Sequence Input in KDE

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Can we take sequence input into account in KDE?

There is straightforward approach, using multivariate KDE

- Treat each sequence as a vector variable
- Learn an estimator as usual

Individual sequences in the new dataset are treated as independent:

- This is due to the basic assumptions behind KDE
- In practice, for a sufficiently high window length
- ...The dependencies become negligible

Does it sound familiar?

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Does it sound familiar?

This is simply the Markov property!

Bandwidth Choice in Multivariate KDE

We now need to learn our multivariate KDE estimator

First, we need to choose a bandwidth

- We cannot use the (univariate) rule of thumb
- ...But we can use a more general approach

The basic intuition is that a good bandwidth

...Will make the actual data register as more likely

- Therefore we can pick a validation set
- ...And tune the bandwidth for maximum likelihood

To avoid overfitting, there should be no overlap with the training data

Bandwidth Choice in Multivariate KDE

Formally, let $\tilde{x}$ be a validation set of m examples:

Assuming independent observations, its likelihood is:

$$L(\tilde{\mathbf{x}}, \hat{\mathbf{x}}, h) = \prod_{i=1}^{m} f(\tilde{x}_i, \hat{\mathbf{x}}, h)$$

The likelihood is the estimated probability of a sample

- ...As a function of the model parameters
- f is the density estimator (which outputs a probability)
- $\hat{\mathbf{x}}$ the training set, h is the bandwidth

Bandwidth Choice in Multivariate KDE

We can then choose h so as to maximize the likelihood

Meaning that the training problem is given by:

$$\arg\max_h \mathbb{E}_{\tilde{x}\sim D}\left[L(\tilde{\mathbf{x}}, \hat{\mathbf{x}}, h)\right]$$

- Where D is ideally the true distribution

As many training problem, it cannot be solved in an exact fashion

- Instead we will approximate $\mathbb{E}$ by sampling multiple $\tilde{x}$
- ...And pick the bandwidth h^* leading to the maximum average likelihood

In a pinch, we could even use a single $\tilde{x}$

Bandwidth Choice in Multivariate KDE

A simple approach consist in using grid search

- It's the same approach that we used for optimizing the threshold
- scikit learn provides a convenient implementation
- ...Which resorts to cross-fold validation to define $\tilde{x}$

First, we separate the training set as usual:

```
In [2]: wdata_tr = wdata[wdata.index < train_end]
```

Then we specify the values we want to consider for each parameter:

```
In [3]: params = {'bandwidth': np.linspace(1000, 1200, 20)}
```

Training Multivariate KDE

Finally, we can run the grid search routine

```
In [4]: gs_kde = GridSearchCV(KernelDensity(kernel='gaussian'), params, cv = 5)
        gs_kde.fit(wdata_tr)
        gs_kde.best_params_

Out[4]: {'bandwidth': 1010.5263157894736}
```

- `cv` is the number of folds
- After training, `GridSearchCV` acts as a proxy for the best estimator

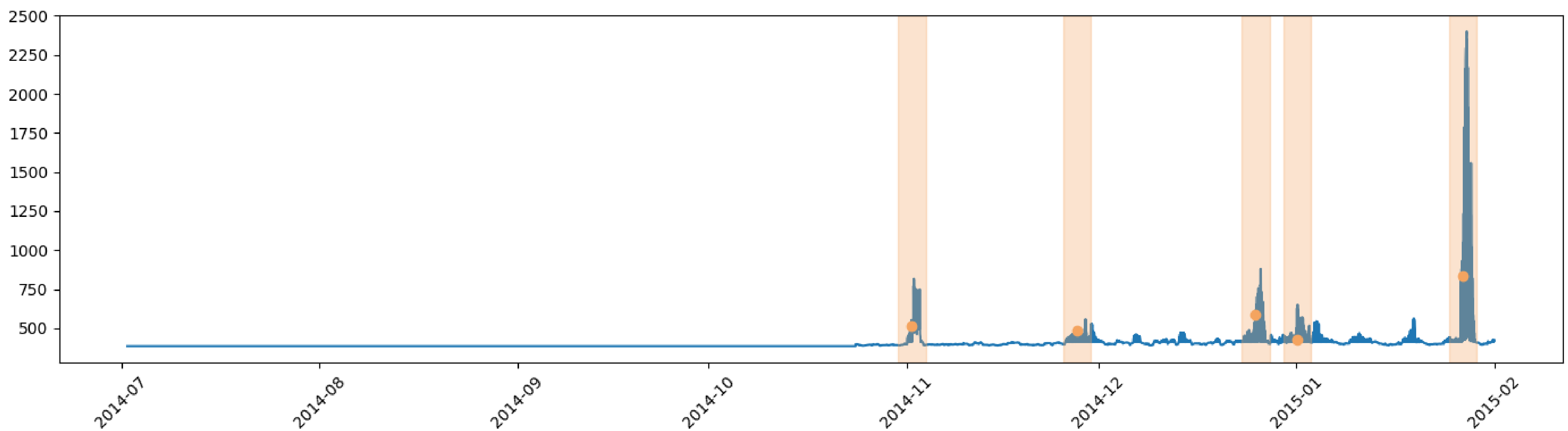
This is an expensive operation

- We need to test multiple bandwidth values
- For each one, we need to perform cross-validation
- ...And finally adding dimensions makes KDE slower

Sequences via Multivariate KDE

Now we can use the best estimator to generate the alarm signal

In [5]:
```
ldens = gs_kde.score_samples(wdata)
signal = pd.Series(index=wdata.index, data=-ldens)
nab.plot_series(signal, labels, windows, figsize=figsize)
```



- The signal seems much better than before (but a bit noisy)

Threshold Optimization

Finally, we can do threshold optimization as usual

In [6]:
```
signal_opt = signal[signal.index < val_end]
labels_opt = labels[labels < val_end]
windows_opt = windows[windows['end'] < val_end]
thr_range = np.linspace(300, 600, 100)

best_thr, best_cost = nab.opt_thr(signal_opt, labels_opt, windows_opt,  cmodel, thr_range)
print(f'Best threshold: {best_thr}, corresponding cost: {best_cost}')
```

```
Best threshold: 527.2727272727273, corresponding cost: 5
```

Cost on the whole dataset

In [7]:
```
ctst = cmodel.cost(signal, labels, windows, best_thr)
print(f'Cost on the whole dataset {ctst}')
```

```
Cost on the whole dataset 22
```