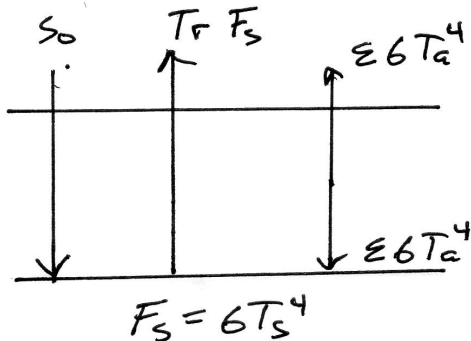


# Radiative equilibrium

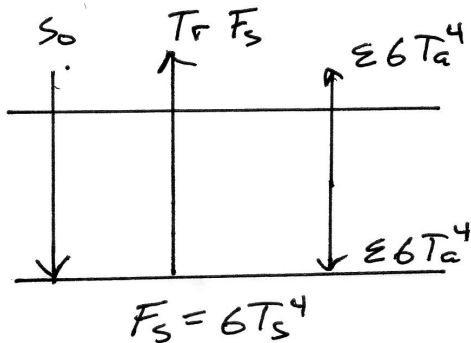
1. Coverage: W&H 116-122
2. Introduce heating rate, radiative equilibrium, brightness temperature
3. Extend the discussion of black atmospheric layers in WH Fig 4.9 page 122 to include *grey* layers.

## One layer grey atmosphere



This figure shows a grey atmosphere with a transmissivity of  $T_r$ , an emissivity of  $\epsilon$  and temperature  $T_a$  over a black surface with temperature  $T_S$ . There is a downward shortwave flux of  $S_0$  and no atmospheric absorption of solar photons

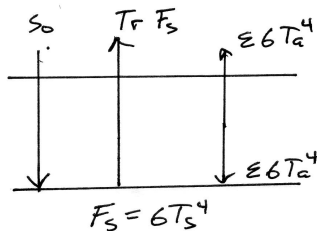
## One layer atmosphere – cont.



If we ignore the  $S_0$  contribution for the moment and want to know how fast the atmosphere and surface are cooling in the longwave, we need the the upward flux leaving the top of the atmosphere:

$$\begin{aligned}\text{longwave flux leaving the top} &= F_{T,\uparrow} = T_r F_s + \epsilon \sigma T_a^4 \\ &= (1 - \epsilon) \sigma T_s^4 + \epsilon \sigma T_a^4 = \sigma T_s^4 - \epsilon \sigma (T_s^4 - T_a^4)\end{aligned}$$

## Greenhouse effect



The *Greenhouse effect* is defined as the difference in the upward flux to space caused by the presence of an absorbing/emitting atmosphere with a temperature that's different from the surface:

$$G = F_{B,\uparrow} - F_{T,\uparrow} = \sigma T_s^4 - (\sigma T_s^4 - \epsilon \sigma (T_s^4 - T_a^4))$$
$$G \text{ (W m}^{-2}\text{)} = \epsilon \sigma (T_s^4 - T_a^4)$$

Note that  $G > 0$  (positive, heating) only if the atmosphere is colder than the surface. Also note that increasing the emissivity/absorptivity ( $\epsilon$ ) increases the heating.

## Heating rate

- ▶ How fast is the atmosphere heating/cooling? That depends on the *net flux divergence*,  $\frac{dF_N}{dz}$ , where  $F_N = F_{\uparrow} - F_{\downarrow}$  at a particular height.
- ▶ For this one layer atmosphere with no shortwave absorption,

$$F_{N,T} = F_{T,\uparrow} - F_{T,\downarrow} = TrF_s + \epsilon\sigma T_a^4 = (1 - \epsilon)\sigma T_s^4 + \epsilon\sigma T_a^4$$

$$F_{N,B} = F_{B,\uparrow} - F_{B,\downarrow} = \sigma T_s^4 - \epsilon\sigma T_a^4$$

$$\Delta F_N = F_{N,T} - F_{N,B} = (1 - \epsilon)\sigma T_s^4 + \epsilon\sigma T_a^4 - (\sigma T_s^4 - \epsilon\sigma T_a^4) =$$
$$\Delta F_N = 2\epsilon\sigma T_a^4 - \epsilon\sigma T_s^4$$

where positive  $\Delta F_N$  means the layer is cooling

some representative numbers:

$$T_a = 250 \text{ K}, T_s = 280 \text{ K}, \epsilon = 0.8$$

which gives  $\Delta F_N = 75 \text{ W m}^{-2}$

## Heating rate – cont.

- ▶ What does  $\Delta F_N = 75 \text{ W m}^{-2}$  mean in terms of temperature change? Suppose the average air density is  $\bar{\rho} = 1 \text{ kg m}^{-3}$  and the total mass of a  $1 \text{ m}^2$  column is  $M=10,000 \text{ kg}$ . The column is losing  $75 \text{ J/s}$ , and since the heat capacity of air is  $c_p = 1004 \text{ J, kg}^{-1} \text{ K}^{-1}$ , the cooling rate is:

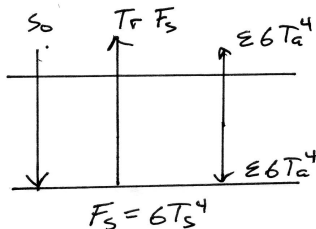
$$\text{cooling rate} = \Delta F_N = -M \times c_p \times \frac{dT}{dt} = -\bar{\rho} \times \Delta z \times c_p \times \frac{dT}{dt}$$

or rearranging :

$$\frac{dT}{dt} = -\frac{1}{\bar{\rho} c_p} \frac{\Delta F_N}{\Delta z}$$

So this atmosphere loses  $75 \text{ W m}^{-2}$  and cools at  $75 \text{ W m}^{-2} / 1004 \text{ J, kg}^{-1} \text{ K}^{-1} / 10000 \text{ m} = 0.6 \text{ K/day}$

# Equilibrium temperature



- Note that the atmosphere and surface will stop cooling when the outgoing longwave balances the incoming shortwave at each level.

# Summary

- ▶ We can make a rough calculation of the heating/cooling in an isothermal layer if we know the transmission, absorption and reflection of the flux through the layer
- ▶ A layer that is neither heating or cooling is in equilibrium, at a temperature at which absorption and emission are in balance.