Basic Approaches for Missing Values

Basic Approaches for Missing Values

We will now discuss a few simple approaches to deal with missing values

We will use partially synthetic data

- We will focus on specific (and mosly intact) sections of our series
- The we will remove values artificially
- ...And measure the accuracy of our filling approaches via the Root MSE

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=0}^{n}(x_i - \hat{x}_i)^2}$$

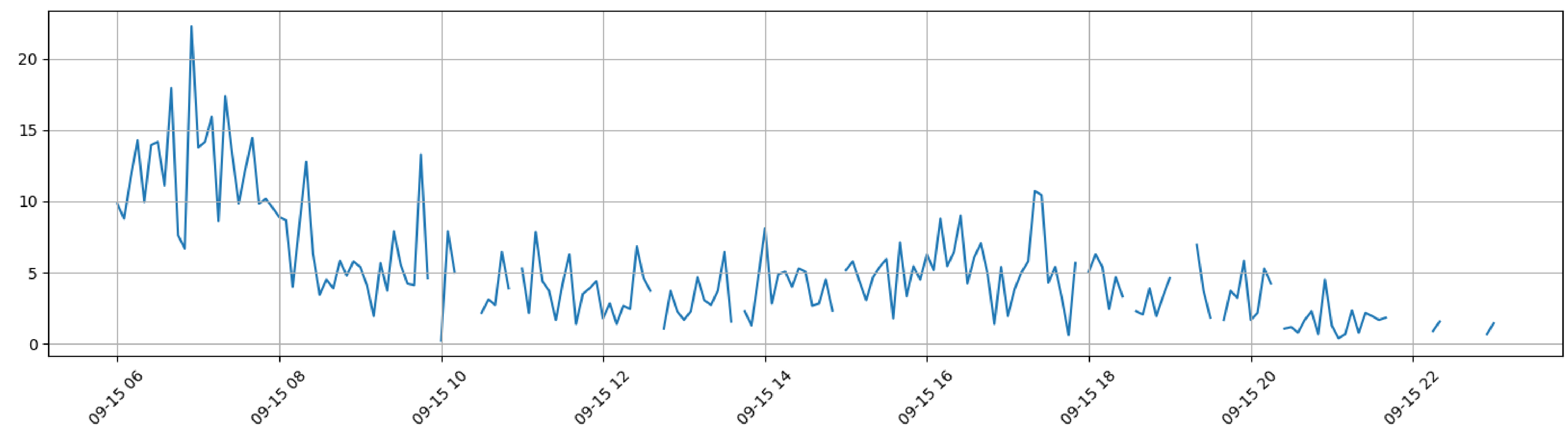
Where x_i is a value from the filled series and $\hat{x}_i$ the ground truth

- $x_i = \hat{x}_i$ if no value is missing
- Hence, any MSE difference is entirely due to missing values

The Benchmark Dataset

Our benchmark dataset will consist of this particular stretch from our traffic series

In [2]:
```
util.plot_series(ddata, figsize=figsize)
```



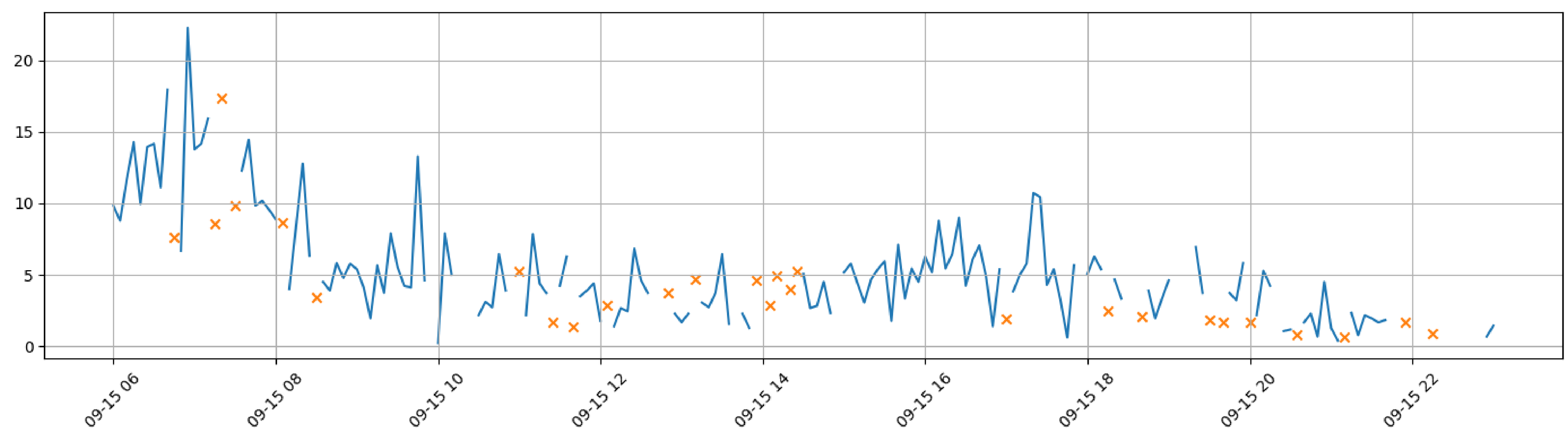
- There are (comparatively) few missing values
- Some of them are isolated, some for contiguous "holes"

The Benchmark Dataset

We now introduce some missing values

...By drawing them at random:

In [3]:
```python
np.random.seed(42) # seed (to get reproducible results)
mv_idx = np.random.choice(range(len(ddata.index)), size=30, replace=False)
ddata_mv = ddata.copy()
ddata_mv.iloc[mv_idx] = np.NaN
util.plot_series(ddata_mv, figsize=figsize)
plt.scatter(ddata.index[mv_idx], ddata.iloc[mv_idx], color='tab:orange', marker='x');
```



- The orange markers represent the value that have been removed

Forward/Backward Filling

The easiest approach for missing values consists in replicating nearby observations

- Forward filling: propagate forward the last valid observation
- Backward filling: propagate backward the next valid observation

An important observation:

- When filling missing values, we have access to the whole series
- ...So we can reason both forward and backward

Forward/backward filling are simple methods, but they can work well

- Rationale: most time series have a certain "inertia"
- ...I.e.: a strong level of local correlation
- For this reason (e.g.) the last observation is often a good predictor for the next one

Forward/Backward Filling

Forward and backward filling are pre-implemented in pandas

They are available through the `fillna` method:

```
DataFrame.fillna(..., method=None, ...)
```

- `fillna` replaces `NaN` values in a `DataFrame` or `Series`
- The `method` parameter can take the values:
  - "pad" or "ffill": these correspond to forward filling
  - "backfill" or "bfill": these correspond to backward filling

They are generally applied to datasets with a dense index

- Remember that our benchmark dataset already has a dense index

Forward/Backward Filling on the Benchmark

We can finally test forward/backward filling

In [4]:
```
nan_mask = ddata['value'].isnull()
ffseries = ddata_mv.fillna(method='ffill')
ffseries[nan_mask] = np.NaN # We empty the values that were originally empty
bfseries = ddata_mv.fillna(method='bfill')
bfseries[nan_mask] = np.NaN # We empty the values that were originally empty
```

We can check the corresponding RMSE:

In [5]:
```
rmse_ff = np.sqrt(mean_squared_error(ddata[~nan_mask], ffseries[~nan_mask]))
rmse_bf = np.sqrt(mean_squared_error(ddata[~nan_mask], bfseries[~nan_mask]))
print(f'RMSE for forwad filling: {rmse_ff:.2f}, for backward filling {rmse_bf:.2f}')
```

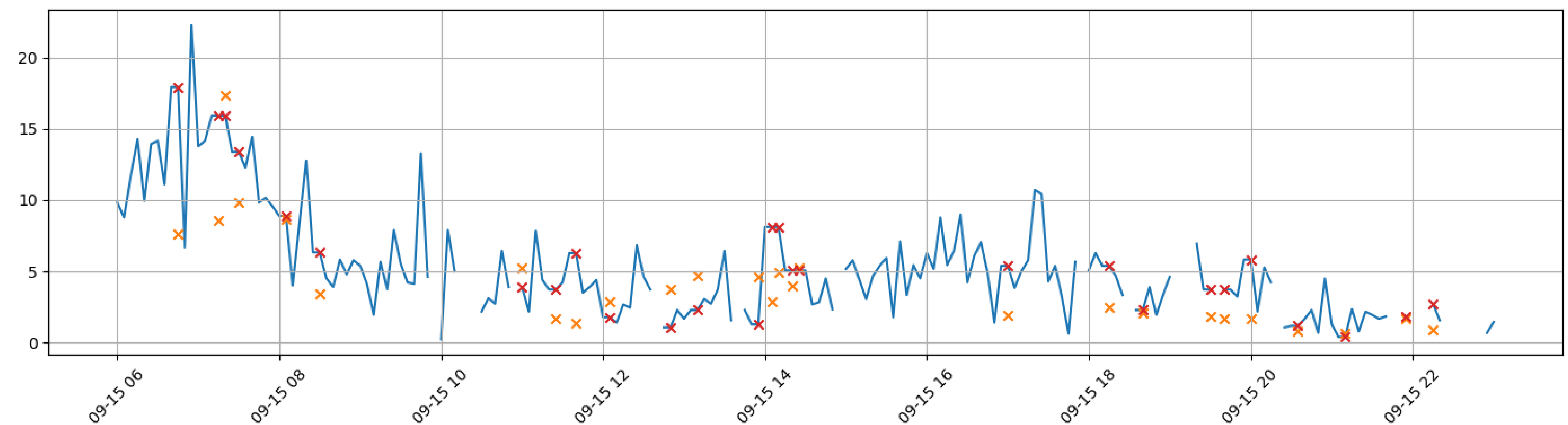
```
RMSE for forwad filling: 1.33, for backward filling 0.87
```

- In this case backward filling seems to work better
- The results are of course application-dependent

Forward/Backward Filling on the Benchmark

Let's have a close look at the results for forward filling

In [6]:
```
util.plot_series(ffseries, figsize=figsize)
plt.scatter(ddata.index[mv_idx], ddata.iloc[mv_idx], color='tab:orange', marker='x');
plt.scatter(ddata.index[mv_idx], ffseries.iloc[mv_idx], color='tab:red', marker='x');
```

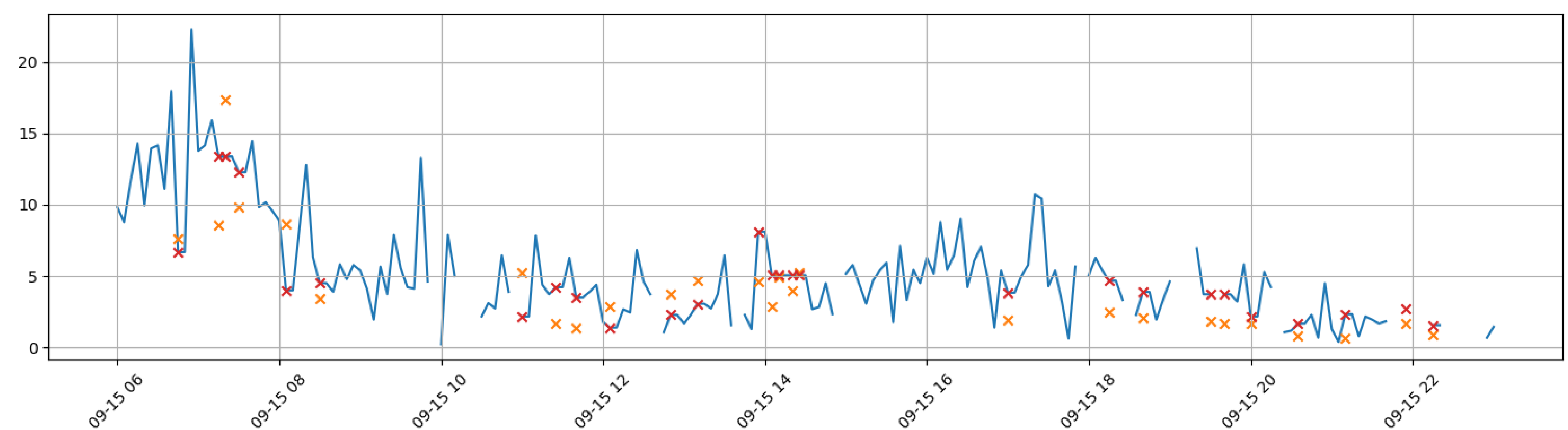


- The red marks represent the values that have been filled

Forward/Backward Filling on the Benchmark

Let's have a close look at the results for backward filling

```
In [7]: util.plot_series(bfseries, figsize=figsize)
        plt.scatter(ddata.index[mv_idx], ddata.iloc[mv_idx], color='tab:orange', marker='x');
        plt.scatter(ddata.index[mv_idx], bfseries.iloc[mv_idx], color='tab:red', marker='x');
```



- Forward/backward filling tend to work well for low variance sections
- ...And conversely work worse for high variance sections

(Geometric) Interpolation

A few more options are available via the `interpolate` method

```
DataFrame/Series.interpolate(method='linear', ...)
```

The `method` parameter determines how NaNs are filled:

- "linear" uses a linear interpolation, assuming uniformly spaced samples
- "time" uses a linear interpolation, but supports non-uniformly spaced samples
- "nearest" uses the closest value
- "polynomial" uses a polynomial interpolation
- Even "ffill" and "bfill" are available

Both "polynomial" and "spline" require to specify the additional parameter `order`

- E.g. `df.interpolate(method='polynomial', order='3')`

(Geometric) Interpolation

Let us check the performance of some approaches

In [8]:
```
args = [{'method':'linear'}, {'method':'time'}, {'method':'nearest'},
        {'method':'polynomial', 'order': 2}, {'method':'spline', 'order': 4}]
filling_res = {}
for a in args:
    filling_res[a['method']] = ddata_mv.interpolate(**a)
    rmse = np.sqrt(mean_squared_error(ddata[~nan_mask], filling_res[a['method']][~nan_mask]))
    print(f'RMSE for {a["method"]}: {rmse:.2f}')
```

```
RMSE for linear: 0.98
RMSE for time: 0.98
RMSE for nearest: 1.35
RMSE for polynomial: 1.10
RMSE for spline: 1.03
```

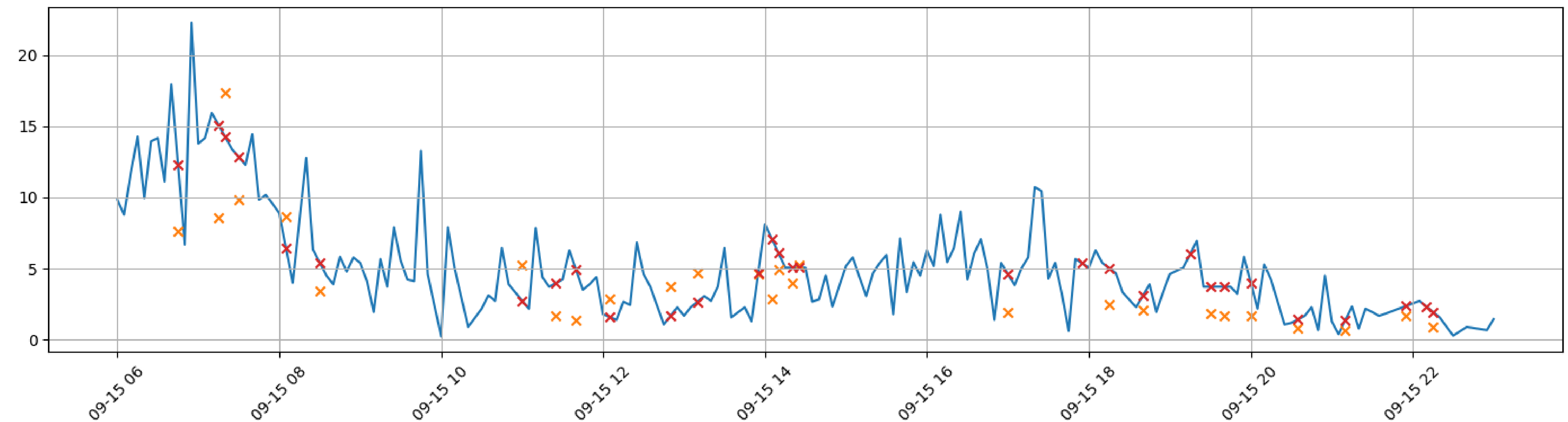
- "linear" and "time" are equivalent (we have uniformly-spaced samples)
- "polynomial" is the most complex, and in this case also the worst

All perform worse than backward filling (at least in this case)!

Linear Filling on the Benchmark

Let's have a close look at the results for linear filling

In [9]:
```
util.plot_series(filling_res['linear'], figsize=figsize)
plt.scatter(ddata.index[mv_idx], ddata.iloc[mv_idx], color='tab:orange', marker='x');
plt.scatter(ddata.index[mv_idx], filling_res['linear'].iloc[mv_idx], color='tab:red', marker
```

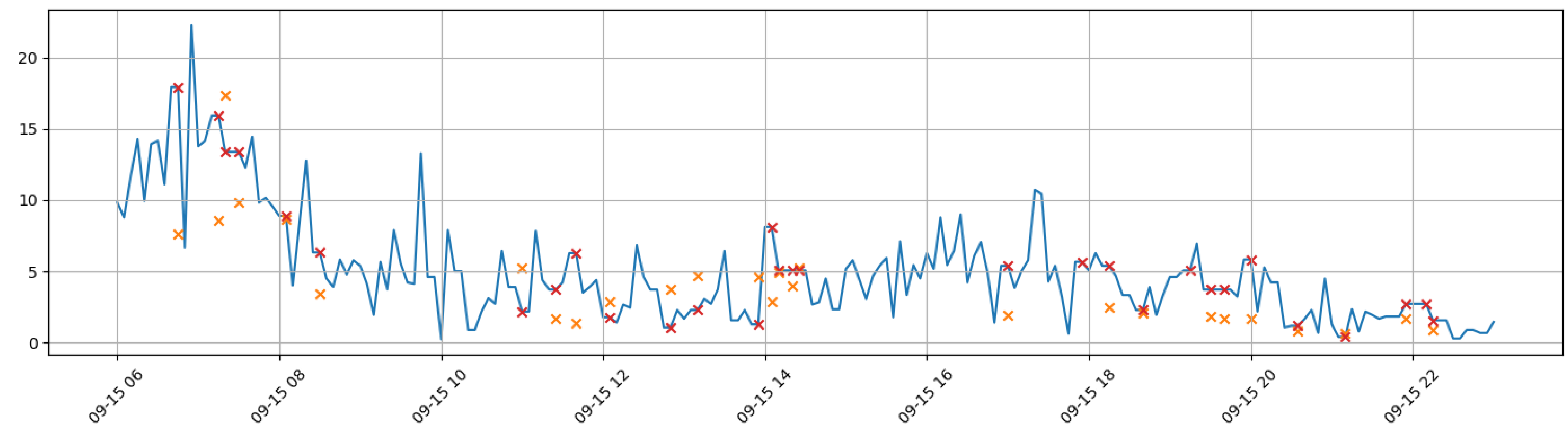


- Linear filling works well for series with slower dynamics
- ...But does not work well for series that faster dynamics

Nearest Filling on the Benchmark

Let's have a close look at the results for nearest filling

```
In [10]: util.plot_series(filling_res['nearest'], figsize=figsize)
plt.scatter(ddata.index[mv_idx], ddata.iloc[mv_idx], color='tab:orange', marker='x');
plt.scatter(ddata.index[mv_idx], filling_res['nearest'].iloc[mv_idx], color='tab:red', marke
```

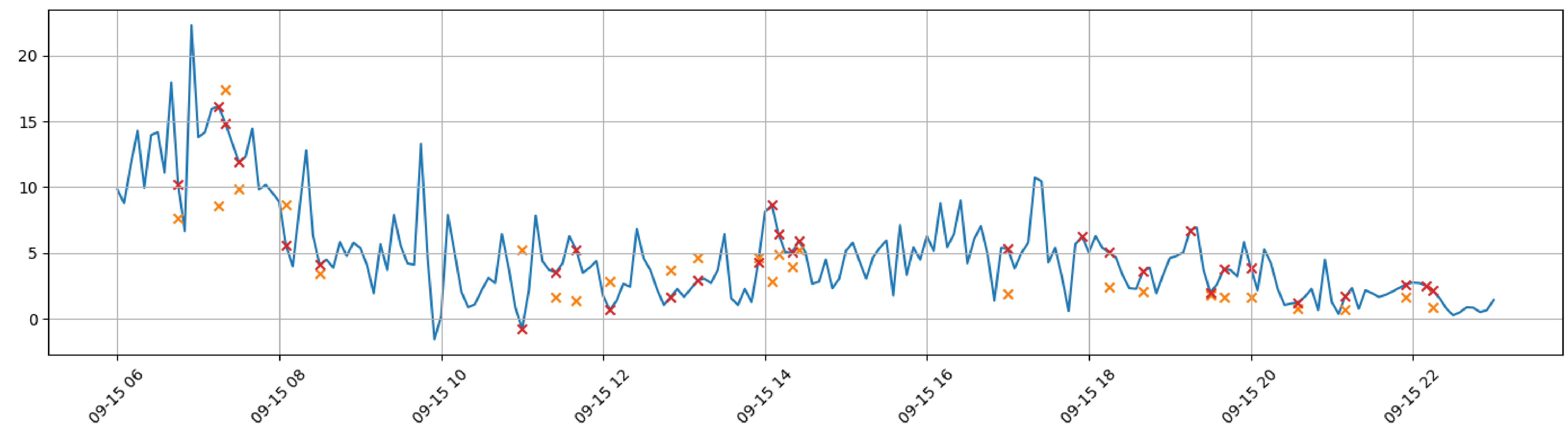


- Nearest filling is a compromise between forward and backward filling
- ...And in our case its performance is the average of the two

Polynomial Filling on the Benchmark

Let's have a close look at the results for polynomial filling

```
In [11]: util.plot_series(filling_res['polynomial'], figsize=figsize)
         plt.scatter(ddata.index[mv_idx], ddata.iloc[mv_idx], color='tab:orange', marker='x');
         plt.scatter(ddata.index[mv_idx], filling_res['polynomial'].iloc[mv_idx], color='tab:red', ma
```

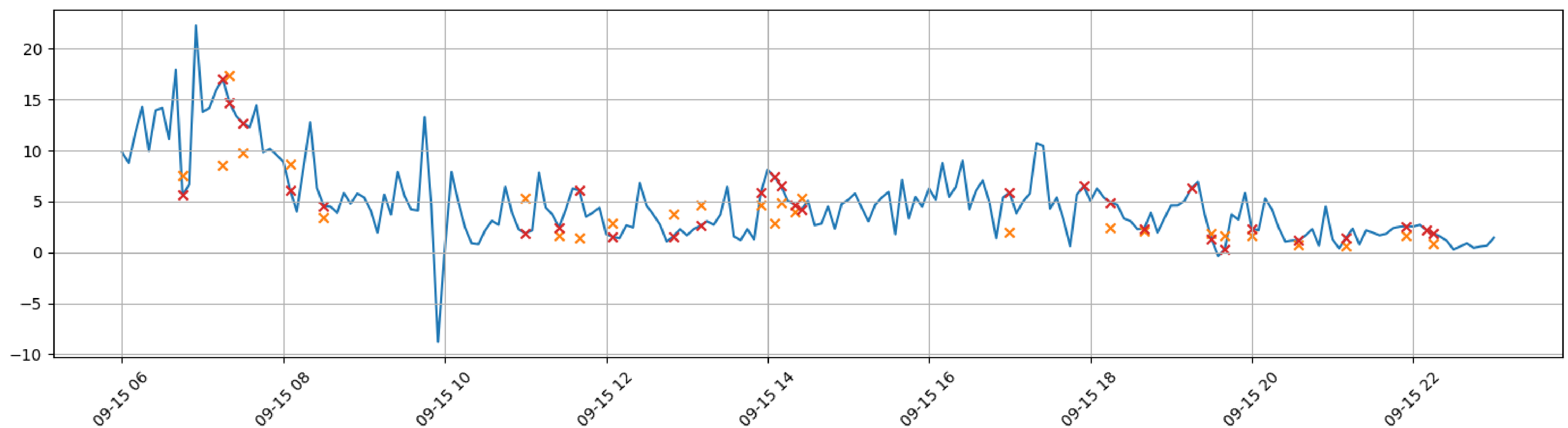


- Polynomial interpolation relies on nearby values to fit a polynomial
- High-order polinomial often vary too much and work less well

Spline Filling on the Benchmark

Let's have a close look at the results for spline filling

In [12]:
```
util.plot_series(filling_res['spline'], figsize=figsize)
plt.scatter(ddata.index[mv_idx], ddata.iloc[mv_idx], color='tab:orange', marker='x');
plt.scatter(ddata.index[mv_idx], filling_res['spline'].iloc[mv_idx], color='tab:red', marker
```



- Spline interpolation relies on piecewise polynomial curves
- ...And it's often more robust than polynomial interpolation

How do we choose which method is best? Is the (R)MSE enough?