Distance from a point to a convex hull and $\boldsymbol{QP}$ problem

Distance from a reduced d-1 dimension point to a reduced dimension d-1 dimension region (represented as a list of d dimension vectors)

Given $\boldsymbol{H} = [\overrightarrow{h_1}\ \overrightarrow{h_2}\ \dots\ \overrightarrow{h_a}]$, $,\vec{w} = \begin{bmatrix} w[1] \\ w[2] \\ \dots \\ w[d-1] \end{bmatrix}$, $\overrightarrow{h_\iota}$ are all d dimension column vectors

Minimize $\mathrm{obj}^2 = |w - x|^2 = (w-x)^T \cdot (w-x) = x^T \cdot x - 2w^T \cdot x + w^T \cdot w$

Subject to, $\overrightarrow{h_{\iota,\ 1:d-1}}^T \cdot x \le h_{i,d}$, $I_{d\times d} \cdot x \ge \vec{\mathbf{0}}_d$, $\vec{\mathbf{1}}_d^T \cdot x < 1$

Where:

$\boldsymbol{H}$ **is the matrix consisting of constrains (half-spaces);**

$\boldsymbol{w}$ **is the point;**

$\overrightarrow{h_{\iota,\ 1:d-1}}^T \cdot x \le h_{i,d}$ **represents a half-space constrain;**

all element of $\boldsymbol{x}$ **should be greater than 0,** $I_{d\times d} \cdot x \ge \vec{\mathbf{0}}_{d}$,;

sum of all element of $\boldsymbol{x}$ **should be no greater than 1,** $\vec{\mathbf{1}}_d^T \cdot x < 1$.

$\boldsymbol{QP}$:

Minimize $0.5x^T P x + q^T x$ subject to $l \le Ax \le u$

Transform between Dominate Radius and $\boldsymbol{QP}$ **Problem:**

$\vec{l} = \begin{bmatrix} -\infty \\ \overrightarrow{-\infty}_{a\times 1} \\ \vec{\mathbf{0}}_{d\times 1} \end{bmatrix}$, $\vec{u} = \begin{bmatrix} 1 \\ \overrightarrow{-\infty}_{a\times 1} \\ H_{d,1:a}^T \\ \overrightarrow{\infty}_{d\times 1} \end{bmatrix}$, $A = \left(\vec{\mathbf{1}}_d,\ H,\ I_d\right)^T$

$P = I_d$, $q = -w$

$obj = \sqrt{2obj_{QP} + w^T \cdot w}$,

where $I_{d\times d}$ is the d-dimension identity matrix.