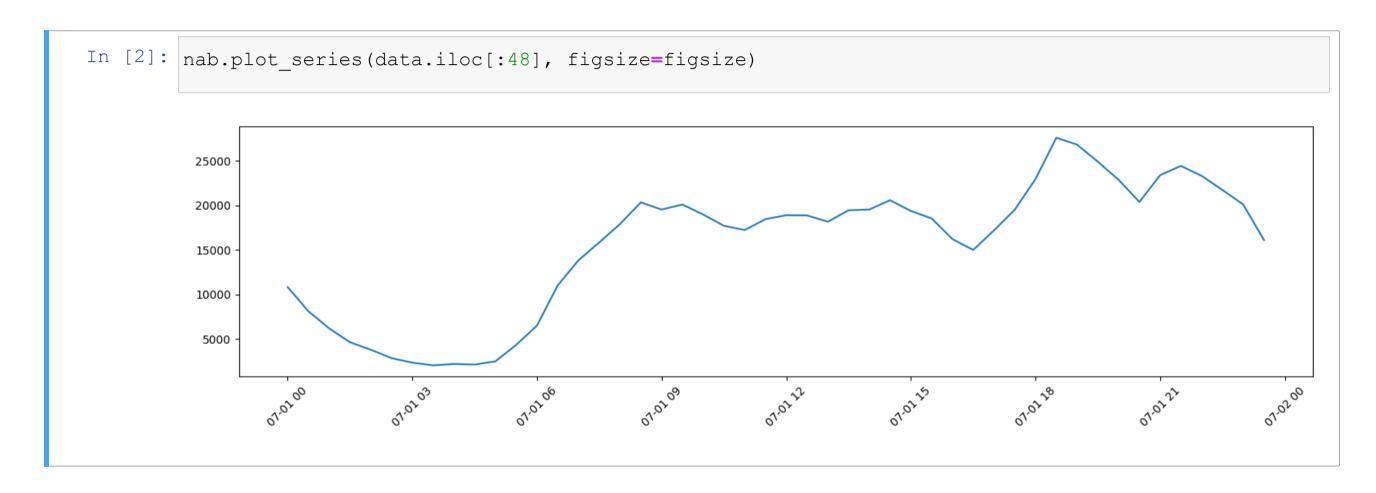


#### Let's have a closer look at our time series



- Nearby points tend to have similar values
- ...Meaning they are correlated

### **Determine the Period**

## How can we study such correlation?

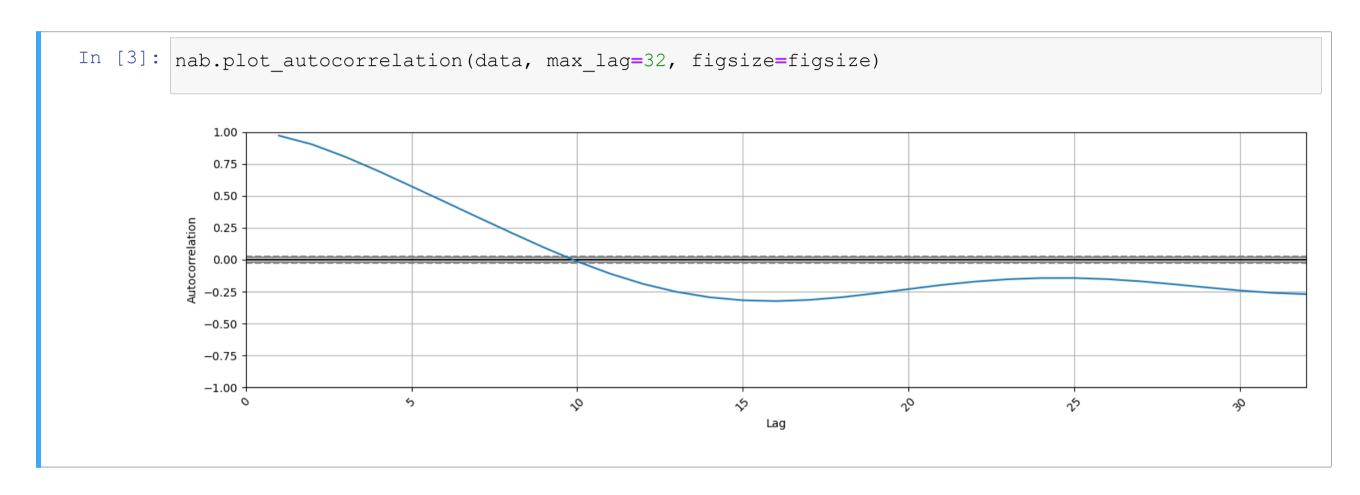
A useful tool: <u>autocorrelation</u> plots

- Consider a range of possible lags
- lacksquare For each lag value l:
  - lacksquare Make a copy of the series and shift it by  $m{l}$  time steps
  - Compute the <u>Pearson Correlation Coefficient</u> with the original series
- Plot the correlation coefficients over the lag values

### Then we look at the resulting plot:

- Where the curve is far from zero, there is a significant correlation
- Where it gets close to zero, no significant correlation exists

### Let's have a look at our plot



■ The correlation is strong up to 4-5 lags

#### These correlations are a source of information

- They could be exploited to improve our estimated probabilities
- ...But our models so far make no use of them

How can we take advantage of them?

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### How can we take advantage of them?

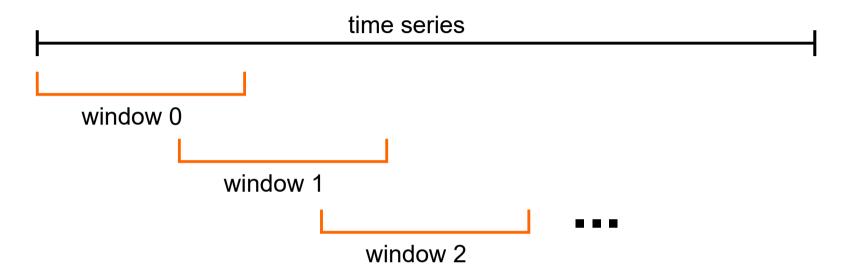
### For example, rather then feeding our model with individual observations

We can use sequences of observations as input

- This is a very common approach in time series
- ...And in many cases it's a good idea

## **Sliding Window**

## A common approach consist in using a sliding window



- $\blacksquare$  We choose a window length w, i.e. the length of each sub-sequence
- We place the "window" at the beginning of the series
- ...We extract the corresponding observations
- Then, we move the forward by a certain stride and we repeat

# **Sliding Window**

#### The result is a table

Let m be the number of examples and w be the window length

	$S_0$	$\mathbf{S}_{1}$	• • •	$S_{W-1}$
$t_{w-1}$	$x_0$	$x_1$	• • •	$x_{w-1}$
$t_{\rm w}$	$x_1$	$x_2$	• • •	$x_w$
$t_{w+1}$	$x_2$	$x_3$	• • •	$x_{w+1}$
•	•	•	•	•
$t_{m-1}$	$x_{m-w}$	$x_{m-w+1}$	•	$x_{m-1}$

- The first window includes observations from  $x_0$  to  $x_{w-1}$
- lacksquare The second from  $x_1$  to  $x_w$  and so on
- $\blacksquare$   $t_i$  is the time window index (where it was applied)
- lacksquare s<sub>j</sub> is the position of an observation within a window

### pandas provides a sliding window iterator

```
DataFrame.rolling(window, ...)
```

```
In [4]: wlen = 10
        for i, w in enumerate(data['value'].rolling(wlen)):
           print(w)
            if i == 2: break # We print the first three windows
        timestamp
        2014-07-01
                     10844
        Name: value, dtype: int64
        timestamp
        2014-07-01 00:00:00
                               10844
        2014-07-01 00:30:00
                               8127
        Name: value, dtype: int64
        timestamp
        2014-07-01 00:00:00
                               10844
        2014-07-01 00:30:00
                                8127
        2014-07-01 01:00:00
                                 6210
        Name: value, dtype: int64
```

Notice how the first windows are not full (shorter than wlen)

### We can build our dataset using the rolling iterator

- We discard the first wlen-1 (incomplete) applications
- Then we store each window in a list, and we wrap everything in a DataFrame

```
In [5]: %%time
  rows = []
  for i, w in enumerate(data['value'].rolling(wlen)):
      if i >= wlen-1: rows.append(w.values)

wdata_index = data.index[wlen-1:]
  wdata = pd.DataFrame(index=wdata_index, columns=range(wlen), data=rows)

CPU times: user 449 ms, sys: 5.18 ms, total: 455 ms
  Wall time: 454 ms
```

- The values field allows access to the series content as a numpy array
- We use it to discard the index
- ...Since the series for multiple iterations have inconsistent indexes

### This method works, but it's a bit slow

- We are building our table by rows...
- ...But it is usually faster to do it by columns!
- After all, there are usually fewer columns than rows

## Let us look again at our table:

	$\mathbf{S_0}$	$\mathbf{s}_1$	• • •	$s_{w-1}$
$t_{w-1}$	$x_0$	$x_1$	• • •	$x_{w-1}$
$t_{\rm w}$	$x_1$	$x_2$	• • •	$x_w$
$t_{w+1}$	$x_2$	$x_3$	• • •	$x_{w+1}$
•	•	•	•	•
$t_{m-1}$	$x_{m-w}$	$x_{m-w+1}$	•	$x_{m-1}$

### We can build the columns by slicing the original DataFrame

```
In [6]: m = len(data)
        c0 = data.iloc[0:m-wlen+1] # first column
        c1 = data.iloc[1:m-wlen+1+1] # second column
       print(c0.iloc[0:3])
       print(c1.iloc[0:3])
                             value
        timestamp
        2014-07-01 00:00:00 10844
        2014-07-01 00:30:00
                             8127
        2014-07-01 01:00:00
                            6210
                             value
        timestamp
        2014-07-01 00:30:00
                              8127
        2014-07-01 01:00:00
                              6210
        2014-07-01 01:30:00
                              4656
```

■ iloc in pandas allows to address a DataFrame by position

#### Now we collect all columns in a list and we stack them

**2014-07-0105:00:00** 8127 3820 2873 2369 2064 2221 2158 2515 6210 4656 **2014-07-0105:30:00** 6210 4656 3820 2873 2369 2064 2221 2158 2515 4364 **2014-07-0106:00:00** 4656 3820 2873 2369 2064 2221 2158 2515 4364 6526 **2014-07-0106:30:00** 3820 2873 2369 2064 2221 2158 2515 4364 6526 11039

### We can wrap this approach in a function:

```
def sliding_window_1D(data, wlen):
    m = len(data)
    lc = [data.iloc[i:m-wlen+i+1] for i in range(0, wlen)]
    wdata = np.hstack(lc)
    wdata = pd.DataFrame(index=data.index[wlen-1:], data=wdata, columns=range(wlen))
    return wdata
```

```
In [8]: %%time
   wdata = nab.sliding_window_1D(data, wlen=wlen)

CPU times: user 1.71 ms, sys: 0 ns, total: 1.71 ms
   Wall time: 1.25 ms
```

- This is available in the (updated)) nab module
- The function works for univariate data (but the approach is general)