# Probability

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The chances of winning the lottery is 50%. You either win or don't

# What is probability?

Definition: Probability is the likelihood of an outcome.

ChatGPT's response: Probability is a measure of how likely an event is to occur. It quantifies uncertainty and is expressed as a number between 0 and 1.

Event:

An event is a set of outcomes of an experiment (a subset of the sample space) to which a probability is assigned.

Sample space:

The sample space , denoted S, is the collection of all possible outcomes of a random study.

# How to calculate probability?

$$P(\text{Event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

What are my favourable outcomes?

What are the possible outcomes?

# Example

A 4 digit PIN is selected. What is the probability that there are no repeated digits?

# Problem: 5 minutes

I need to choose a password for a computer account. The rule is that the password must consist of two lowercase letters (a to z) followed by one capital letter (A to Z) followed by four digits (0,1,⋯,9). For example, the following is a valid password: **ejT3018**

1. Find the total number of possible passwords, N.

2. A hacker has been able to write a program that randomly and independently generates 10^8 passwords according to the above rule. Note that the same password could be generated more than once. If one of the randomly chosen passwords matches my password, then he can access my account information. What is the probability that he is successful in accessing my account information?

# Problem: 5 minutes

10 passengers get on an airport shuttle at the airport. The shuttle has a route that includes 5 hotels, and each passenger gets off the shuttle at his/her hotel. The driver records how many passengers leave the shuttle at each hotel. How many different possibilities exist?

# Problem: 5 minutes

Fifteen dogs are available to use in a study to compare three different diets. Each of the diets (let's say, A, B, C) is to be used on five randomly selected dogs.. In how many ways can the diets be assigned to the dogs?

# Types of events:

**Independent Events**

The probability one event occurs in no way affects the probability of the other event occurring. Eg: Owning a dog and growing your own herb garden.

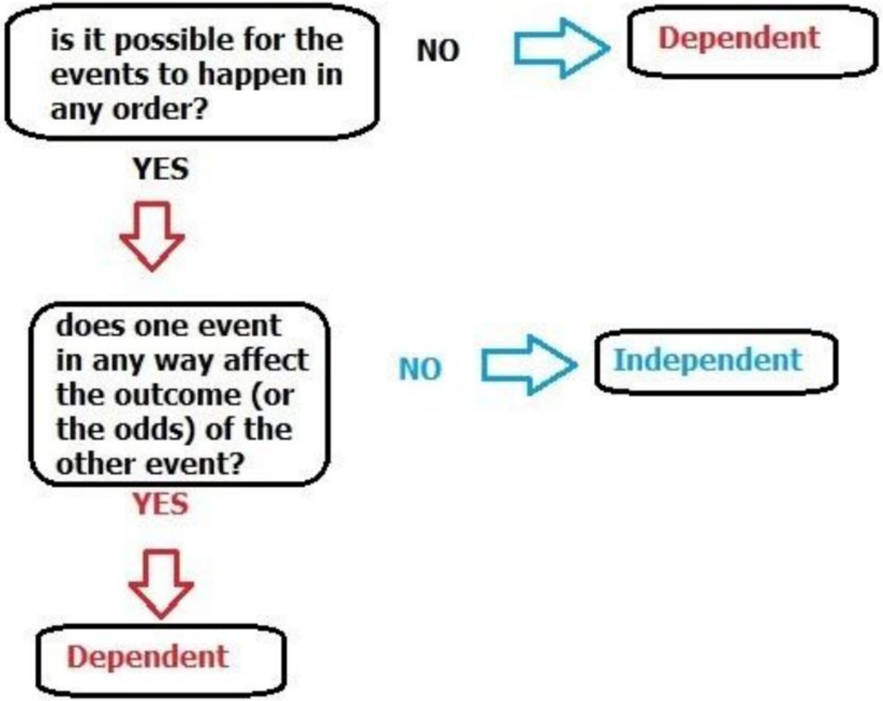
**Dependent Events**

When two events are said to be dependent, the probability of one event occurring influences the likelihood of the other event. Eg: Robbing a bank and going to jail.

**Mutual Exclusivity**

Two events are said to be mutually exclusive if they can't occur at the same time. For a given sample space, its either one or the other but not both. Eg: Head or Tail.

## Dependent or Independent?



# Example:

| | No wind | Some wind | Strong wind | Storm |
|---|---|---|---|---|
| No rain | 0.1 | 0.2 | 0.05 | 0.01 |
| Light rain | 0.05 | 0.1 | 0.15 | 0.04 |
| Heavy rain | 0.05 | 0.1 | 0.1 | 0.05 |

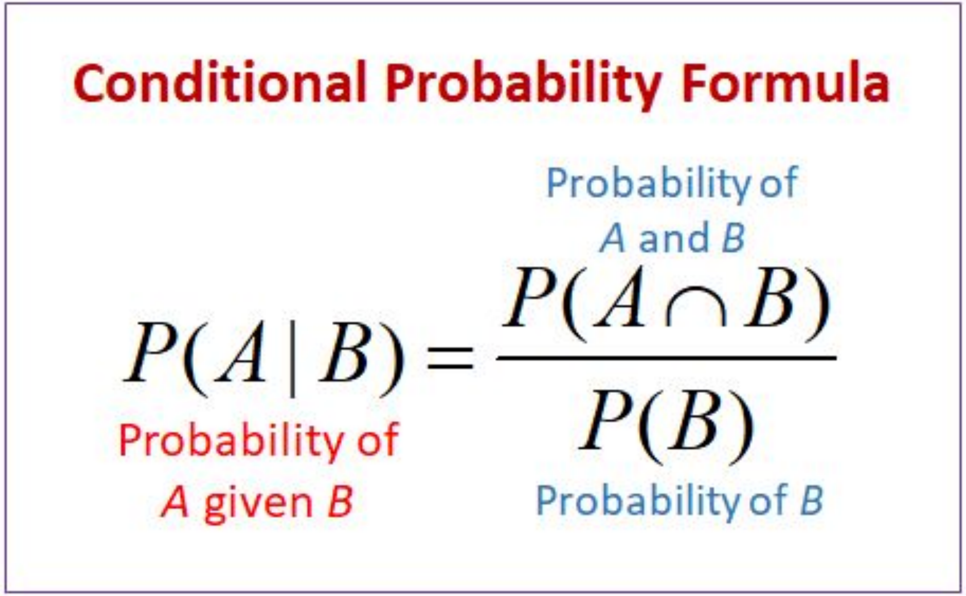
First we marginalize them and then combine to get conditional probabilities.

To find:

P(No wind | Light rain)

P(Light rain | No wind)

# Conditional probability: P(A | B)



# Problem: 5 minutes

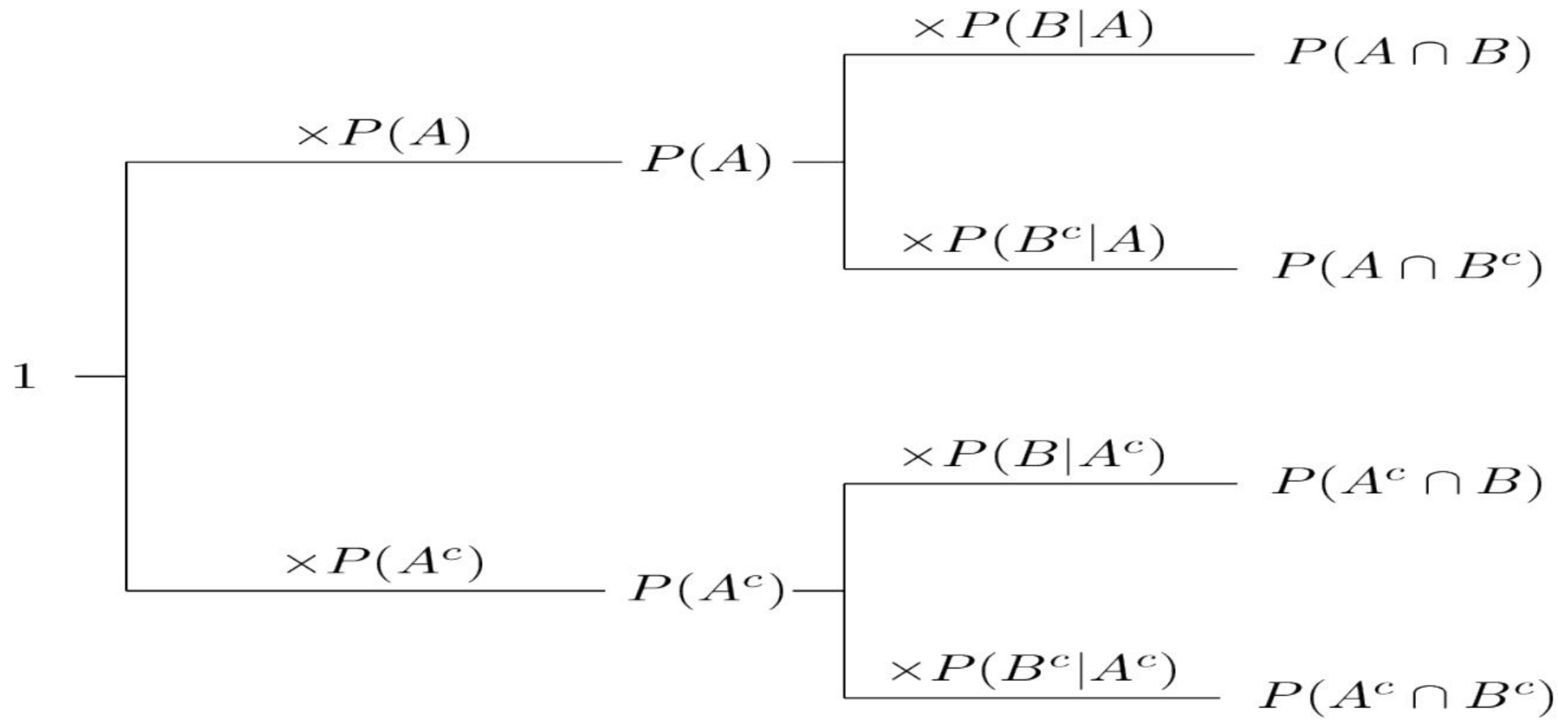
Consider a family that has two children. We are interested in the children's genders.

Our sample space is S={(G,G),(G,B),(B,G),(B,B)}. Also assume that all four possible outcomes are equally likely.

1. What is the probability that both children are girls given that the first child is a girl?

2. We ask the father: "Do you have at least one daughter?" He responds "Yes!" Given this extra information, what is the probability that both children are girls? In other words, what is the probability that both children are girls given that we know at least one of them is a girl?

# Chain rule:


$1$
$\times P(A)$
$P(A)$
$\times P(B|A)$
$P(A \cap B)$
$\times P(B^c|A)$
$P(A \cap B^c)$
$\times P(A^c)$
$P(A^c)$
$\times P(B|A^c)$
$P(A^c \cap B)$
$\times P(B^c|A^c)$
$P(A^c \cap B^c)$

# Problem: 5 minutes

In a factory there are 100 units of a certain product, 55 of which are defective. We pick three units from the 100 units at random. What is the probability that none of them are defective?

# Bayes' theorem

**LIKELIHOOD**
The probability of "B" being True, given "A" is True

**PRIOR**
The probability "A" being True. This is the knowledge.

$$P(A|B) = \frac{P(B|A).P(A)}{P(B)}$$

**POSTERIOR**
The probability of "A" being True, given "B" is True

**MARGINALIZATION**
The probability "B" being True.

# Example

In a particular pain clinic, 10% of patients are prescribed narcotic pain killers. Overall, five percent of the clinic's patients are addicted to narcotics (including pain killers and illegal substances). Out of all the people prescribed pain pills, 8% are addicts. If a patient is an addict, what is the probability that they will be prescribed pain pills?

# Problem: 10 minutes

Data:

1. 100 out of 10,000 women aged forty who participate in a routine screening have breast cancer
2. 80 of every 100 women with breast cancer will get positive tests
3. 950 out of 9,900 women without breast cancer will also get positive tests

Problem:

If 10,000 women in this age group undergo a routine screening, about what

fraction of women with positive tests will actually have breast cancer?

# Problem: 10 minutes

A factory production line is manufacturing bolts using three machines, A, B and C. Of the total output, machine A is responsible for 25%, machine B for 35% and machine C for the rest. It is known from previous experience with the machines that 5% of the output from machine A is defective, 4% from machine B and 2% from machine C. A bolt is chosen at random from the production line and found to be defective. What is the probability that it came from :

1. Machine A.
2. Machine B
3. Machine C

# Applications:

1. Sensor fusion
2. Localisation
3. Path planning and navigation
4. Object detection and recognition

# Summary

- Probability.
- Events and sample space.
- Probability of an event.
- Types of events.
- Conditional probability.
- Chain rule.
- Bayes’ theorem.
- Applications of probability.

# Some extras:

What is a random variable.

What is a distribution.

What are mostly used distributions.

What is PDF, PMF, CDF.

What is mean, variance , covariance.

# References:

- These slides have been adapted from the “Probability Overview” slides by Sathiya Ramesh.
- ChatGPT has been used for some problems, definitions and formulae.