Introduction

This lecture – look at Beer's law in a hydrostatic atmosphere

- We're only glancing over the absorption line material on p. 129-130 for now, will come back to it when we need absorption cross sections in particular satellite channels.
- Scattering: some nice examples of Light scattering by particles
- This time, introduce the complication of an absorbing gas with a vertically varying density

Vertical absorption profile

Focus p. 131 - 132, especially Fig. 4.23 and Fig. 4.24

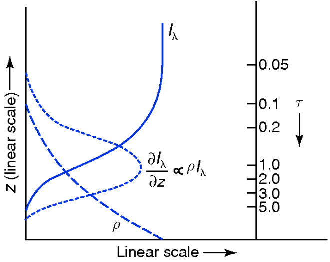
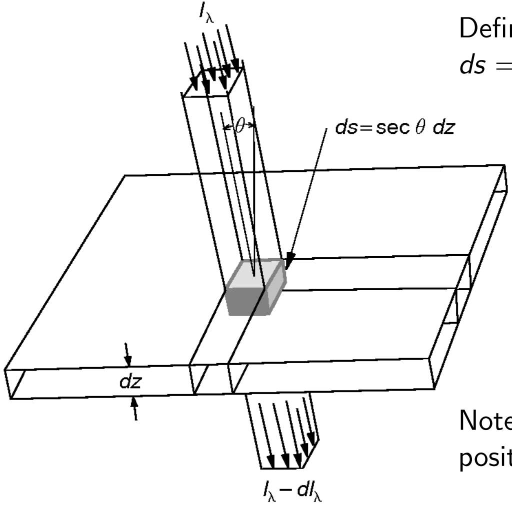


Fig. 4.23 Vertical profiles of the monochromatic intensity of incident radiation, the rate of absorption of incident radiation per unit height, air density and optical depth, for k_λ and r independent of height.

Big question: Why does the heating rate (i.e. the rate at which radiance is building up in the atmosphere) peak at $\tau = 1$

Reason we're interested: It will also turn out that emission peaks at τ=1, which allows us to infer emission at different heights by measuring radiance at different wavelengths

Review: Beers law for direct beam



Define the "slant path" as $ds = dz/\cos\theta$

Note that in this case z is positive downward!

Fig. 4.10 Extinction of incident parallel beam solar radiation as it passes through an infinitesimally thin atmospheric layer containing absorbing gases and/or aerosols.

Absorption: Lorentz & doppler line shapes

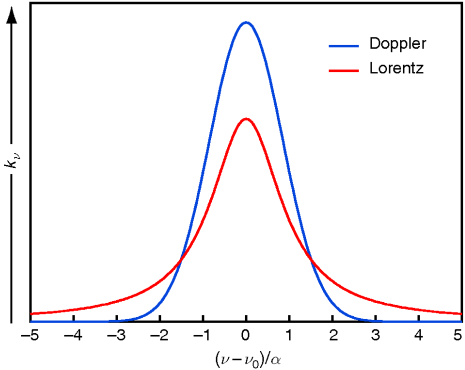
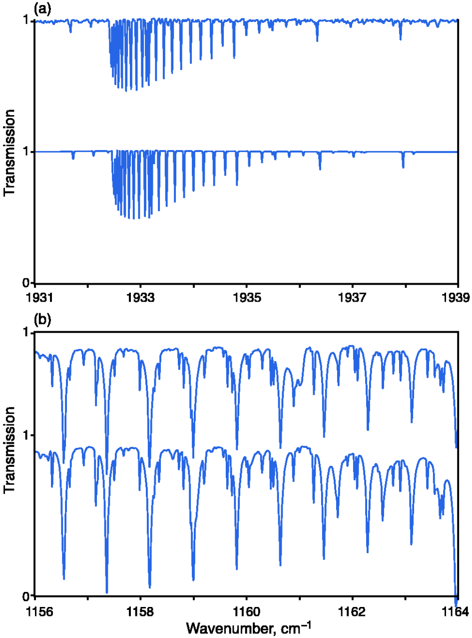


Fig. 4.21 Contrasting absorption line shapes associated with Doppler broadening and pressure broadening. Areas under the two profiles, indicative of the line intensity S, are the same. [Courtesy of Qiang Fu.]

- The width of the lorentz line depends on collison frequency, which is proportional to pressure.
- The width of the doppler line depends on molecular speed, which is proportional to $\sqrt{\text{temperature}}$

Absorption lines are very narrow



- Top figure goes from $\lambda = 5.18\ \ \mu\mathrm{m}$ to $\lambda = 5.16\ \ \mu\mathrm{m}$
- Bottom figure goes from $\lambda = 8.65\ \ \mu\mathrm{m}$ to $\lambda = 8.59\ \ \mu\mathrm{m}$

But these lines are smooshed into wider bands by Doppler and Lorentz broadening

More review:

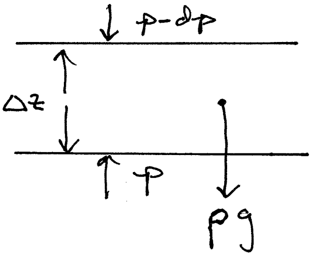
- The slant optical thickness (or slant optical depth) is defined as:

$$d\sigma_\lambda = dTr = \frac{dF_\lambda}{F_\lambda} = -k_\lambda \rho_g ds$$

(where k_λ is the mass absorption coefficient ($\mathrm{m}^2\,\mathrm{kg}^{-1}$))

- In words, it's a measure of how "thick" a gas with density ρ_g is for photons with wavelength λ in a layer of thickness ds.
- The "vertical optical depth" $d\tau_\lambda$ uses dz instead of ds. Since $dz = ds * \cos\theta$, the vertical optical depth is $d\tau_\lambda = \sigma_\lambda \cos\theta$

What is ρ_g? Start with: What is ρ?



- Start with a layer with density ρ_{air}
- Assume hydrostatic balance for a layer of thickness Δz, area A, volume $V = A\Delta z$
  - Force down= ma $= \rho_{air} Vg = \rho_{air} A\Delta zg$
  - pressure force up= dp A
  - Balance requires $dpA = -\rho_{air} A\Delta zg$ or
  - $dp = -\rho_{air} gdz$

Hydrostatic atmosphere, cont. (also see W&H p. 67

- We want to integrate $dp' = -\rho_{air} g dz'$ from the surface to a pressure level p.
- Neglecting water vapor, the equation of state gives $p = \rho_{air} R_d T$
- So use this. Start from the surface with pressure $= p_0$, $z = 0$, $\rho_{air} = \rho_0$ and integrate to height z, pressure p, density ρ_{air}

$$dp' = -\frac{p}{R_d T} g dz'$$

$$\int_{p_0}^{p} \frac{dp'}{p'} = -\int_0^z \frac{g}{R_d T} dz' = -\int_0^z \frac{1}{H} dz'$$

define the pressure scale height: H_p as:

$$\frac{1}{H_p} = \frac{1}{z} \int_0^z \frac{1}{H} dz'$$

which leaves: $\displaystyle\int_{p_0}^{p} d \ln p' = -\frac{z}{H_p}$

Hydrostatic atmosphere, cont.

- Finish the integration:

$$\int_{p_0}^{p} d\ln p' = -\frac{z}{H_p}$$

$$\ln\left(\frac{p}{p_0}\right) = -\frac{z}{H_p}$$

$$p = p_0 exp\left(-\frac{z}{H_p}\right)$$

- and use the equation of state to get density:

$$\rho_{air} R_d T = \rho_0 R_d T_0 exp\left(-\frac{z}{H_p}\right)$$

$$\rho_{air} = \rho_0 \frac{T_0}{T} exp\left(-\frac{z}{H_p}\right) \approx \rho_0 \exp\left(-\frac{z}{H_\rho}\right)$$

where H_ρ is the density scale height. Typical values are $H_p =$ 10 km and H_ρ=8 km

Finally, what is ρ_g?

- If the gas is well mixed, then the mixing ratio:

$$r_g = \frac{\text{kg gas}}{\text{kg air}} = \frac{\rho_g}{\rho_{air}} = constant$$

- Which means that

$$\rho_g = r_g \rho_0 \exp\left(-\frac{z}{H_\rho}\right)$$

- and so the vertical optical depth is:

$$d\tau_\lambda = k_\lambda \rho_g dz = k_\lambda r_g \rho_0 \exp\left(-\frac{z}{H_\rho}\right) dz$$

now use ρ_g to get τ_λ

- Integrate the optical thickness between some height $z' = z$ and $\tau'_\lambda = \tau_\lambda$ and the top of the atmosphere where $z' = \infty$ and $\tau'_\lambda = 0$. We need to introduce a minus sign because we are moving downward (negative z direction) from the top:

$$\int_{\tau_\lambda}^{0} d\tau'_\lambda = -\int_z^\infty k_\lambda \rho_g dz' = -\int_z^\infty k_\lambda r_g \rho_0 \exp\left(-\frac{z'}{H_\rho}\right) dz'$$

$$0 - \tau_\lambda = H_\rho k_\lambda r_g \rho_0 \left[0 - \exp\left(-\frac{z}{H_\rho}\right)\right]$$

$$\tau_\lambda = H_\rho k_\lambda r_g \rho_0 \exp\left(-\frac{z}{H_\rho}\right) = H_\rho k_\lambda r_g \rho = H_\rho \beta_a$$

  where $\beta_a = k_\lambda \rho_g = k_\lambda r_g \rho_{air}$ is the volume absorption coefficient.
- So the vertical optical depth measured from the top of the atmosphere is proportional to the density for a well-mixed absorber.

Level of maximum heating from ozone absorption

- The stratosphere is produced by solar heating due to ozone absorption of ultraviolet photons.
- Recall the heating rate equation from (day 8, part II):

$$\frac{dT}{dt} = -\frac{1}{\overline{\rho_{air}} c_p} \frac{dF_N}{dz}$$

  where ΔF_N is the net *upward* flux.
- Since we know τ, we know the direct beam flux from Beer's law (Lecture 8)

$$F_\lambda = F_{\lambda 0} exp(-\tau_\lambda)$$

  and since F_λ from the sun is downwards, and we are assuming no reflection, $F_\lambda = -F_N$ and $\frac{dT}{dt} \propto \frac{dF_\lambda}{dz}$
- So the heating rate will be maximum where $\frac{d^2 F_\lambda}{dz^2} = 0$

heating rate: find τ where $\frac{d^2F}{dz^2} = 0$

- since $F = F_0 \exp(-\tau)$ and $\tau = C \exp(-z/H)$:

$$\frac{dF}{dz} = -F_0 \frac{d\tau}{dz} \exp(-\tau) = F_0 \frac{C}{H} \exp(-z/H) \exp(-\tau)$$

$$\frac{d^2F}{dz^2} = -\frac{FC}{H^2} \exp(-z/H) \exp(-\tau) + \frac{FC}{H} \exp(-z/H) \exp(-\tau) \frac{d\tau}{dz} = 0$$

$$\frac{1}{H} + \frac{d\tau}{dz} = 0$$

$$\frac{1}{H} - \frac{C}{H} \exp(-z/H) = 0$$

$$\frac{1}{H} - \frac{\tau}{H} = 0$$

$$\tau = 1$$

- So the heating rate is maxium where $\tau = 1$.

Vertical absorption profile for downward direct solar intensity

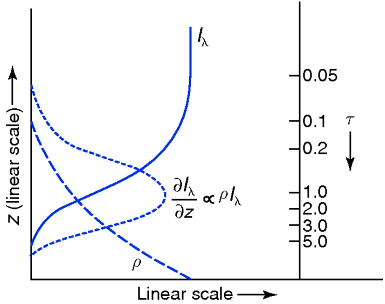


Fig. 4.23 Vertical profiles of the monochromatic intensity of incident radiation, the rate of absorption of incident radiation per unit height, air density and optical depth, for k_λ and r independent of height.

- Why does the direct beam change fastest at $\tau_\lambda = 1$?
- What height is this?

Beers law for ozone

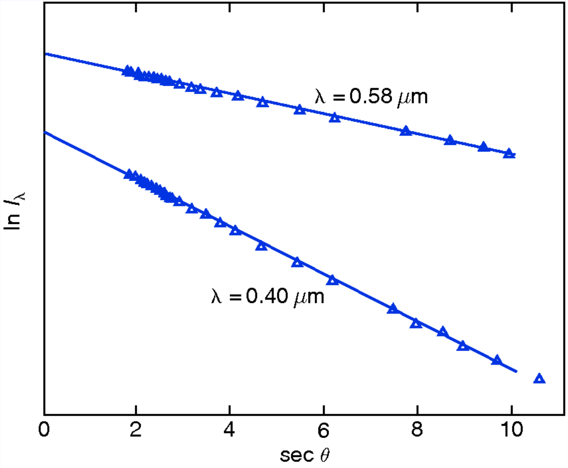


Fig. 4.24 Monochromatic intensity of solar radiation measured at the ground as a function of solar zenith angle under clear, stable conditions at Tucson, Arizona on 12 December 1970. [From *J. Appl. Meteor.*, **12**, 376 (1973).]

Why are we getting straight lines of different slopes?

- Beer's law:

$$I_\lambda = I_{\lambda 0} \exp\left(-\tau_\lambda / \cos\theta\right)$$
$$\ln(I_\lambda) = \ln(I_{\lambda 0}) - \tau_\lambda / \cos\theta$$

(assuming a plane parallel atmosphere)

- If k_λ is constant:

$$\tau_\lambda = \int_z^\infty k_\lambda \rho_g dz' = k_\lambda \int_z^\infty \rho_g dz' = k_\lambda \times Constant$$

- so the slope of the line is proportional to the mass absorption coefficient, which is strongest for ozone in the ultraviolet (0.4 $\mu\mathrm{m}$)

Discovery of the ozone hole

- The ozone hole was discovered using land based sun photometers in 1985.
- The hole was also detected by the TOMS instrument
- The cause wasn't fully understood until aircraft measurements were made in 1987.

Summary

- Coverage: pp. 122-132 on scattering, absorption emission
- Main points:
  - Scattering direction and efficience depends on wavelength λ and particile size (r) via the size parameter
  - Three distinct scattering regimes: Rayleigh, Mie, Geometric
  - Absorption depends on line strength and line width (function of temperature, pressure, wavelength)
  - Vertical optical thickness relates the physical propoerties of absorbers/scatterers to driect beam transmission