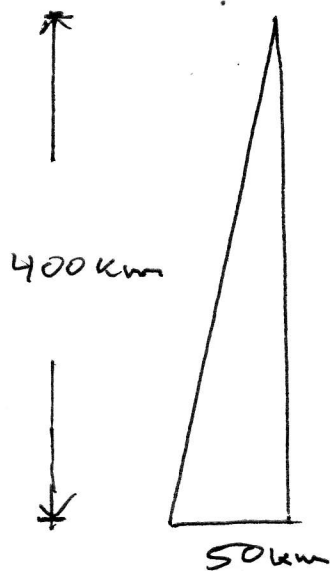


Quiz 1 lecture 3 solution



1) Find the solid angle
using $\int_0^{2\pi} \int_0^{\theta} \sin\theta' d\theta' d\phi' = \omega_{\text{exact}}$

2) compare to the approximate

$$\omega_{\text{approx}} = \frac{\text{disk area}}{r^2}$$

$$\theta = \tan^{-1}\left(\frac{25}{400}\right) = 6.242 \times 10^{-2} \text{ rad}$$

$$\omega_{\text{exact}} = 2\pi \int_0^{6.242 \times 10^{-2}} \sin\theta d\theta = -2\pi \cos\theta \Big|_0^{6.242 \times 10^{-2}}$$

$$\omega_{\text{exact}} = 2\pi(1 - \cos(6.242 \times 10^{-2})) = 0.012236 \text{ sr}$$

$$\text{disk area} = \pi \left(\frac{50}{2}\right)^2 = \pi 25^2 = 1963.5 \text{ km}^2$$

$$\omega_{\text{approx}} = \frac{\text{area}}{r^2} = \frac{1963.5}{(400 \text{ km})^2} = 0.12227$$

$$\frac{\omega_{\text{exact}} - \omega_{\text{approx}}}{\omega_{\text{exact}}} = -0.3\%$$