

Solved Example

Find the local elevation angle of the sun at 3 PM local time on 5 March in Vancouver, Canada.

Solution

Assume: Not a leap year. Not daylight savings time.

Also, Vancouver is in the Pacific time zone,

where $t_{UTC} = t + 8$ h during standard time.

Given: $t = 3$ PM = 15 h. Thus, $t_{UTC} = 23$ h.

$\phi = 49.25^\circ\text{N}$, $\lambda_e = 123.1^\circ\text{W}$ for Vancouver.

$\delta_s = -7.05^\circ$ from previous solved example.

Find: $\Psi = ?^\circ$

$$\begin{aligned} \text{Use eq. (2.6): } \sin(\Psi) &= \\ &= \sin(49.25^\circ) \cdot \sin(-7.05^\circ) - \\ &\quad \cos(49.25^\circ) \cdot \cos(-7.05^\circ) \cdot \cos[360^\circ \cdot (23\text{h}/24\text{h}) - 123.1^\circ] \\ &= 0.7576 \cdot (-0.1227) - 0.6527 \cdot 0.9924 \cdot \cos(221.9^\circ) \\ &= -0.09296 + 0.4821 = 0.3891 \\ \Psi &= \arcsin(0.3891) = \underline{22.90^\circ} \end{aligned}$$

Check: Units OK. Physics OK.

Discussion: The sun is above the local horizon, as expected for mid afternoon.

Beware of other situations such as night that give negative elevation angle.

Daily Effects

As the Earth rotates about its axis, the **local elevation angle** Ψ of the sun above the local horizon rises and falls. This angle depends on the latitude ϕ and longitude λ_e of the location:

$$\sin(\Psi) = \sin(\phi) \cdot \sin(\delta_s) - \cos(\phi) \cdot \cos(\delta_s) \cdot \cos\left[\frac{C \cdot t_{UTC}}{t_d} - \lambda_e\right] \quad \bullet(2.6)$$

Time of day t_{UTC} is in UTC, $C = 2\pi$ radians = 360° as before, and the length of the day is t_d . For t_{UTC} in hours, then $t_d = 24$ h. Latitudes are positive north of the equator, and longitudes are positive west of the prime meridian. The $\sin(\Psi)$ relationship is used later in this chapter to calculate the daily cycle of solar energy reaching any point on Earth.

[CAUTION: Don't forget to convert angles to radians if required by your spreadsheet or programming language.]

Solved Example(S)

Use a spreadsheet to plot elevation angle vs. time at Vancouver, for 22 Dec, 23 Mar, and 22 Jun. Plot these three curves on the same graph.

Solution

Given: Same as previous solved example, except

$d = 355, 82, 173$.

Find: $\Psi = ?^\circ$

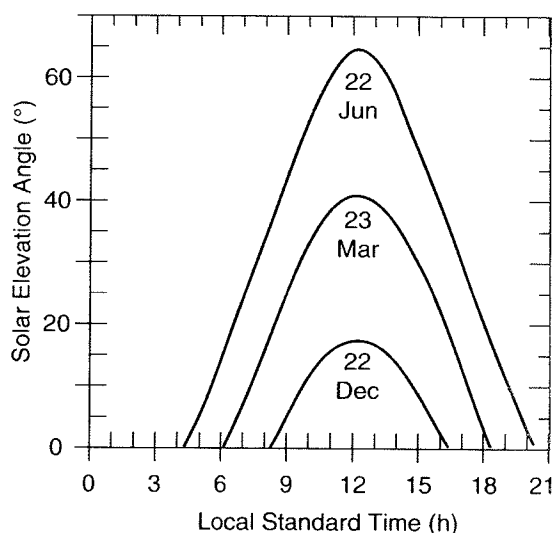
A portion of the tabulated results are shown below, as well as the full graph.

t (h)	Ψ ($^\circ$)		
	22 Dec	23 Mar	22 Jun
3	0.0	0.0	0.0
4	0.0	0.0	0.0
5	0.0	0.0	6.6
6	0.0	0.0	15.6
7	0.0	7.8	25.1
8	0.0	17.3	34.9
9	5.7	25.9	44.5
10	11.5	33.2	53.4
11	15.5	38.4	60.5
12	17.2	40.8	64.1
13	16.5	39.8	62.6

(continues in next column)

Solved Example

(continuation)



Check: Units OK. Physics OK. Graph OK.

Discussion: Summers are pleasant with long days. The peak elevation does not happen precisely at local noon, because Vancouver is not centered within its time zone.