3*h13+N} > 251.1886$ *or* $\frac{p2*h12}{p1*h11+p3*h13+N} > 251.1886$ *or* $\frac{p3*h13}{p1*h11+p2*h12+N} > 251.1886$
AND
If $\frac{p1*h21}{p2*h22+p3*h23+N} > 251.1886$ *or* $\frac{p2*h22}{p1*h21+p3*h23+N} > 251.1886$ *or* $\frac{p3*h23}{p1*h21+p2*h22+N} > 251.1886$
AND
If $\frac{p1*h31}{p2*h32+p3*h33+N} > 251.1886$ *or* $\frac{p2*h32}{p1*h31+p3*h33+N} > 251.1886$ *or* $\frac{p3*h33}{p1*h31+p2*h32+N} > 251.1886$

THEN
add the MS into the valid table,
ELSE
print "invalid"

where p1, p2, and p3 are linear powers representing between -30 dBm to +30 dBm. And the justification for the *or* statements is because any one of the hxy-values in row x can be the largest and most dominating (assuming minimal interference). This is very similar to the last design project where instead of finding pairs, the algorithm is finding triplets in this case.

The logic stated above assumes that all MSs chosen for the timeslot wil have 24 dBs of SINR so as to proceed to the next timeslot, otherwise it fails and informs the developer to change a parameter – scan or physical parameter. A scan would be more preferable to tweak because the impact of changing the scan is localized to that timeslot between the BSs, whereas changing the physical paramaters such as locations and antenna configuration will have an influence throughout the 5 timeslots. Consequently if changing the scan anywhere between -90 and 90 for any BS at a given time does not work, then it will be another round of painstaking work that is required to find the "goldilocks" configuration that is just right. After several days of trial-and-error, the only time this logic passed was when the BS1 and BS3 antenna configurations were at 100 and 80 degrees – a very counterintuitive direction. That particular time slot consisted of MS8, 15, and 6, with scans 90, 90, -90 for BS1, BS2 and BS3 respectively. This is consistent with Figure 10 and 11 showing the 5 gain plots. However, it did not pass for other scans and unfortunately this physical configuration had to be abandoned.

Eventually the decision was made to prioritize the VIPs over the regular public and provide the best SINR possible from the test cases recorded in a large excel sheey. This logic is as follows:

MAXVEC	idx = 1	idx+1	...	226,981
powe1	-30			30
power2	-30			30
power3	-30			30
maxof(h1x)				
maxof(h2x)				
maxof(h3x)				

i. Let the 9 inequalities stated above from the original logic be: a, b, c, d, e, f, g, h, i, where a-i are just the LHS (the fraction part)
ii. Create a 6 x 2266,981 table of power1, power2, power3, max(a-c), max(d-f), max(g-i)
iii. Take the 3 max values from the table by performing max operations horizonally, and get the 3 index values, idxA, idxB and idxC as well
iv. The 10*log(max) are the three SINRs, and MAXVEC(row1, idxA) is power1, MAXVEC(row2, idxB) is power2 and MAXVEC(row3, idxC) is power3 for the 3 SINRs respectively.

Note from the SINR calculations table in Table 2 that BS2 is always the highest. That is because instead of making a 3-way comparison for gains and nulls, a 2 way comparison was made with BS1 and BS3 such that the interference from BS1 and BS3 are minimum at the MS that BS2 will be transmitting to (13, 14, 15, 3, 4). The algorithm is made to be biased as a result of the time constraint, but hopefully with more trial-and-error a better set of SINR calculations can be produced after the completion of the report. Please see code for the modified algorithm (biased).

```
for power1 = -30:30
        p1 = 10^((power1-30)/10);
        for power2 = -30:30
            p2 = 10^((power2-30)/10);
            for power3 = -30:30
                p3 = 10^((power3-30)/10);

                ptable = [p1*table(1,1) / addrms([p2*table(1,2) p3*table(1,3) NOISE])
                    p2*table(1,2) / addrms([p1*table(1,1) p3*table(1,3) NOISE])
                    p3*table(1,3) / addrms([p2*table(1,2) p1*table(1,1) NOISE])
                    p1*table(2,1) / addrms([p2*table(2,2) p3*table(2,3) NOISE])
                    p2*table(2,2) / addrms([p1*table(2,1) p3*table(2,3) NOISE])
                    p3*table(2,3) / addrms([p2*table(2,2) p1*table(2,1) NOISE])
                    p1*table(3,1) / addrms([p2*table(3,2) p3*table(3,3) NOISE])
                    p2*table(3,2) / addrms([p1*table(3,1) p3*table(3,3) NOISE])
                    p3*table(3,3) / addrms([p2*table(3,2) p1*table(3,1) NOISE])];

                maxvec(1,idx) = power1;
                maxvec(2,idx) = power2;
                maxvec(3,idx) = power3;
                maxvec(4,idx) = max(ptable(1:3));
                maxvec(5,idx) = max(ptable(4:6));
                maxvec(6,idx) = max(ptable(7:9));
                idx = idx + 1;
             end
        end
    end
    [m,i] = max(maxvec(4:6,:),[],2);
    SINR = (10.*log10(m))';
    cow.antpower.(basestations{1}) = maxvec(1,i(1));
    cow.antpower.(basestations{2}) = maxvec(2,i(2));
    cow.antpower.(basestations{3}) = maxvec(3,i(3));
```

APPENDIX A

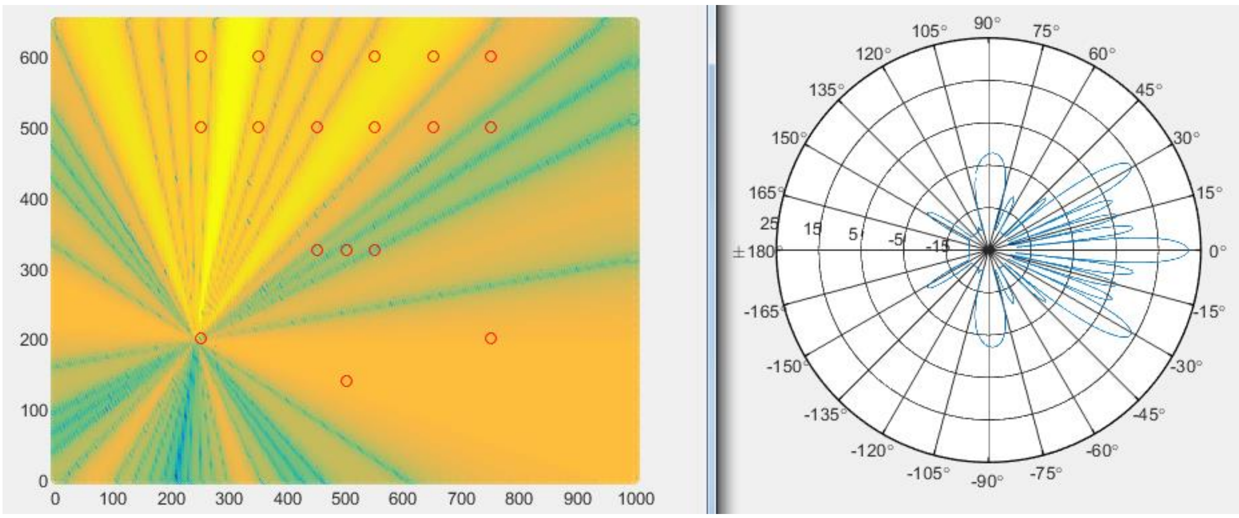

Figure 3: BS1, Antenna Count 5, Spacing 30cm, Alpha 0 degrees

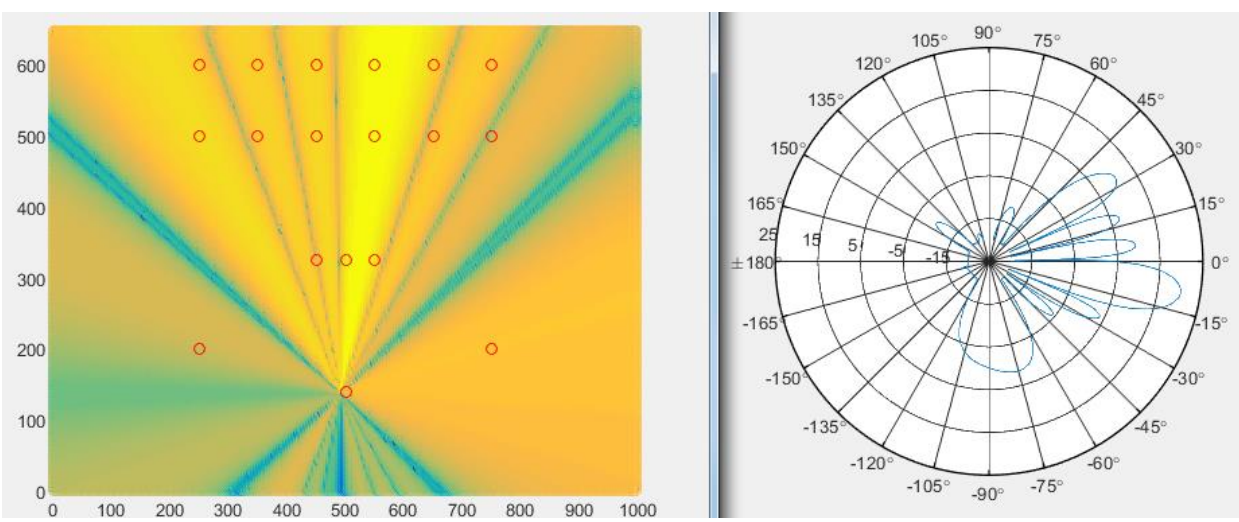

Figure 4: BS2, Antenna Count 4, Spacing 20.1cm, Alpha 80 degrees

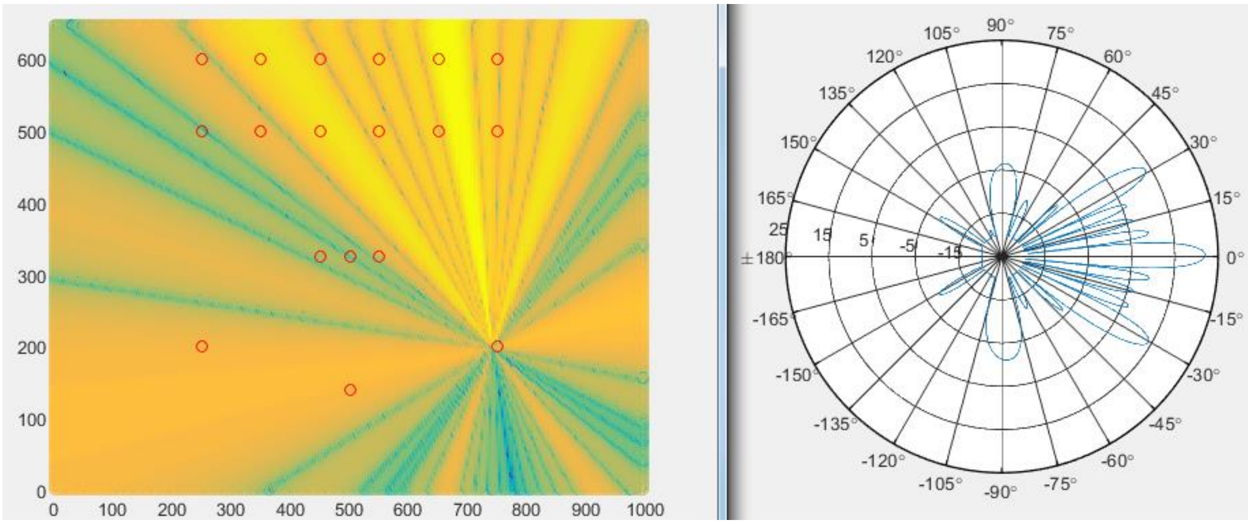

Figure 5: BS3, Antenna Count 5, Spacing 30cm, Alpha -5 degrees

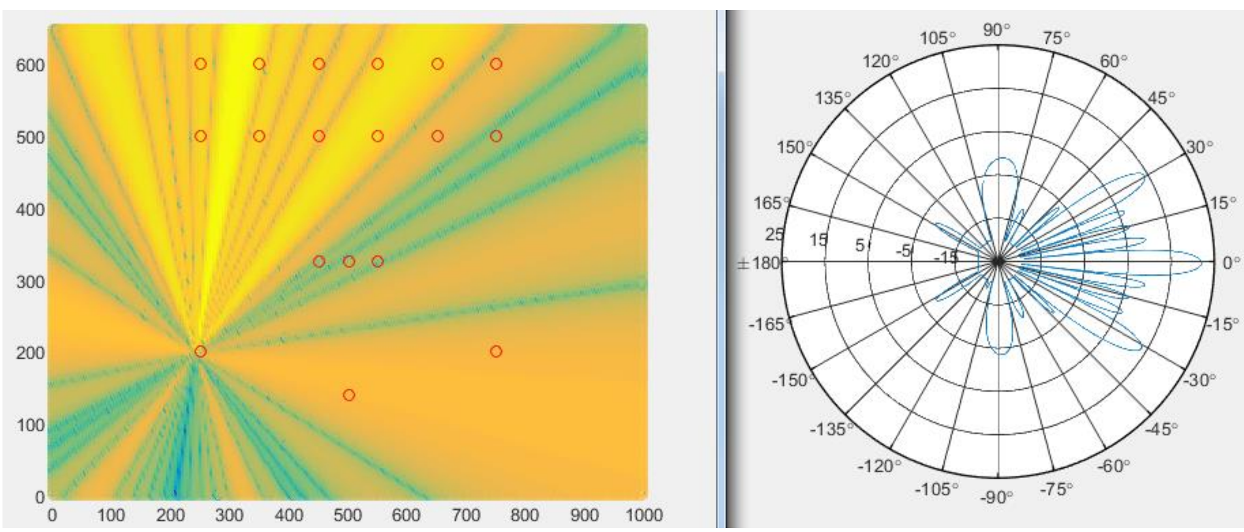

Figure 6: BS1, Antenna Count 5 Spacing 30cm, Alpha 5 degrees

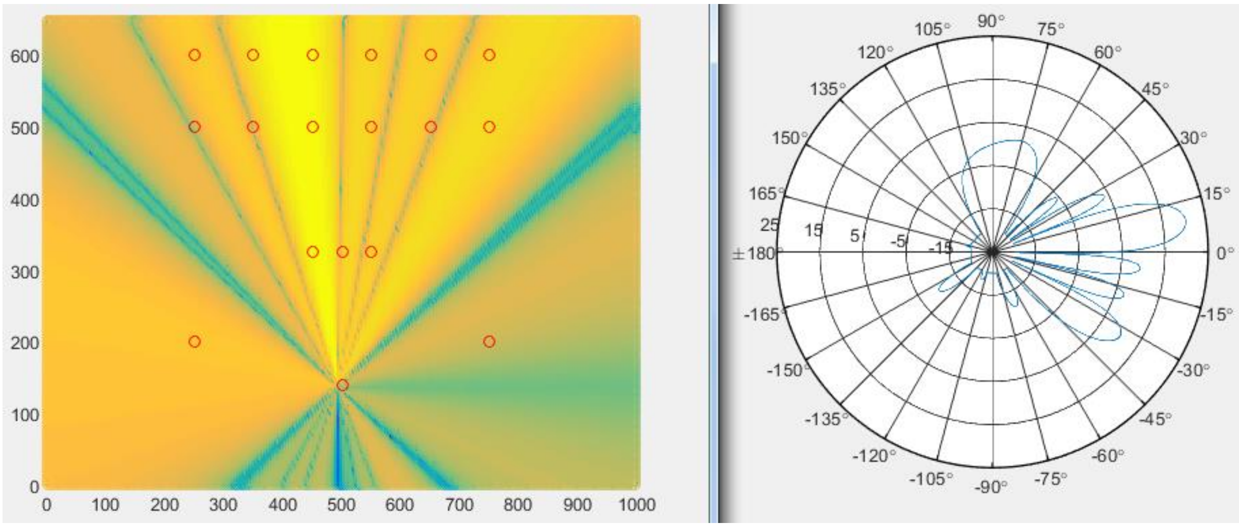

Figure 7: BS2, Antenna Count 4, Spacing 20.1cm, Alpha -80 degrees

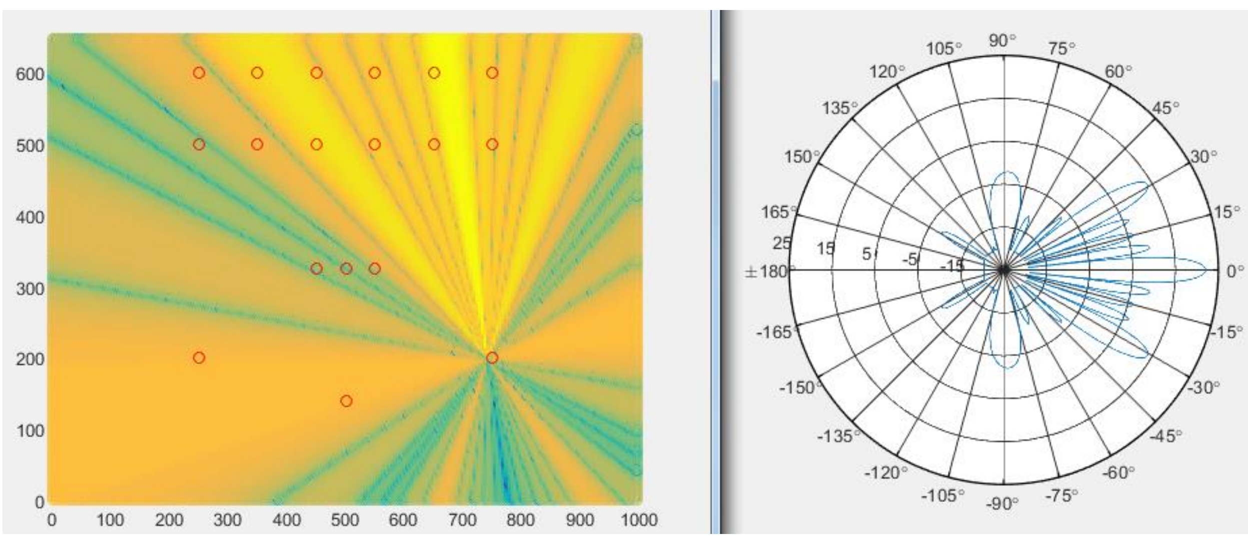

Figure 8: BS3, Antenna Count 5, Spacing 30cm, Alpha 0 degrees

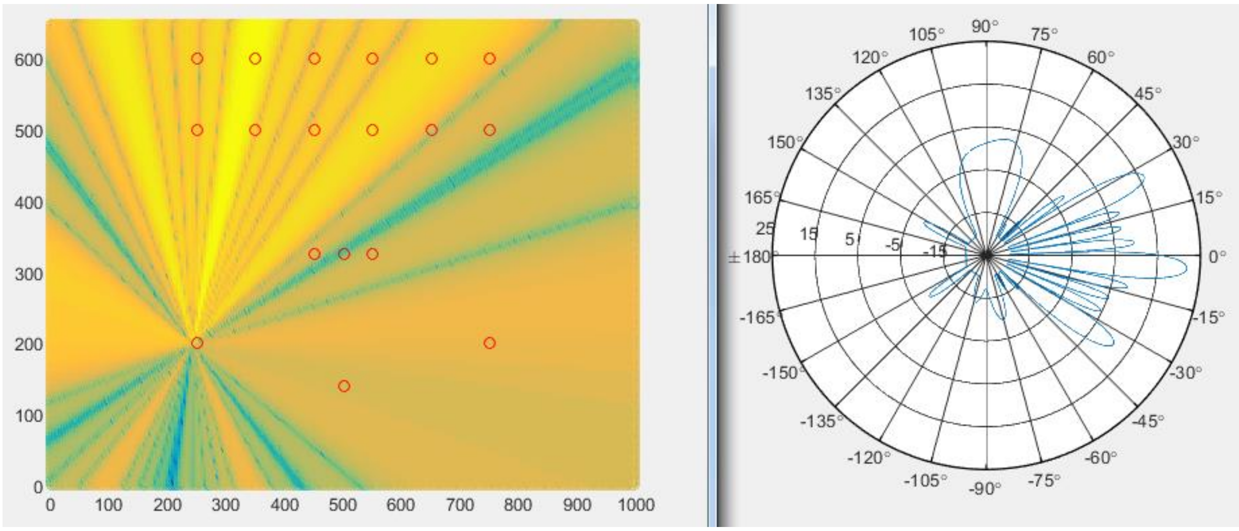

Figure 9: BS1, Antenna Count 5 Spacing 30cm, Alpha 45 degrees

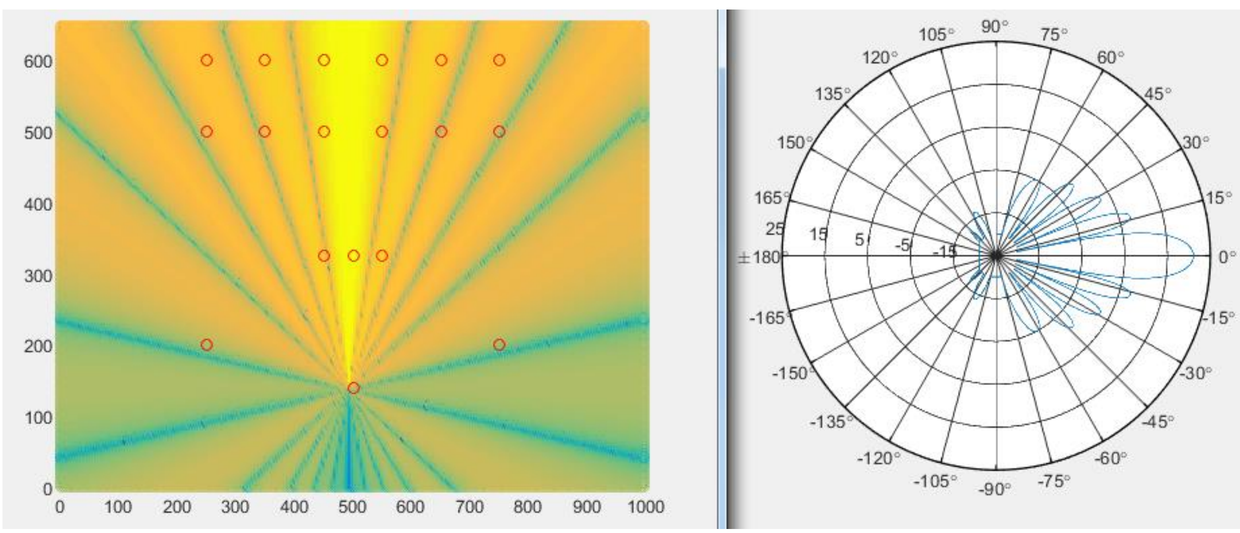

Figure 10: BS2, Antenna Count 4, Spacing 20.1cm, Alpha 0 degrees

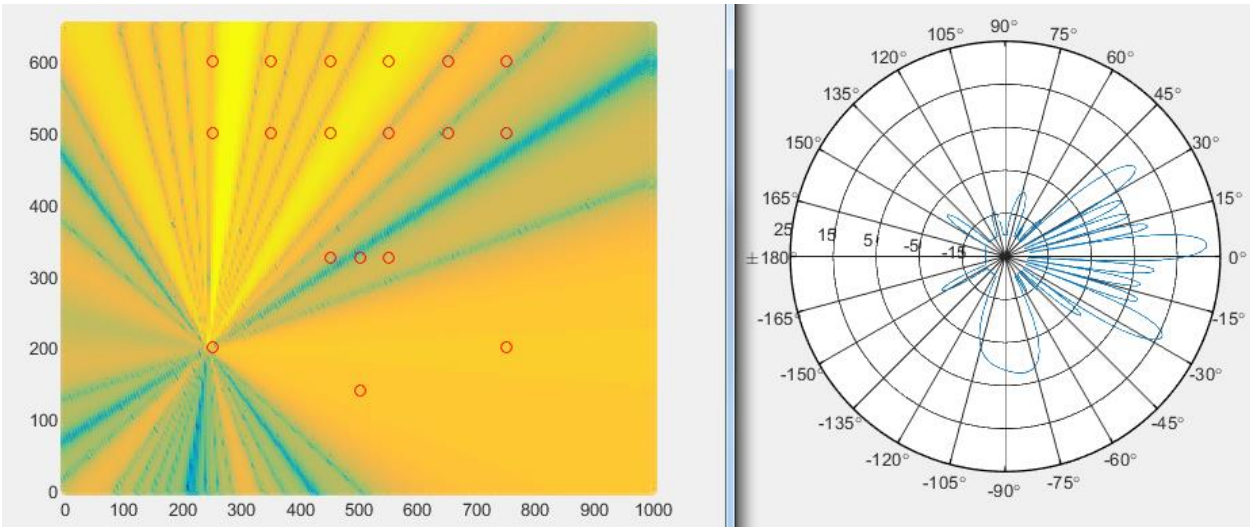

Figure 12: BS1, Antenna Count 5 Spacing 30cm, Alpha -45 degrees

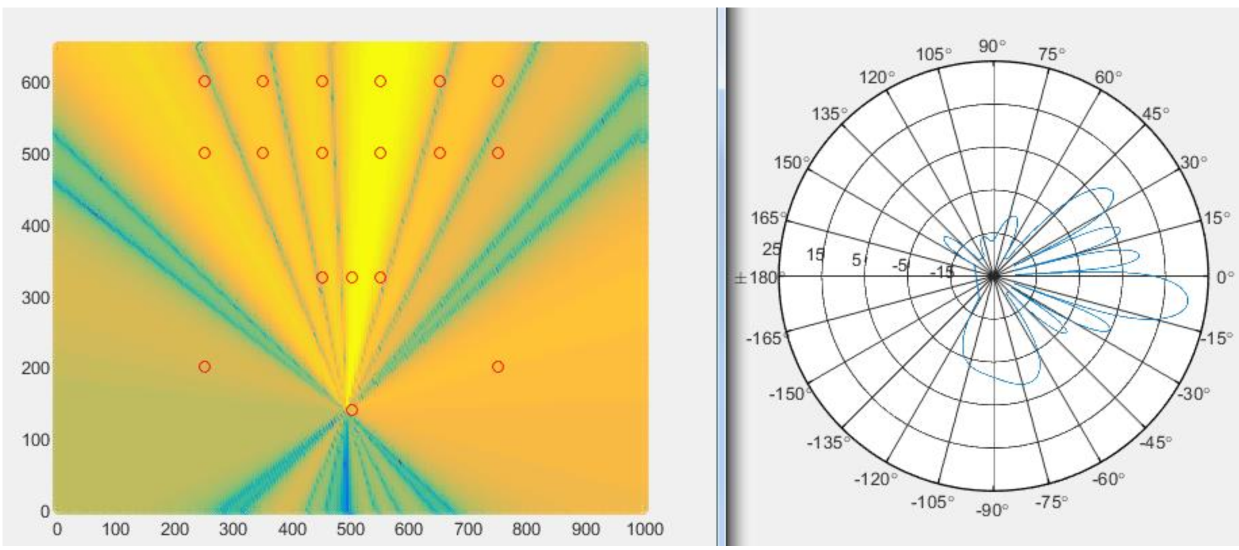

Figure 11: BS2, Antenna Count 5 Spacing 20.1cm, Alpha 65 degrees

Appendix B

BS1	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90
1	7.611045	6.151219	4.002189	0.730999	-5.03822	-32.4273	-6.05492	-0.13149	3.031731	5.005308	6.260492	6.993465	7.293119	7.191356	6.677608	5.694228	4.107816	1.618555	-2.58985
2	9.32622	8.751875	7.627138	5.706192	2.357035	-4.97027	-10.4553	2.035327	7.26145	10.64495	13.13219	15.06906	16.62138	17.88148	18.9058	19.73099	20.38178	20.87514	21.22262
3	1.720416	3.673279	4.818263	5.407786	5.554652	5.304464	4.658274	3.573695	1.945712	-0.4558	-4.20381	-11.4386	-21.7694	-7.57661	-2.75899	-0.0087	1.733393	2.828632	3.434188
4	8.663008	9.564053	10.28195	10.8364	11.24092	11.50451	11.63271	11.62811	11.49062	11.21745	10.80285	10.23754	9.507636	8.592922	7.46377	6.075586	4.357967	2.191189	-0.65324
5	-7.79017	-7.74015	-8.10414	-8.95729	-10.4688	-13.0366	-17.8864	-36.4342	-18.9895	-11.9703	-7.93712	-5.10165	-2.94046	-1.22677	0.158408	1.285127	2.19778	2.925877	3.489557
6	-25.9502	-30.0178	-39.3048	-39.217	-29.6592	-25.3383	-22.6599	-20.8567	-19.6414	-18.8849	-18.531	-18.5699	-19.0343	-20.0149	-21.7144	-24.6273	-30.458	-49.0835	-27.7055
7	7.611045	6.151219	4.002189	0.730999	-5.03822	-32.4273	-6.05492	-0.13149	3.031731	5.005308	6.260492	6.993465	7.293119	7.191356	6.677608	5.694228	4.107816	1.618555	-2.58985
8	14.45692	16.2107	17.62639	18.77776	19.71124	20.45759	21.03774	21.46608	21.75229	21.90241	21.91949	21.80388	21.55324	21.16233	20.62251	19.92075	19.03802	17.94655	16.60473
9	4.002051	5.117935	5.740187	5.941172	5.741959	5.121668	4.008215	2.240302	-0.55454	-5.48249	-20.7591	-8.74906	-1.72473	1.961154	4.330966	5.93829	7.007137	7.63982	7.87712
10	7.640793	5.081088	1.568279	-3.97678	-19.0299	-9.03027	-2.44867	0.695286	2.514219	3.573375	4.098088	4.189473	3.886356	3.184211	2.033846	0.318479	-2.21931	-6.24225	-14.4958
11	8.663008	9.564053	10.28195	10.8364	11.24092	11.50451	11.63271	11.62811	11.49062	11.21745	10.80285	10.23754	9.507636	8.592922	7.46377	6.075586	4.357967	2.191189	-0.65324
12	-7.15903	-8.80665	-11.622	-17.1601	-44.2682	-15.432	-9.1895	-5.41567	-2.71457	-0.63828	1.015101	2.353957	3.443011	4.32367	5.023615	5.561784	5.95115	6.200325	6.314511
13	-31.9498	-47.0994	-29.1771	-23.8045	-20.6822	-18.6164	-17.214	-16.305	-15.8124	-15.7114	-16.019	-16.802	-18.2133	-20.6056	-25.0099	-38.2984	-28.4226	-20.6253	-16.361
14	-29.968	-29.402	-29.2522	-29.5028	-30.1804	-31.3655	-33.2362	-36.2121	-41.6097	-64.5271	-42.6534	-36.3348	-32.8722	-30.6193	-29.0892	-28.0799	-27.4987	-27.3109	-27.5243
15	-64.2893	-31.482	-25.6665	-22.5405	-20.5871	-19.3455	-18.6233	-18.333	-18.444	-18.9677	-19.963	-21.5659	-24.0821	-28.3507	-38.6695	-35.8208	-27.1039	-22.9687	-20.3771
	-90	-85	-80	-75	-70	-65	-60	-55	-50	-45	-40	-35	-30	-25	-20	-15	-10	-5	
1	18.96023	18.00123	16.81952	15.36602	13.56219	11.2716	8.224961	3.759593	-4.79539	-10.4882	0.543612	4.644006	6.908988	8.24544	8.975242	9.239303	9.098508	8.565525	
2	6.648577	7.299817	7.526882	7.353659	6.762047	5.683976	3.966104	1.25726	-3.45944	-16.7771	-7.89685	-0.44977	3.358979	5.791158	7.439008	8.538649	9.197726	9.46017	
3	17.53806	17.59963	17.5287	17.32382	16.98079	16.49217	15.84656	15.02718	14.0096	12.75764	11.21566	9.292872	6.826766	3.480445	-1.65692	-13.5643	-9.27906	-1.69449	
4	-3.00303	-1.70274	-0.87617	-0.45794	-0.43132	-0.82147	-1.70751	-3.26685	-5.92027	-10.9999	-34.6828	-11.0781	-4.36144	-0.42564	2.360072	4.49006	6.181688	7.549982	
5	-12.4169	-10.9877	-10.1209	-9.70542	-9.69573	-10.0909	-10.9337	-12.3315	-14.5207	-18.0982	-25.3611	-33.6016	-20.2645	-15.328	-12.3805	-10.4156	-9.0838	-8.23164	
6	-19.1358	-20.065	-21.4929	-23.5954	-26.783	-32.3262	-53.125	-34.3094	-28.1061	-24.8493	-22.8256	-21.5368	-20.7775	-20.4548	-20.5342	-21.0238	-21.9782	-23.5252	
7	18.96023	18.00123	16.81952	15.36602	13.56219	11.2716	8.224961	3.759593	-4.79539	-10.4882	0.543612	4.644006	6.908988	8.24544	8.975242	9.239303	9.098508	8.565525	
8	-3.40591	1.878162	4.966055	7.012142	8.400957	9.299059	9.781892	9.873209	9.55499	8.758769	7.328842	4.904392	0.420718	-13.5842	-2.54257	5.092464	9.310757	12.23856	
9	-7.46438	0.450412	3.946212	5.937844	7.10725	7.714993	7.877301	7.641713	7.01082	5.944053	4.339455	1.97381	-1.70396	-8.70128	-20.9492	-5.51309	-0.57016	2.230681	
10	11.68338	12.88324	13.85743	14.63926	15.25132	15.70911	16.02315	16.20015	16.24372	16.15474	15.93142	15.56913	15.05997	14.39189	13.54724	12.50029	11.21278	9.625498	
11	-3.00303	-1.70274	-0.87617	-0.45794	-0.43132	-0.82147	-1.70751	-3.26685	-5.92027	-10.9999	-34.6828	-11.0781	-4.36144	-0.42564	2.360072	4.49006	6.181688	7.549982	
12	-7.68209	-7.73778	-8.20209	-9.12636	-10.6325	-12.992	-16.9191	-25.5641	-27.5389	-17.226	-12.7558	-10.0095	-8.16484	-6.91929	-6.13766	-5.7606	-5.77578	-6.21309	
13	-18.1254	-19.9603	-22.6904	-27.1248	-37.249	-35.7614	-26.9742	-23.0148	-20.6401	-19.1245	-18.1932	-17.723	-17.6617	-18.0031	-18.7838	-20.1004	-22.1656	-25.5013	
14	-34.1617	-30.9293	-29.0659	-27.978	-27.4314	-27.3212	-27.6064	-28.2894	-29.4171	-31.1023	-33.5926	-37.5182	-45.3934	-51.8504	-39.8627	-35.3231	-32.6887	-31.0147	
15	-14.4168	-17.4861	-21.9998	-30.7498	-35.5841	-24.9534	-20.9172	-18.6799	-17.36	-16.6421	-16.3878	-16.5376	-17.0798	-18.0447	-19.5177	-21.6862	-24.9927	-30.8598	

BS2	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90
1	-9.38935	-16.5059	-24.3096	-10.7764	-5.47749	-2.08378	0.414416	2.378129	3.978645	5.31089	6.432964	7.382933	8.186983	8.863761	9.426861	9.886329	10.2496	10.52213	10.70775
2	-12.7096	-22.1277	-8.26607	-3.2943	-0.30397	1.746314	3.220594	4.286166	5.030288	5.499972	5.717951	5.689028	5.401184	4.82211	3.888945	2.483291	0.364726	-3.05375	-9.97823
3	12.34532	10.42836	8.002007	4.737343	-0.24703	-11.6006	-8.13811	-0.14122	3.554368	5.804735	7.290712	8.280588	8.904132	9.229904	9.293627	9.109813	8.675959	7.972058	6.955119
4	12.34532	13.91231	15.21887	16.32026	17.25293	18.04206	18.70565	19.25682	19.7053	20.05828	20.32101	20.49718	20.58917	20.59818	20.52434	20.36668	20.12308	19.79016	19.36303
5	-12.7096	-5.42453	-1.65758	0.799489	2.535588	3.792875	4.69135	5.295384	5.638731	5.734822	5.580258	5.153733	4.409706	3.262473	1.5464	-1.10074	-5.71832	-19.2125	-9.41108
6	-9.38935	-5.91799	-3.77378	-2.35235	-1.40796	-0.82004	-0.52463	-0.48873	-0.69983	-1.16228	-1.89825	-2.95391	-4.41505	-6.44573	-9.40122	-14.2865	-27.4267	-19.2475	-11.77
7	2.749133	3.087645	3.178991	3.019452	2.587334	1.836519	0.680254	-1.04936	-3.72077	-8.3996	-22.4502	-11.7388	-4.17836	-0.02232	2.868938	5.07788	6.849508	8.310415	9.534597
8	6.715927	6.066855	5.115356	3.790301	1.958186	-0.65633	-4.75445	-13.3362	-16.3034	-5.71871	-1.21465	1.581235	3.520522	4.919689	5.928296	6.625261	7.052901	7.230757	7.160933
9	16.19877	15.04124	13.66467	12.00582	9.959754	7.332676	3.699896	-2.22718	-23.7472	-4.64366	1.365251	4.509858	6.491787	7.811992	8.683768	9.214055	9.459817	9.44973	9.193078
10	16.19877	17.17824	18.00762	18.70673	19.28995	19.76791	20.14846	20.4374	20.63883	20.7555	20.78895	20.73963	20.60689	20.38896	20.08285	19.68415	19.18672	18.58223	17.85943
11	6.715927	7.100366	7.236541	7.123755	6.743884	6.056076	4.982949	3.376587	0.922301	-3.22951	-13.4491	-10.1014	-1.12073	3.398062	6.459176	8.768815	10.60848	12.11941	13.38264
12	2.749133	2.150365	1.258048	0.008566	-1.71659	-4.15602	-7.887	-15.0449	-25.3726	-10.9543	-5.92105	-2.90625	-0.84227	0.641162	1.713614	2.46339	2.938084	3.160815	3.136707
13	7.21268	7.74002	7.983223	7.970751	7.711721	7.198545	6.405074	5.279091	3.723834	1.551113	-1.66194	-7.21676	-29.1899	-8.7042	-2.38553	1.090112	3.401078	5.045486	6.23681
14	21.08415	21.04276	20.91806	20.70839	20.41089	20.02129	19.53363	18.93983	18.22894	17.38615	16.39101	15.21455	13.81416	12.1237	10.03247	7.333404	3.562631	-2.75732	-306.187
15	7.21268	6.344585	5.029455	3.055528	-0.07344	-6.0337	-26.1469	-3.78442	2.179723	5.825885	8.460021	10.51054	12.17252	13.55145	14.71086	15.69189	16.52261	17.2229	17.80718
	-90	-85	-80	-75	-70	-65	-60	-55	-50	-45	-40	-35	-30	-25	-20	-15	-10	-5	
1	-11.77	-19.2475	-27.4267	-14.2865	-9.40122	-6.44573	-4.41505	-2.95391	-1.89825	-1.16228	-0.69983	-0.48873	-0.52463	-0.82004	-1.40796	-2.35235	-3.77378	-5.91799	
2	-9.41108	-19.2125	-5.71832	-1.10074	1.5464	3.262473	4.409706	5.153733	5.580258	5.734822	5.638731	5.295384	4.69135	3.792875	2.535588	0.799489	-1.65758	-5.42453	
3	19.36303	19.79016	20.12308	20.36668	20.52434	20.59818	20.58917	20.49718	20.32101	20.05828	19.7053	19.25682	18.70565	18.04206	17.25293	16.32026	15.21887	13.91231	
4	6.955119	7.972058	8.675959	9.109813	9.293627	9.229904	8.904132	8.280588	7.290712	5.804735	3.554368	-0.14122	-8.13811	-11.6006	-0.24703	4.737343	8.002007	10.42836	
5	-9.97823	-3.05375	0.364726	2.483291	3.888945	4.82211	5.401184	5.689028	5.717951	5.499972	5.030288	4.286166	3.220594	1.746314	-0.30397	-3.2943	-8.26607	-22.1277	
6	10.70775	10.52213	10.2496	9.886329	9.426861	8.863761	8.186983	7.382933	6.432964	5.31089	3.978645	2.378129	0.414416	-2.08378	-5.47749	-10.7764	-24.3096	-16.5059	
7	3.136707	3.160815	2.938084	2.46339	1.713614	0.641162	-0.84227	-2.90625	-5.92105	-10.9543	-25.3726	-15.0449	-7.887	-4.15602	-1.71659	0.008566	1.258048	2.150365	
8	13.38264	12.11941	10.60848	8.768815	6.459176	3.398062	-1.12073	-10.1014	-13.4491	-3.22951	0.922301	3.376587	4.982949	6.056076	6.743884	7.123755	7.236541	7.100366	
9	17.85943	18.58223	19.18672	19.68415	20.08285	20.38896	20.60689	20.73963	20.78895	20.7555	20.63883	20.4374	20.14846	19.76791	19.28995	18.70673	18.00762	17.17824	
10	9.193078	9.44973	9.459817	9.214055	8.683768	7.811992	6.491787	4.509858	1.365251	-4.64366	-23.7472	-2.22718	3.699896	7.332676	9.959754	12.00582	13.66467	15.04124	
11	7.160933	7.230757	7.052901	6.625261	5.928296	4.919689	3.520522	1.581235	-1.21465	-5.71871	-16.3034	-13.3362	-4.75445	-0.65633	1.958186	3.790301	5.115356	6.066855	
12	9.534597	8.310415	6.849508	5.07788	2.868938	-0.02232	-4.17836	-11.7388	-22.4502	-8.3996	-3.72077	-1.04936	0.680254	1.836519	2.587334	3.019452	3.178991	3.087645	
13	17.80718	17.2229	16.52261	15.69189	14.71086	13.55145	12.17252	10.51054	8.460021	5.825885	2.179723	-3.78442	-26.1469	-6.0337	-0.07344	3.055528	5.029455	6.344585	
14	-306.187	-2.75732	3.562631	7.333404	10.03247	12.1237	13.81416	15.21455	16.39101	17.38615	18.22894	18.93983	19.53363	20.02129	20.41089	20.70839	20.91806	21.04276	
15	6.23681	5.045486	3.401078	1.090112	-2.38553	-8.7042	-29.1899	-7.21676	-1.66194	1.551113	3.723834	5.279091	6.405074	7.198545	7.711721	7.970751	7.983223	7.74002	

BS3	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90
1	-25.9502	-23.5252	-21.9782	-21.0238	-20.5342	-20.4548	-20.7775	-21.5368	-22.8256	-24.8493	-28.1061	-34.3094	-53.125	-32.3262	-26.783	-23.5954	-21.4929	-20.065	-19.1358
2	-7.79017	-8.23164	-9.0838	-10.4156	-12.3805	-15.328	-20.2645	-33.6016	-25.3611	-18.0982	-14.5207	-12.3315	-10.9337	-10.0909	-9.69573	-9.70542	-10.1209	-10.9877	-12.4169
3	8.663008	7.549982	6.181688	4.49006	2.360072	-0.42564	-4.36144	-11.0781	-34.6828	-10.9999	-5.92027	-3.26685	-1.70751	-0.82147	-0.43132	-0.45794	-0.87617	-1.70274	-3.00303
4	1.720416	-1.69449	-9.27906	-13.5643	-1.65692	3.480445	6.826766	9.292872	11.21566	12.75764	14.0096	15.02718	15.84656	16.49217	16.98079	17.32382	17.5287	17.59963	17.53806
5	9.32622	9.46017	9.197726	8.538649	7.439008	5.791158	3.358979	-0.44977	-7.89685	-16.7771	-3.45944	1.25726	3.966104	5.683976	6.762047	7.353659	7.526882	7.299817	6.648577
6	7.611045	8.565525	9.098508	9.239303	8.975242	8.24544	6.908988	4.644006	0.543612	-10.4882	-4.79539	3.759593	8.224961	11.2716	13.56219	15.36602	16.81952	18.00123	18.96023
7	-7.15903	-6.21309	-5.77578	-5.7606	-6.13766	-6.91929	-8.16484	-10.0095	-12.7558	-17.226	-27.5389	-25.5641	-16.9191	-12.992	-10.6325	-9.12636	-8.20209	-7.73778	-7.68209
8	8.663008	7.549982	6.181688	4.49006	2.360072	-0.42564	-4.36144	-11.0781	-34.6828	-10.9999	-5.92027	-3.26685	-1.70751	-0.82147	-0.43132	-0.45794	-0.87617	-1.70274	-3.00303
9	7.640793	9.625498	11.21278	12.50029	13.54724	14.39189	15.05997	15.56913	15.93142	16.15474	16.24372	16.20015	16.02315	15.70911	15.25132	14.63926	13.85743	12.88324	11.68338
10	4.002051	2.230681	-0.57016	-5.51309	-20.9492	-8.70128	-1.70396	1.97381	4.339455	5.944053	7.01082	7.641713	7.877301	7.714993	7.10725	5.937844	3.946212	0.450412	-7.46438
11	14.45692	12.23856	9.310757	5.092464	-2.54257	-13.5842	0.420718	4.904392	7.328842	8.758769	9.55499	9.873209	9.781892	9.299059	8.400957	7.012142	4.966055	1.878162	-3.40591
12	7.611045	8.565525	9.098508	9.239303	8.975242	8.24544	6.908988	4.644006	0.543612	-10.4882	-4.79539	3.759593	8.224961	11.2716	13.56219	15.36602	16.81952	18.00123	18.96023
13	-64.2893	-30.8598	-24.9927	-21.6862	-19.5177	-18.0447	-17.0798	-16.5376	-16.3878	-16.6421	-17.36	-18.6799	-20.9172	-24.9534	-35.5841	-30.7498	-21.9998	-17.4861	-14.4168
14	-29.968	-31.0147	-32.6887	-35.3231	-39.8627	-51.8504	-45.3934	-37.5182	-33.5926	-31.1023	-29.4171	-28.2894	-27.6064	-27.3212	-27.4314	-27.978	-29.0659	-30.9293	-34.1617
15	-31.9498	-25.5013	-22.1656	-20.1004	-18.7838	-18.0031	-17.6617	-17.723	-18.1932	-19.1245	-20.6401	-23.0148	-26.9742	-35.7614	-37.249	-27.1248	-22.6904	-19.9603	-18.1254
	-90	-85	-80	-75	-70	-65	-60	-55	-50	-45	-40	-35	-30	-25	-20	-15	-10	-5	
1	-27.7055	-49.0835	-30.458	-24.6273	-21.7144	-20.0149	-19.0343	-18.5699	-18.531	-18.8849	-19.6414	-20.8567	-22.6599	-25.3383	-29.6592	-39.217	-39.3048	-30.0178	
2	3.489557	2.925877	2.19778	1.285127	0.158408	-1.22677	-2.94046	-5.10165	-7.93712	-11.9703	-18.9895	-36.4342	-17.8864	-13.0366	-10.4688	-8.95729	-8.10414	-7.74015	
3	-0.65324	2.191189	4.357967	6.075586	7.46377	8.592922	9.507636	10.23754	10.80285	11.21745	11.49062	11.62811	11.63271	11.50451	11.24092	10.8364	10.28195	9.564053	
4	3.434188	2.828632	1.733393	-0.0087	-2.75899	-7.57661	-21.7694	-11.4386	-4.20381	-0.4558	1.945712	3.573695	4.658274	5.304464	5.554652	5.407786	4.818263	3.673279	
5	21.22262	20.87514	20.38178	19.73099	18.9058	17.88148	16.62138	15.06906	13.13219	10.64495	7.26145	2.035327	-10.4553	-4.97027	2.357035	5.706192	7.627138	8.751875	
6	-2.58985	1.618555	4.107816	5.694228	6.677608	7.191356	7.293119	6.993465	6.260492	5.005308	3.031731	-0.13149	-6.05492	-32.4273	-5.03822	0.730999	4.002189	6.151219	
7	6.314511	6.200325	5.95115	5.561784	5.023615	4.32367	3.443011	2.353957	1.015101	-0.63828	-2.71457	-5.41567	-9.1895	-15.432	-44.2682	-17.1601	-11.622	-8.80665	
8	-0.65324	2.191189	4.357967	6.075586	7.46377	8.592922	9.507636	10.23754	10.80285	11.21745	11.49062	11.62811	11.63271	11.50451	11.24092	10.8364	10.28195	9.564053	
9	-14.4958	-6.24225	-2.21931	0.318479	2.033846	3.184211	3.886356	4.189473	4.098088	3.573375	2.514219	0.695286	-2.44867	-9.03027	-19.0299	-3.97678	1.568279	5.081088	
10	7.87712	7.63982	7.007137	5.93829	4.330966	1.961154	-1.72473	-8.74906	-20.7591	-5.48249	-0.55454	2.240302	4.008215	5.121668	5.741959	5.941172	5.740187	5.117935	
11	16.60473	17.94655	19.03802	19.92075	20.62251	21.16233	21.55324	21.80388	21.91949	21.90241	21.75229	21.46608	21.03774	20.45759	19.71124	18.77776	17.62639	16.2107	
12	-2.58985	1.618555	4.107816	5.694228	6.677608	7.191356	7.293119	6.993465	6.260492	5.005308	3.031731	-0.13149	-6.05492	-32.4273	-5.03822	0.730999	4.002189	6.151219	
13	-20.3771	-22.9687	-27.1039	-35.8208	-38.6695	-28.3507	-24.0821	-21.5659	-19.963	-18.9677	-18.444	-18.333	-18.6233	-19.3455	-20.5871	-22.5405	-25.6665	-31.482	
14	-27.5243	-27.3109	-27.4987	-28.0799	-29.0892	-30.6193	-32.8722	-36.3348	-42.6534	-64.5271	-41.6097	-36.2121	-33.2362	-31.3655	-30.1804	-29.5028	-29.2522	-29.402	
15	-16.361	-20.6253	-28.4226	-38.2984	-25.0099	-20.6056	-18.2133	-16.802	-16.019	-15.7114	-15.8124	-16.305	-17.214	-18.6164	-20.6822	-23.8045	-29.1771	-47.0994	