RUL Prediction as Regression

Say we want to define a RUL-based maintenance policy

How would you tackle that problem?

RUL Prediction as Regression

Let's start from the simpler formulation of a RUL-based policy

- We will predict the RUL using a regression approach
- ...And trigger maintenance when the estimated RUL becomes too low, i.e.:

$$f(x, \theta) \leq \varepsilon$$

- f is the regressor, with parameter vector θ
- The threshold ε must account for possible estimation errors

We will focus on the hardest of the four datasets (to reduce training times):

In [3]:
```
data_by_src = util.split_by_field(data, field='src')
dt = data_by_src['train_FD004']
```

We now need to define our training and test data

How do we proceed?

Training and Test Data

We now need to define our training and test data

In a practical setting:

- Some run-to-failure experiments will form the training set
- Others run-to-failure experiments will be used for testing

I.e. we split whole experiments rather than individual examples!

Each run-to-failure experiment in our data is associated to a machine

Let's check how many we have:

```
In [4]: print(f'Number of machines: {len(dt.machine.unique())}')
```

```
Number of machines: 249
```

- This is actually a very large number
- In most practical setting, much fewer experiments will be available

Training and Test Data

Let's use 75% of the machine for training, the rest for testing

First, we partition the machine indexes:

In [5]:
```
tr_ratio = 0.75
np.random.seed(42)
machines = dt.machine.unique()
np.random.shuffle(machines)

sep = int(tr_ratio * len(machines))
tr_mcn = machines[:sep]
ts_mcn = machines[sep:]
```

Then, we partition the dataset itself (via a helper function):

In [6]:
```
tr, ts = util.partition_by_machine(dt, tr_mcn)
```

Training and Test Data

Let's have a look at the training data

In [7]:
```
tr
```

Out[7]:

	src	machine	cycle	p1	p2	p3	s1	s2	s3	s4	...	s13	s14	s15	s1
0	train_FD004	461	1	42.0049	0.8400	100.0	445.00	549.68	1343.43	1112.93	...	2387.99	8074.83	9.3335	0.0
1	train_FD004	461	2	20.0020	0.7002	100.0	491.19	606.07	1477.61	1237.50	...	2387.73	8046.13	9.1913	0.0
2	train_FD004	461	3	42.0038	0.8409	100.0	445.00	548.95	1343.12	1117.05	...	2387.97	8066.62	9.4007	0.0
3	train_FD004	461	4	42.0000	0.8400	100.0	445.00	548.70	1341.24	1118.03	...	2388.02	8076.05	9.3369	0.0
4	train_FD004	461	5	25.0063	0.6207	60.0	462.54	536.10	1255.23	1033.59	...	2028.08	7865.80	10.8366	0.0
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
60989	train_FD004	708	180	35.0019	0.8409	100.0	449.44	556.28	1377.65	1148.96	...	2387.77	8048.91	9.4169	0.0
60990	train_FD004	708	181	0.0023	0.0000	100.0	518.67	643.95	1602.98	1429.57	...	2388.27	8122.44	8.5242	0.0
60991	train_FD004	708	182	25.0030	0.6200	60.0	462.54	536.88	1268.01	1067.09	...	2027.98	7865.18	10.9790	0.0
60992	train_FD004	708	183	41.9984	0.8414	100.0	445.00	550.64	1363.76	1145.72	...	2387.48	8069.84	9.4607	0.0
60993	train_FD004	708	184	0.0013	0.0001	100.0	518.67	643.50	1602.12	1430.34	...	2388.33	8120.43	8.4998	0.0

45385 rows × 28 columns

Training and Test Data

...And at the test data

In [8]:
```
ts
```

Out[8]:

	src	machine	cycle	p1	p2	p3	s1	s2	s3	s4	...	s13	s14	s15	s1
321	train_FD004	462	1	41.9998	0.8400	100.0	445.00	548.99	1341.82	1113.16	...	2387.98	8082.37	9.3300	0.0
322	train_FD004	462	2	9.9999	0.2500	100.0	489.05	604.23	1498.00	1299.54	...	2388.07	8125.46	8.6088	0.0
323	train_FD004	462	3	42.0079	0.8403	100.0	445.00	549.11	1351.47	1126.43	...	2387.93	8082.11	9.2965	0.0
324	train_FD004	462	4	42.0077	0.8400	100.0	445.00	548.77	1345.81	1116.64	...	2387.88	8079.41	9.3200	0.0
325	train_FD004	462	5	24.9999	0.6200	60.0	462.54	537.00	1259.55	1043.95	...	2028.13	7867.08	10.8841	0.0
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
61244	train_FD004	709	251	9.9998	0.2500	100.0	489.05	605.33	1516.36	1315.28	...	2388.73	8185.69	8.4541	0.0
61245	train_FD004	709	252	0.0028	0.0015	100.0	518.67	643.42	1598.92	1426.77	...	2388.46	8185.47	8.2221	0.0
61246	train_FD004	709	253	0.0029	0.0000	100.0	518.67	643.68	1607.72	1430.56	...	2388.48	8193.94	8.2525	0.0
61247	train_FD004	709	254	35.0046	0.8400	100.0	449.44	555.77	1381.29	1148.18	...	2388.83	8125.64	9.0515	0.0
61248	train_FD004	709	255	42.0030	0.8400	100.0	445.00	549.85	1369.75	1147.45	...	2388.66	8144.33	9.1207	0.0

15864 rows × 28 columns

Standardization/Normalization

We will use a Neural Network regressor

...Therefore, we need to make the range of each columns more uniform

- We will standardize all parameters and sensor inputs:

```
In [9]: trmean = tr[dt_in].mean()
        trstd = tr[dt_in].std().replace(to_replace=0, value=1) # handle static fields

        ts_s = ts.copy()
        ts_s[dt_in] = (ts_s[dt_in] - trmean) / trstd
        tr_s = tr.copy()
        tr_s[dt_in] = (tr_s[dt_in] - trmean) / trstd
```

- We will normalize the RUL values (i.e. our regression target)

```
In [10]: trmaxrul = tr['rul'].max()

         ts_s['rul'] = ts['rul'] / trmaxrul
         tr_s['rul'] = tr['rul'] / trmaxrul
```

Standardization/Normalization

Let's check the results

In [11]:
```
tr_s.describe()
```

Out[11]:

	machine	cycle	p1	p2	p3	s1	s2	s3
count	45385.000000	45385.000000	4.538500e+04	4.538500e+04	4.538500e+04	4.538500e+04	4.538500e+04	4.538500e+04
mean	582.490955	133.323896	2.894775e-16	1.302570e-16	1.178889e-16	4.664830e-15	2.522791e-15	1.727041e-15
std	71.283034	89.568561	1.000000e+00	1.000000e+00	1.000000e+00	1.000000e+00	1.000000e+00	1.000000e+00
min	461.000000	1.000000	-1.623164e+00	-1.838222e+00	-2.381839e+00	-1.055641e+00	-1.176507e+00	-1.646830e+00
25%	521.000000	62.000000	-9.461510e-01	-1.031405e+00	4.198344e-01	-1.055641e+00	-8.055879e-01	-6.341243e-01
50%	585.000000	123.000000	6.868497e-02	4.154560e-01	4.198344e-01	-3.917563e-01	-6.336530e-01	-4.718540e-01
75%	639.000000	189.000000	1.218855e+00	8.661917e-01	4.198344e-01	6.926385e-01	7.407549e-01	7.495521e-01
max	708.000000	543.000000	1.219524e+00	8.726308e-01	4.198344e-01	1.732749e+00	1.741030e+00	1.837978e+00

8 rows × 27 columns

Regression Model

We can now define a regression model

We will use a feed-forward neural network (MLP):

```
def build_nn_model(input_shape, output_shape, hidden, output_activation='linear'):
    model_in = keras.Input(shape=input_shape, dtype='float32')
    x = model_in
    for h in hidden:
        x = layers.Dense(h, activation='relu')(x)
    model_out = layers.Dense(output_shape, activation=output_activation)(x)
    model = keras.Model(model_in, model_out)
    return model
```

- The `hidden` argument is a list of sizes for the hidden layers
- ...E.g. `hidden = [64, 32]`

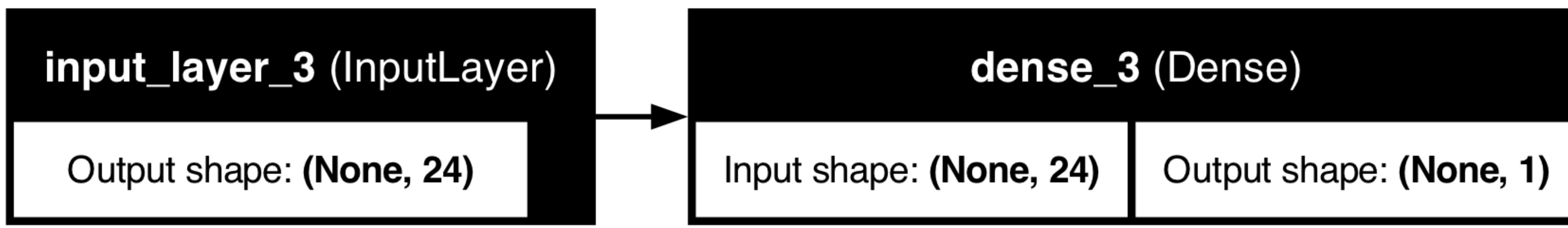
Regression Model

We will start with the simplest possible Neural Network

...Meaning a Linear Regressor!

In [15]:
```
nn1 = util.build_nn_model(input_shape=(len(dt_in), ), output_shape=1, hidden=[])
util.plot_nn_model(nn1)
```

Out[15]:



- We just need to specify that there are no hidden layers
- Why the simplest? As usual, due to Occam's razor

Training

It's useful to define a generic training function

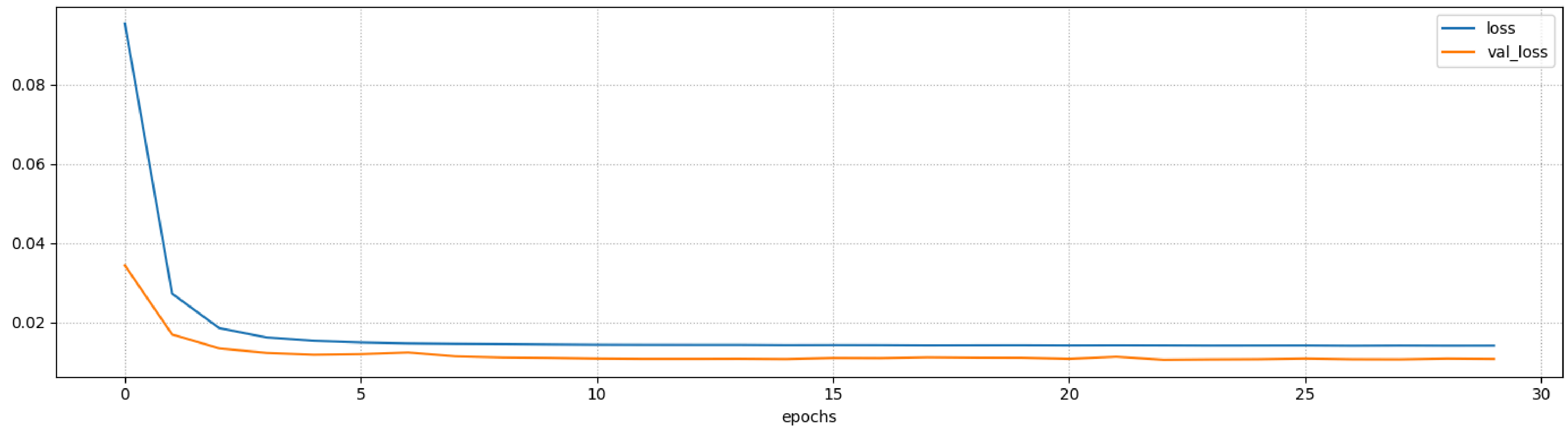
As usual, you can find it in the `util` module:

```
def train_nn_model(model, X, y, loss,
        verbose=0, patience=10,
        validation_split=0.0, **fit_params):
    # Compile the model
    model.compile(optimizer='Adam', loss=loss)
    # Build the early stop callback
    cb = []
    if validation_split > 0:
        cb += [callbacks.EarlyStopping(patience=patience,
            restore_best_weights=True)]
    # Train the model
    history = model.fit(X, y, callbacks=cb, validation_split=validation_split,
            verbose=verbose, **fit_params)
    return history
```

Training

We can now train our model

```
In [16]: nn1 = util.build_nn_model(input_shape=(len(dt_in), ), output_shape=1, hidden=[])
         history = util.train_nn_model(nn1, tr_s[dt_in], tr_s['rul'], loss='mse', epochs=30, validat
         util.plot_training_history(history, figsize=figsize)
```

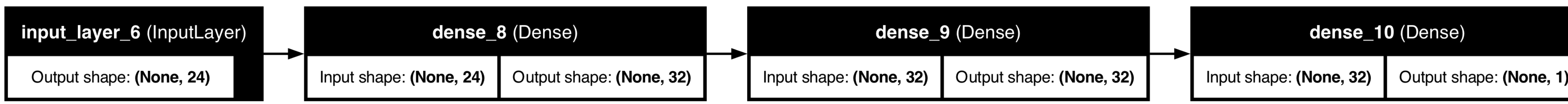



```
Final loss: 0.0142 (training), 0.0108 (validation)
```

Let's try with a more complex model

In [18]:
```python
nn2 = util.build_nn_model(input_shape=(len(dt_in), ), output_shape=1, hidden=[32, 32])
util.plot_nn_model(nn2)
```

Out[18]:

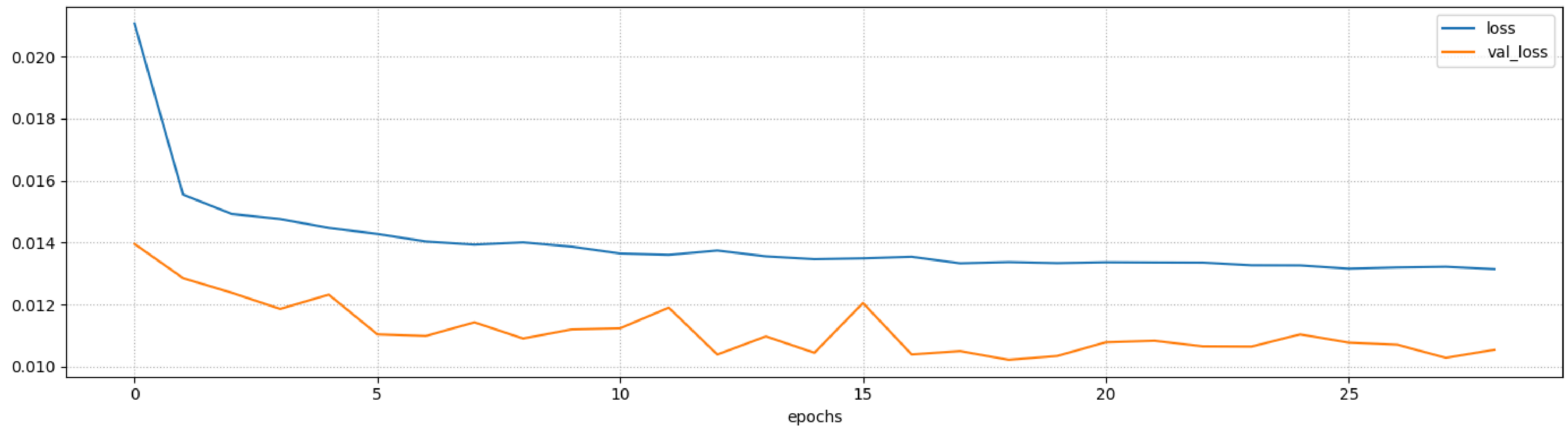


- Now we have two hidden layers
- ...Each with 32 ReLU neurons

Training

Let's check the loss behavior and compare it to Linear Regression

In [23]:
```
nn2 = util.build_nn_model(input_shape=(len(dt_in), ), output_shape=1, hidden=[32, 32])
history = util.train_nn_model(nn2, tr_s[dt_in], tr_s['rul'], loss='mse', epochs=30, validat
util.plot_training_history(history, figsize=figsize)
```




```
Final loss: 0.0131 (training), 0.0105 (validation)
```

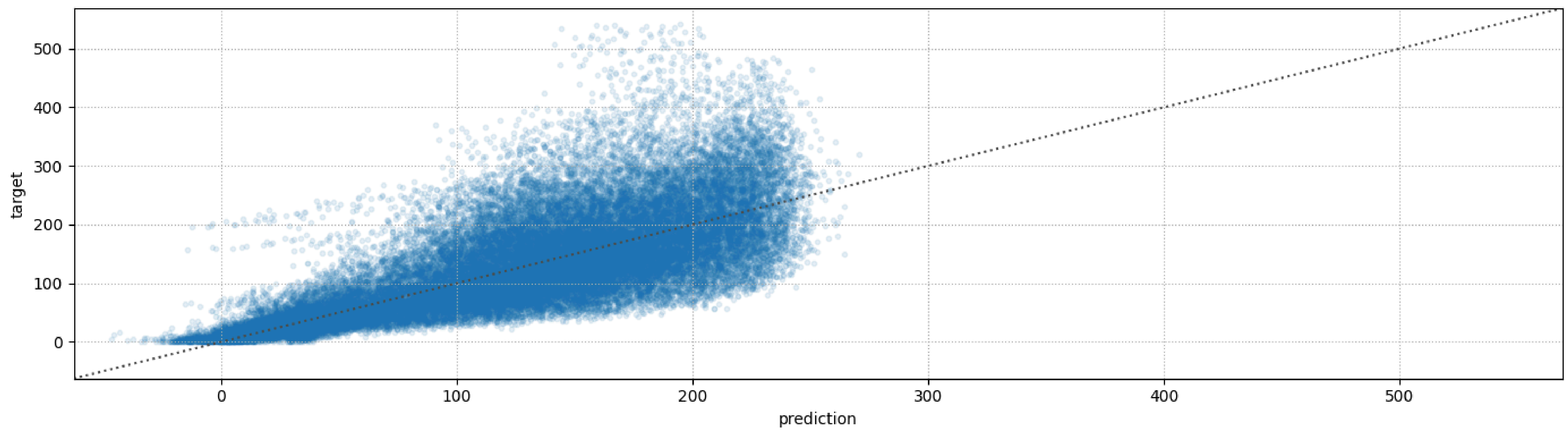
- There is a modest improvement w.r.t. Linear Regression

Predictions

We can now obtain the predictions and evaluate their quality

```
In [24]: tr_pred = nn2.predict(tr_s[dt_in], verbose=0).ravel() * trmaxrul
         util.plot_pred_scatter(tr_pred, tr['rul'], figsize=figsize)
         print(f'R2 score: {r2_score(tr["rul"], tr_pred)}')
```

```
R2 score: 0.5380727648735046
```



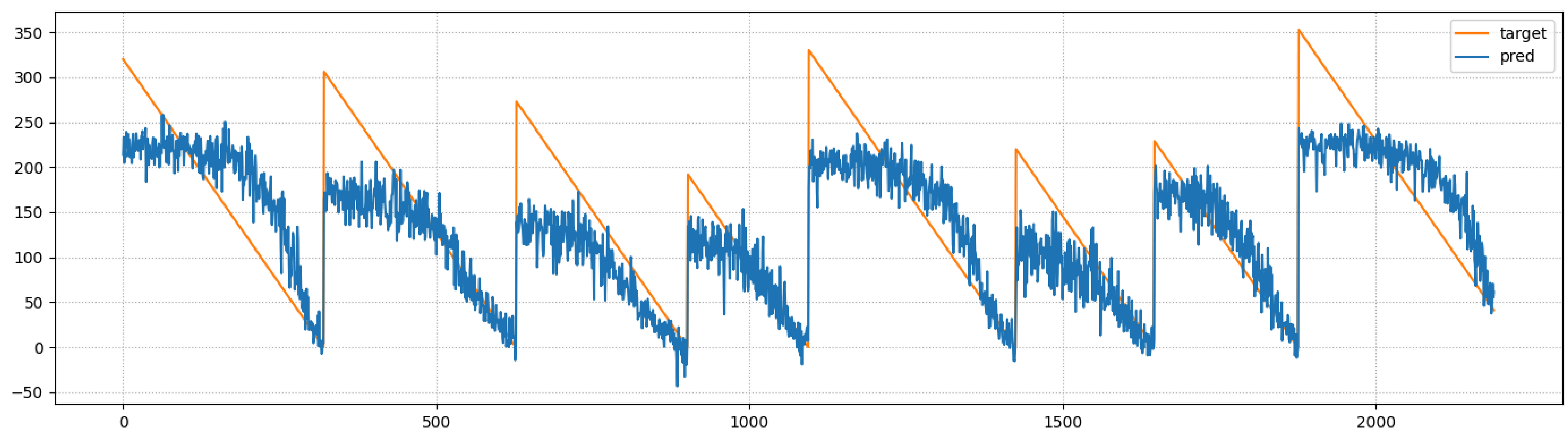
What do you think of these results?

Predictions

The results so far are not comforting

...But it's worth seeing what is going on over time:

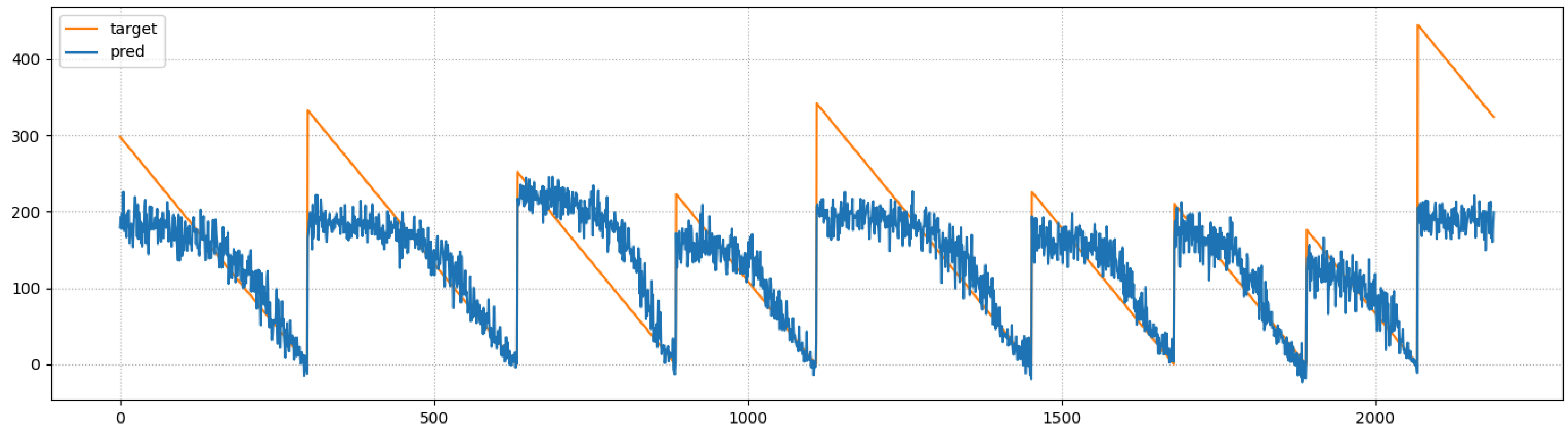
```
In [28]: stop = 2*1095
         util.plot_rul(tr_pred[:stop], tr['rul'][:stop], figsize=figsize)
```



Predictions

The situation is similar on the test set:

```
In [29]: ts_pred = nn2.predict(ts_s[dt_in], verbose=0).ravel() * trmaxrul
         util.plot_rul(ts_pred[:stop], ts['rul'][:stop], figsize=figsize)
```



Quality Evaluation

Let's try to recap the situation

- Our accuracy is quite poor
- ...Especially for large RUL values

Possible reasons:

- Large RUL value are somewhat scarce on the dataset
- Fault effects become noticeable only after a while

Quality Evaluation

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Possible reasons:

- Large RUL value are somewhat scarce on the dataset
- Fault effects become noticeable only after a while

But perhaps we don't care! Our goal is not a high accuracy

- We just need to stop at the right time
- ...And our model may still be good enough for that

For a proper evaluation, we need a cost model

Cost Model

We will assume that:

We consider one step of operation as our value unit

- ...So we can express the failure cost in terms of operating steps

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Cost Model

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- ...We can diseregard the maintenance cost

A traditional preventive maintenance policy is also available

- We will never trigger maintenance ealier that such policy
- We only gain value if we beat such policy

Cost Model

Slighly more formally:

- One step of operation brings 1 unit of profit
- A failure costs C units more than maintenance
- We only count what happens after s steps

Formally, let x_k be the times series for machine k, and I_k its set of time steps

- The time step when our policy triggers maintenance is given by:

$$\min\{i \in I_k \mid f(x_{ki}) < \varepsilon\}$$

- A failure occurs if:

$$f(x_{ki}) \geq \varepsilon \quad \forall i \in I_k$$

Cost Model

The whole cost formula for a single machine will be:

$$cost(f(x_k), \varepsilon) = op\_profit(f(x_k), \varepsilon) + fail\_cost(f(x_k), \varepsilon)$$

Where:

$$op\_profit(f(x_k), \varepsilon) = -\max(0, \min\{i \in I_k \mid f(x_{ki}) < \varepsilon\} - s)$$

$$fail\_cost(f(x_k), \varepsilon) = \begin{cases} C \text{ if } f(x_{ki}) \geq \varepsilon & \forall i \in I_k \\ 0 \text{ otherwise} \end{cases}$$

- s units of machine operation are guaranteed
- ...So we gain over the default policy only if we stop after that
- Profit is modeled as a negative cost

For the total cost, we need to sum over all machines

Cost Model

Normally, we would proceed as follows

- s is determined by the preventive maintenance schedule
- C must be determined by discussing with the customer

In our example, we will derive both from data

First, we collect all failure times

In [30]:
```
failtimes = dt.groupby('machine')['cycle'].max()
failtimes.head()
```

Out[30]:
```
machine
461    321
462    299
463    307
464    274
465    193
Name: cycle, dtype: int64
```

Cost Model

Then, we define s and C based on statistics

In [31]:
```
print(failtimes.describe())
safe_interval = failtimes.min()
maintenance_cost = failtimes.max()
```

```
count    249.00000
mean     245.97992
std       73.11080
min      128.00000
25%      190.00000
50%      234.00000
75%      290.00000
max      543.00000
Name: cycle, dtype: float64
```

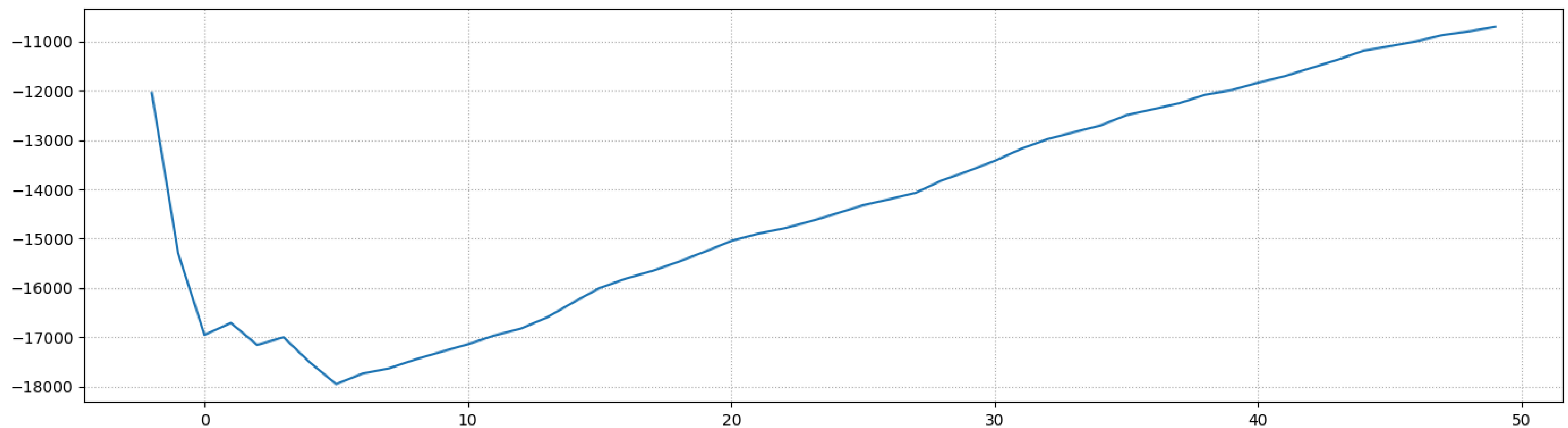
- For the safe interval s, we choose the minimum failure time
- For the maintenance cost C we choose the largest failure time

Threshold Choice

We can then choose the threshold θ as usual

In [36]:
```
cmodel = util.RULCostModel(maintenance_cost=maintenance_cost, safe_interval=safe_interval)
th_range = np.arange(-2, 50)
tr_thr = util.opt_threshold_and_plot(tr['machine'].values, tr_pred, th_range, cmodel, figsi
print(f'Optimal threshold for the training set: {tr_thr}')
```

```
Optimal threshold for the training set: 5
```



Evaluation

Let's see how we fare in terms of cost

In [37]:
```
tr_c, tr_f, tr_sl = cmodel.cost(tr['machine'].values, tr_pred, tr_thr, return_margin=True)
ts_c, ts_f, ts_sl = cmodel.cost(ts['machine'].values, ts_pred, tr_thr, return_margin=True)
print(f'Avg. cost: {tr_c/len(tr_mcn):.2f} (training), {ts_c/len(ts_mcn):.2f} (test)')
```

```
Avg. cost: -96.51 (training), -106.02 (test)
```

We can also evaluate the margin for improvement:

In [38]:
```
print(f'Avg. fails: {tr_f/len(tr_mcn)} (training), {ts_f/len(ts_mcn)} (test)')
print(f'Avg. slack: {tr_sl/len(tr_mcn):.2f} (training), {ts_sl/len(ts_mcn):.2f} (test)')
```

```
Avg. fails: 0.0 (training), 0.0 (test)
Avg. slack: 19.59 (training), 17.79 (test)
```

- Slack = distance between when we stop and the failure
- The results are actually quite good!
- ...And we also generalize fairly well

Sequence Input in Neural Models

Sequence Input in Neural Models

Feeding more time steps to our NN might improve the results

- Intuitively, sequences provide information about the trend
- This may allow a better RUL estimate w.r.t. using only the current state
- E.g. we may gauge how quickly the component is deteriorating

We will try to build a model capable to processing such input

But first, we need to apply a sliding window:

In [39]:
```
wlen = 3
tr_sw, tr_sw_m, tr_sw_r = util.sliding_window_by_machine(tr_s, wlen, dt_in)
ts_sw, ts_sw_m, ts_sw_r = util.sliding_window_by_machine(ts_s, wlen, dt_in)
```

- This must be done on a per machine basis
- Windows should not mix data belonging to different machines!

Sliding Window for Multivariate Data

The `sliding_window_by_machine` relies internally on:

```
def sliding_window_2D(data, wlen, stride=1):
    # Get shifted _tables_
    m = len(data)
    lt = [data.iloc[i:m-wlen+i+1:stride, :].values for i in range(wlen)]
    # Reshape to _add a new axis_
    s = lt[0].shape
    for i in range(wlen):
        lt[i] = lt[i].reshape(s[0], 1, s[1])
    # Concatenate
    wdata = np.concatenate(lt, axis=1)
    return wdata
```

- It's similar to our code for the univariate case
- The output is a tensor with shape `(n_windows, w_len, n_dims)`

Sliding Window for Multivariate Data

Let's look in deeper detail at the returned data structures

`tr_sw` contain the actual sliding window data:

```
In [42]: tr_sw[0]
```

```
Out[42]: array([[ 1.21931469,  0.86619169,  0.41983436, -1.05564063, -0.79621447,
                 -0.70080293, -0.74549387, -1.1386061 , -1.08249848, -0.99389823,
                 -0.11421637, -0.6315044 , -0.67586863, -0.36411574, -0.98910425,
                  0.41889575,  0.08700467,  0.05991388, -0.69502688, -0.63793104,
                 -0.11268403,  0.41983436, -1.03117521, -1.03187757],
                [-0.26962527,  0.41609996,  0.41983436,  0.6926385 ,  0.71397375,
                  0.56288953,  0.29808726,  0.36365649,  0.3710279 ,  0.33249075,
                  0.65388538,  0.56210134, -0.20641916,  0.32893584,  0.33156802,
                  0.41687122, -0.24758681, -0.12925879, -0.69502688,  0.47652818,
                  0.65613725,  0.41983436,  0.35321893,  0.35869109],
                [ 1.21924025,  0.86908928,  0.41983436, -1.05564063, -0.8157647 ,
                 -0.70372248, -0.7109787 , -1.1386061 , -1.08433606, -0.98831315,
                 -0.11380415, -0.64524209, -0.67586863, -0.37335643, -0.99026013,
                  0.41874002, -0.00870947,  0.14931194, -0.69502688, -0.67388133,
                 -0.11268403,  0.41983436, -1.04527086, -1.02276728]])
```

Sliding Window for Multivariate Data

Let's look in deeper detail at the returned data structures

`tr_sw_m` contains the corresponding machine values

```
In [43]: tr_sw_m

Out[43]: array([461, 461, 461, ..., 708, 708, 708])
```

- The structure is a plain numpy array

`tr_sw_r` contains the RUL values

```
In [44]: tr_sw_r

Out[44]: array([0.58671587, 0.58487085, 0.58302583, ..., 0.00369004, 0.00184502,
                0.        ])
```

- Again, the structure is a plain numpy array

1D Convolutions in Keras

The chosen format is ideal for 1D convolutions in keras

We have a function to build 1D convolutional model in the `util` module

```
def build_cnn_model(input_shape, output_shape, wlen, conv=[], hidden=[], output_activati
='linear'):
    model_in = keras.Input(shape=input_shape, dtype='float32')
    x = model_in
    for k in conv:
        x = layers.Conv1D(k, kernel_size=3, activation='relu')(x)
    x = layers.Flatten()(x)
    for k in hidden:
        x = layers.Dense(k, activation='relu')(x)
    x = layers.Dense(output_shape, activation=output_activation)(x)
    model = keras.Model(model_in, x)
    return model
```

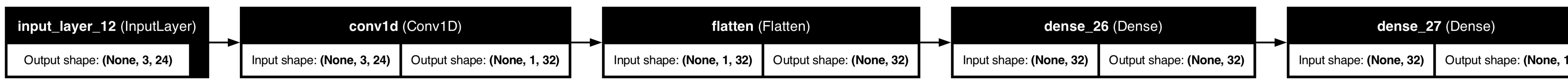
- Each convolution in our code will consider 3 time steps
- We need to "flatten" the input before the fully connected layers

1D Convolutions in Keras

Let's build a 1D convolutional model

In [45]:
```
nn3 = util.build_cnn_model(input_shape=(wlen, len(dt_in)), output_shape=1, wlen=wlen, conv=
util.plot_nn_model(nn3)
```

Out[45]:

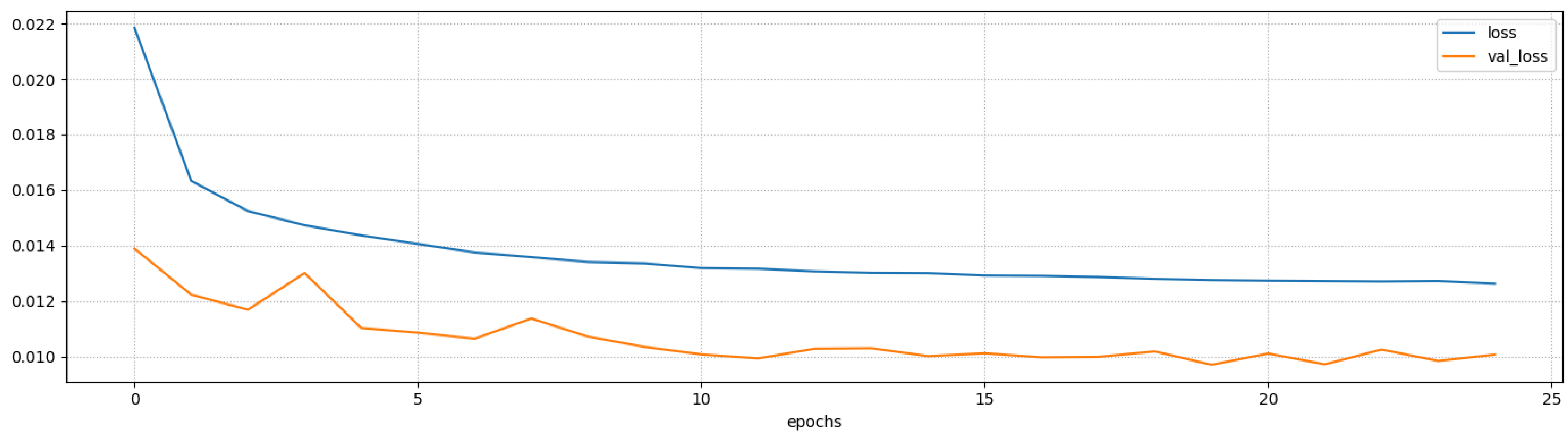


- We have a single convlution with 32 kernels
- Then a hidden layer with 32 ReLU neurons
- ...And finally the output layer

CNN Training

Let's train our CNN

```
In [46]: nn3 = util.build_cnn_model(input_shape=(wlen, len(dt_in)), output_shape=1, wlen=wlen, conv=
         history = util.train_nn_model(nn3, tr_sw, tr_sw_r, loss='mse', epochs=25, validation_split=(
         util.plot_training_history(history, figsize=figsize)
```




```
Final loss: 0.0126 (training), 0.0101 (validation)
```

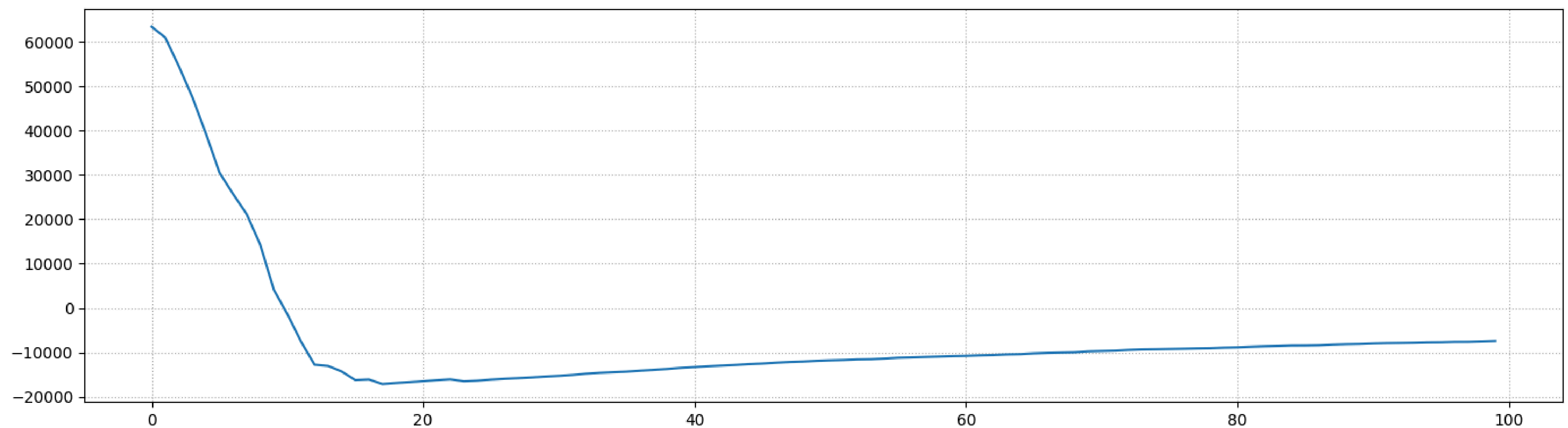
- We obtained a marginal improvement
- This suggest that considering sequence be modestly useful in this case

Threshold Optimization

Now we can proceed by choosing a threshold

In [29]:
```
tr_pred3 = nn3.predict(tr_sw, verbose=0).ravel() * trmaxrul
ts_pred3 = nn3.predict(ts_sw, verbose=0).ravel() * trmaxrul
tr_thr3 = util.opt_threshold_and_plot(tr_sw_m, tr_pred3, th_range, cmodel, figsize=figsize)
print(f'Optimal threshold for the training set: {tr_thr3}')
```

```
Optimal threshold for the training set: 17
```



Evaluation

Let's see how the CNN fares in terms of cost

In [30]:
```python
tr_c3, tr_f3, tr_sl3 = cmodel.cost(tr_sw_m, tr_pred3, tr_thr3, return_margin=True)
ts_c3, ts_f3, ts_sl3 = cmodel.cost(ts_sw_m, ts_pred3, tr_thr3, return_margin=True)
print(f'Cost: {tr_c3/len(tr_mcn):.2f} (training), {ts_c3/len(ts_mcn):.2f} (test)')
print(f'Avg. fails: {tr_f3/len(tr_mcn):.2f} (training), {ts_f3/len(ts_mcn):.2f} (test)')
print(f'Avg. slack: {tr_sl3/len(tr_mcn):.2f} (training), {ts_sl3/len(ts_mcn):.2f} (test)')
```

```
Cost: -92.10 (training), -105.86 (test)
Avg. fails: 0.01 (training), 0.00 (test)
Avg. slack: 18.98 (training), 16.21 (test)
```

Which is (more or less) on par with our MLP approach, for which we had:

In [31]:
```python
print(f'Cost: {tr_c/len(tr_mcn):.2f} (training), {ts_c/len(ts_mcn):.2f} (test)')
print(f'Avg. fails: {tr_f/len(tr_mcn):.2f} (training), {ts_f/len(ts_mcn):.2f} (test)')
print(f'Avg. slack: {tr_sl/len(tr_mcn):.2f} (training), {ts_sl/len(ts_mcn):.2f} (test)')
```

```
Cost: -98.37 (training), -109.06 (test)
Avg. fails: 0.00 (training), 0.00 (test)
Avg. slack: 17.85 (training), 14.87 (test)
```

Which stresses a simple, but important point...

Time Series and Sequence Input

Just because you are dealing with time series

...Do not assume that sequence input is useful!

- Sequences matter only if the output is correlated with patterns
- ...That involve multiple time steps

In many practical problems

...A single "state" encodes most of the useful information

- You can think of that as sort of Markov property

Therefore, before using sequences, it makes sense to think

Do you expect sequences to provide useful information?

- E.g. is there seom kind of inertia?
- ...And does it matter for the considered problem?