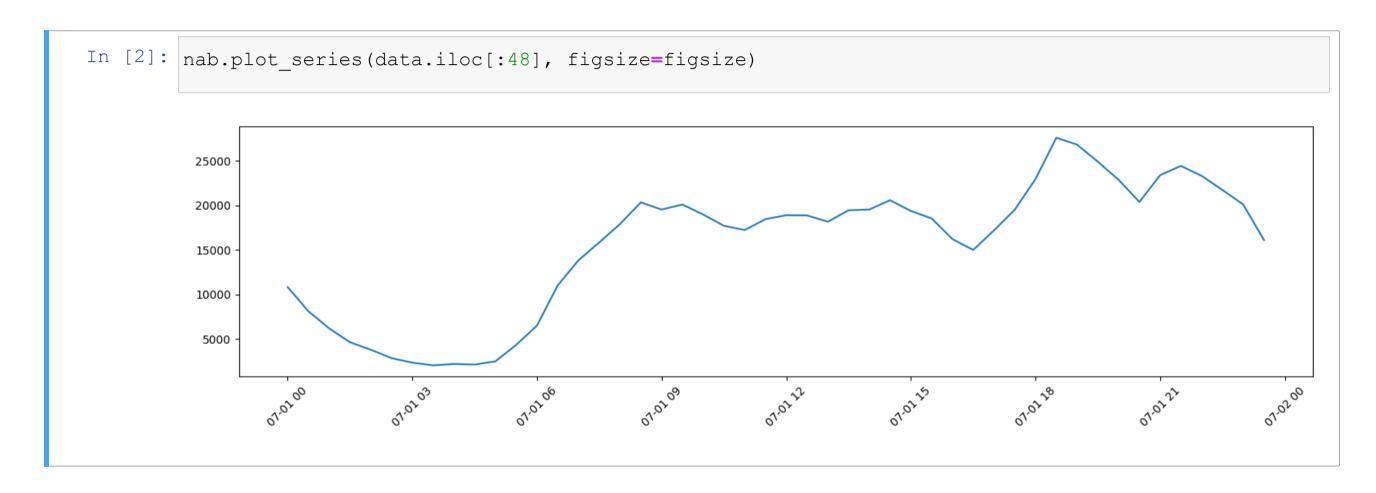


Let's have a closer look at our time series



- Nearby points tend to have similar values
- ...Meaning they are correlated

Determine the Period

How can we study such correlation?

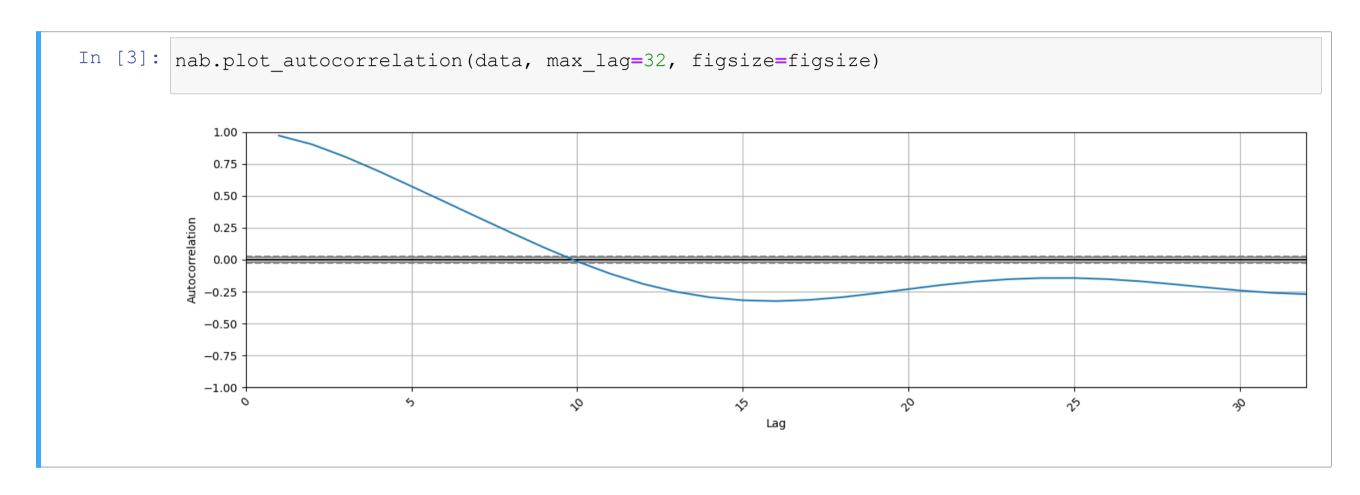
A useful tool: <u>autocorrelation</u> plots

- Consider a range of possible lags
- lacksquare For each lag value l:
 - lacksquare Make a copy of the series and shift it by $m{l}$ time steps
 - Compute the <u>Pearson Correlation Coefficient</u> with the original series
- Plot the correlation coefficients over the lag values

Then we look at the resulting plot:

- Where the curve is far from zero, there is a significant correlation
- Where it gets close to zero, no significant correlation exists

Let's have a look at our plot



■ The correlation is strong up to 4-5 lags

These correlations are a source of information

- They could be exploited to improve our estimated probabilities
- ...But our models so far make no use of them

How can we take advantage of them?

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How can we take advantage of them?

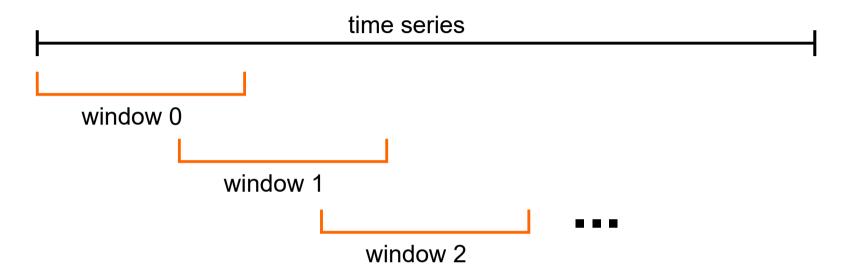
For example, rather then feeding our model with individual observations

We can use sequences of observations as input

- This is a very common approach in time series
- ...And in many cases it's a good idea

Sliding Window

A common approach consist in using a sliding window



- \blacksquare We choose a window length w, i.e. the length of each sub-sequence
- We place the "window" at the beginning of the series
- ...We extract the corresponding observations
- Then, we move the forward by a certain stride and we repeat

Sliding Window

The result is a table

Let m be the number of examples and w be the window length

	S_0	\mathbf{S}_{1}	• • •	S_{W-1}
t_{w-1}	x_0	x_1	• • •	x_{w-1}
$t_{\rm w}$	x_1	x_2	• • •	x_w
t_{w+1}	x_2	x_3	• • •	x_{w+1}
•	•	•	•	•
t_{m-1}	x_{m-w}	x_{m-w+1}	•	x_{m-1}

- The first window includes observations from x_0 to x_{w-1}
- lacksquare The second from x_1 to x_w and so on
- \blacksquare t_i is the time window index (where it was applied)
- lacksquare s_j is the position of an observation within a window

pandas provides a sliding window iterator

```
DataFrame.rolling(window, ...)
```

```
In [4]: wlen = 48
        for i, w in enumerate(data['value'].rolling(wlen)):
           print(w)
            if i == 2: break # We print the first three windows
        timestamp
        2014-07-01
                     10844
        Name: value, dtype: int64
        timestamp
        2014-07-01 00:00:00
                               10844
        2014-07-01 00:30:00
                               8127
        Name: value, dtype: int64
        timestamp
        2014-07-01 00:00:00
                               10844
        2014-07-01 00:30:00
                                8127
        2014-07-01 01:00:00
                                 6210
        Name: value, dtype: int64
```

Notice how the first windows are not full (shorter than wlen)

We can build our dataset using the rolling iterator

- We discard the first wlen-1 (incomplete) applications
- Then we store each window in a list, and we wrap everything in a DataFrame

```
In [5]: %%time
  rows = []
  for i, w in enumerate(data['value'].rolling(wlen)):
      if i >= wlen-1: rows.append(w.values)

  wdata_index = data.index[wlen-1:]
  wdata = pd.DataFrame(index=wdata_index, columns=range(wlen), data=rows)

CPU times: user 558 ms, sys: 4.04 ms, total: 563 ms
  Wall time: 563 ms
```

- The values field allows access to the series content as a numpy array
- We use it to discard the index
- ...Since the series for multiple iterations have inconsistent indexes

This method works, but it's a bit slow

- We are building our table by rows...
- ...But it is usually faster to do it by columns!
- After all, there are usually fewer columns than rows

Let us look again at our table:

	$\mathbf{S_0}$	\mathbf{s}_1	• • •	s_{w-1}
t_{w-1}	x_0	x_1	• • •	x_{w-1}
$t_{\rm w}$	x_1	x_2	• • •	x_w
t_{w+1}	x_2	x_3	• • •	x_{w+1}
•	•	•	•	•
t_{m-1}	x_{m-w}	x_{m-w+1}	•	x_{m-1}

We can build the columns by slicing the original DataFrame

```
In [6]: m = len(data)
        c0 = data.iloc[0:m-wlen+1] # first column
        c1 = data.iloc[1:m-wlen+1+1] # second column
       print(c0.iloc[0:3])
       print(c1.iloc[0:3])
                             value
        timestamp
        2014-07-01 00:00:00 10844
        2014-07-01 00:30:00
                             8127
        2014-07-01 01:00:00
                            6210
                             value
        timestamp
        2014-07-01 00:30:00
                              8127
        2014-07-01 01:00:00
                              6210
        2014-07-01 01:30:00
                              4656
```

■ iloc in pandas allows to address a DataFrame by position

Now we collect all columns in a list and we stack them

```
In [9]: | 1c = [data.iloc[i:m-wlen+i+1].values for i in range(0, wlen)]
         lc = np.hstack(lc)
         wdata = pd.DataFrame(index=wdata index, columns=range(wlen), data=lc)
         wdata.head()
Out[9]:
                                                                        9 ...
                                                            7
                                                                                38
                                                                                       39
                                                                                             40
                                                                                                   41
                                                                                                         42
                                                                                                                43
          timestamp
          2014-07-
          01
                   10844 8127 6210 4656 3820 2873 2369 2064 2221 2158
                                                                         ... 26827 24904 22875 20394 23401 24439 23
          23:30:00
          2014-07-
                                                                          ... 24904 22875 20394 23401 24439
          02
                   8127
                         6210 4656 3820 2873 2369 2064 2221 2158 2515
          00:00:00
          2014-07-
                         4656 3820 2873 2369 2064 2221 2158 2515 4364
                                                                         ... 22875 20394 23401 24439 23318 21733 20
          02
                   6210
          00:30:00
          2014-07-
          02
                   4656
                              2873 2369
                                         2064 2221 2158 2515 4364 6526
                                                                         ... 20394 23401 24439 23318 21733 20104 16
          01:00:00
          2014-07-
          02
                                    2064
                                         2221 2158 2515 4364 6526 11039 ... 23401 24439 23318 21733 20104 16111 13
                   3820
          01:30:00
          5 rows × 48 columns
```

We can wrap this approach in a function:

```
def sliding_window_1D(data, wlen):
    m = len(data)
    lc = [data.iloc[i:m-wlen+i+1] for i in range(0, wlen)]
    wdata = np.hstack(lc)
    wdata = pd.DataFrame(index=data.index[wlen-1:], data=wdata, columns=range(wlen))
    return wdata
```

```
In [10]: %%time
  wdata = nab.sliding_window_1D(data, wlen=wlen)

CPU times: user 6.48 ms, sys: 0 ns, total: 6.48 ms
  Wall time: 5.72 ms
```

- This is available in the (updated)) nab module
- The function works for univariate data (but the approach is general)