Survival Analysis using Neural Models

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- Then, we would need a lot of runs to obtain a good marginalization
- ...And data availability is a critical issue in RUL estimation

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The last observation deserves further attention

Censoring

In many domains, run-to-failure experiments are expensive to obtain

...But partial runs might abunant

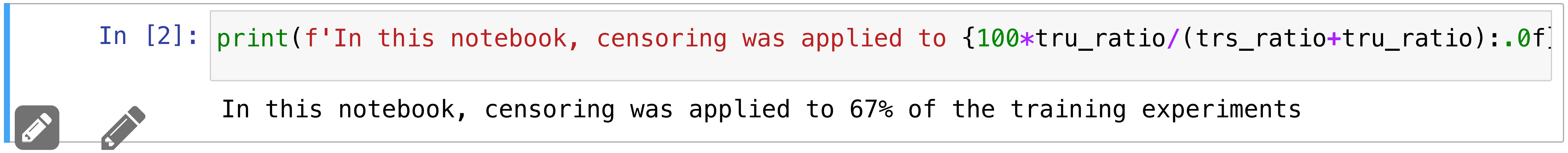
- Broken industrial machines vs regularly maintained ones
- Deaths in organ transplant waiting lists vs alive patients

The C-MAPSS dataset is very unrealistic from this point of view

The simulator is good, but there are way too many experiments

- We can simulate limited availability of supervised data
- ...By randomly truncating a portion of the training set

In survival analysis, the lack of key events is known as censoring



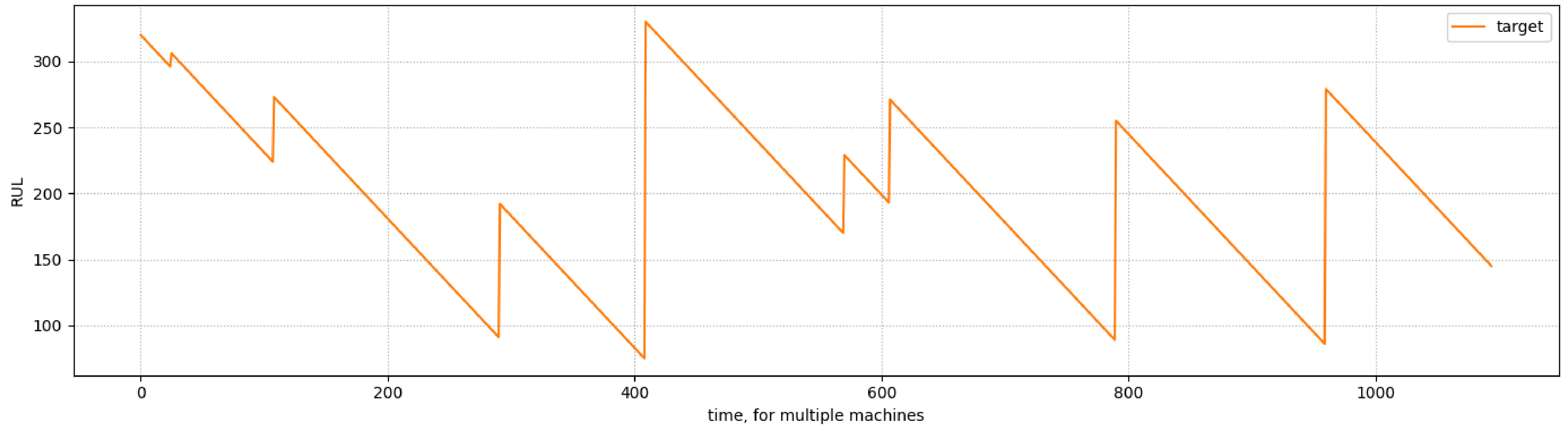
```
In [2]: print(f'In this notebook, censoring was applied to {100*tru_ratio/(trs_ratio+tru_ratio):.0f

        In this notebook, censoring was applied to 67% of the training experiments
```

Censoring

In our plots, censoring will cause irregularities in the sawtooth pattern

```
In [3]: stop = 1095
        util.plot_rul(target=tru['rul'][:stop], figsize=figsize, xlabel='time, for multiple machines
```



- We still can plot the RUL values, but only since we used simulated censoring
- In a real use case, we would have no RUL target for this data

Can we still take advantage of this data? How?

Survival Function

We could study the distribution of T via its survival function

The survival function of a variable T is defined as:

$$S(t) = P(T > t)$$

I.e. it the probability that the entity "survives" at least until time t

- It is the complement of the cumulative probability function $F(t) = P(T \leq t)$

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We can account for conditioning factors

...Which for the survival function only includes the past behavior

$$S(t, X_{\leq t}) = P(T > t \mid X_{\leq t})$$

- This means it cannot account for the future
- ...But also that it cannot overfit due to poor marginalization

...And Hazard Function

If we assume discrete time, then S can be factorized

$$S(t, X_{\leq t}) = (1 - \lambda(t, X_t))(1 - \lambda(t-1, X_{t-1}))\ldots$$

Where λ is called hazard function

The hazard function is a conditional probability

...That of not surving one more step. Formally:

- $\lambda(t, X_t)$ is the probability of not surviving at time t
- ...Given that the entity has survived until time $t - 1$. I.e.:

$$\lambda(t, X_t) = P(T > t \mid T > t-1, X_t)$$

As a side effect, λ only depends on one observation

Our Plan

We will attempt to train an estimator $\hat{\lambda}_\theta(t, x_t)$ for the hazard function

- This requires no assumption on the distribution (besides that of using S)
- It does not risk overfitting due to poor marginalization
- And it makes sense even if we do not observe a "death" event (censoring)

As a side effect, we also cannot account for future behavior

Additionally, S and λ have more limited uses

We can still define a threshold-based policy, e.g. by checking whether:

$$\hat{\lambda}_\theta(t, x_t) \geq \varepsilon$$

...But we'll see that making forecasts is not trivial and requires approximations

Training a Hazard Estimator

Before we get that, we need a way to train our $\hat{\lambda}_\theta$ estimator

We can start by modeling the probability of a survival event

- Say the k-th experiment in our dataset ends at time e_k
- Then the corresponding probability according to our estimator is:

$$\hat{\lambda}_\theta(e_k, x_{k,e_k}) \prod_{t=1}^{e_k-1} (1 - \hat{\lambda}_\theta(t, x_{k,t}))$$

Where $x_{k,t}$ is the available input data for experiment k at time t

This is the probability of:

- Surviving all time steps from 1 to $e_k - 1$
- Not surviving at time e_k

Training a Hazard Estimator

We can now formulate a likelihood maximization problem

Assuming we have m experiments, we get:

$$\operatorname*{argmax}_{\theta} \prod_{k=1}^{m} \hat{\lambda}_\theta(e_k, x_{k,e_k}) \prod_{t=1}^{e_k-1} (1 - \hat{\lambda}_\theta(t, x_{k,t}))$$

Then, let's rewrite the formula:

- Let $d_{kt} = 1$ iff $t = e_k$, i.e. if the experiment ends at time k
- ...And let $d_{kt} = 0$ otherwise. Then we can get:

$$\operatorname*{argmax}_{\theta} \prod_{k=1}^{m} \prod_{t=1}^{e_k} d_{k,t} \hat{\lambda}_\theta(t, x_{k,t}) + (1 - d_{k,t})(1 - \hat{\lambda}_\theta(t, x_{k,t}))$$

Now the two products can be freely swapped

Training a Hazard Estimator

Starting from:

$$\operatorname*{argmax}_{\theta} \prod_{k=1}^{m} \prod_{t=1}^{e_k} d_{k,t} \hat{\lambda}_\theta(t, x_{k,t}) + (1 - d_{k,t})(1 - \hat{\lambda}_\theta(t, x_{k,t}))$$

We obtain an equivalent problem through a log transformation:

$$\operatorname{argmax}_\theta \sum_{k=1}^{m} \sum_{t=1}^{e_k} \log\left(d_{k,t} \hat{\lambda}_\theta(t, x_{k,t}) + (1 - d_{k,t})(1 - \hat{\lambda}_\theta(t, x_{k,t}))\right)$$

Since either $d_{k,t} = 1$ or $d_{k,t} = 0$, we can also split the log argument:

$$\operatorname{argmax}_\theta \sum_{k=1}^{m} \sum_{t=1}^{e_k} d_{k,t} \log \hat{\lambda}_\theta(t, x_{k,t}) + (1 - d_{k,t}) \log(1 - \hat{\lambda}_\theta(t, x_{k,t}))$$

Training a Hazard Estimator

Finally, with a sign switch we get:

$$\operatorname{argmin}_\theta - \sum_{k=1}^{m} \sum_{t=1}^{e_k} d_{k,t} \log \hat{\lambda}_\theta(t, x_{k,t}) + (1 - d_{k,t}) \log(1 - \hat{\lambda}_\theta(t, x_{k,t}))$$

Does this remind you of something?

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Does this remind you of something?

This is a (binary) crossentropy minimization problem!

- $d_{k,t}$ has the same role as a class
- $\hat{\lambda}_\theta(t, x_{k,t})$ is the model output
- We have a sample for every experiment and time step (the double summation)

Training a Hazard Estimator

This means that our $\hat{\lambda}_\theta$ can be seen as a classifier

- We just need to consider all samples in our dataset individually
- Then attach to them a class corresponding to d_{kt}
- ...And finally we can train a neural classifier as usual

The model output will be an estimate of the hazard function

This is almost precisely what we did in our classification approach

...But now we have a much better interpretation

- We know how to define the classes
- We better know how to interpret the output
- We know the semantic for a threshold-based policy
- We know that we can safely deal with censoring

Classes and Models

Let's start by defining the classes

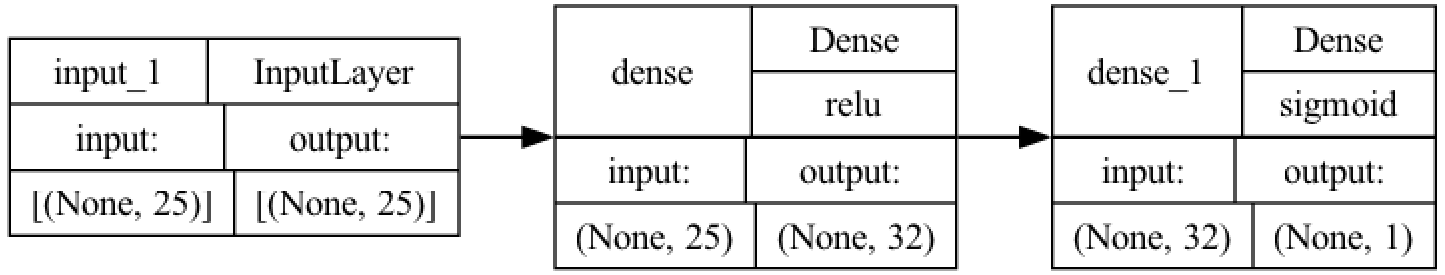
We check when the RUL is 0 (this the same as $t = e_k$)

```
In [4]: tr_lbl = (tr['rul'] == 0)
        ts_lbl = (ts['rul'] == 0)
```

Then we can build a (usual) classification model:

```
In [5]: nnl = util.build_nn_model(input_shape=(len(dt_in), ), output_shape=1, hidden=[32], output_a
        util.plot_nn_model(nnl)
```

Out[5]:



Effect on Censoring on the Distribution

The new approach allows us to use censored data

This is good, but it also has the effect of altering the distribution

- For end-to-failure experiments, are samples follow their natural distribution
- ...But censored data includes no end event, causing a skew

Effect on Censoring on the Distribution

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- For end-to-failure experiments, are samples follow their natural distribution
- ...But censored data includes no end event, causing a skew

We can try to account for that by using sample weights

- Intuitively, if the censored data is equal to 100% of the sample with 0 label
- ...That will make the 0 label apparently twice as likely

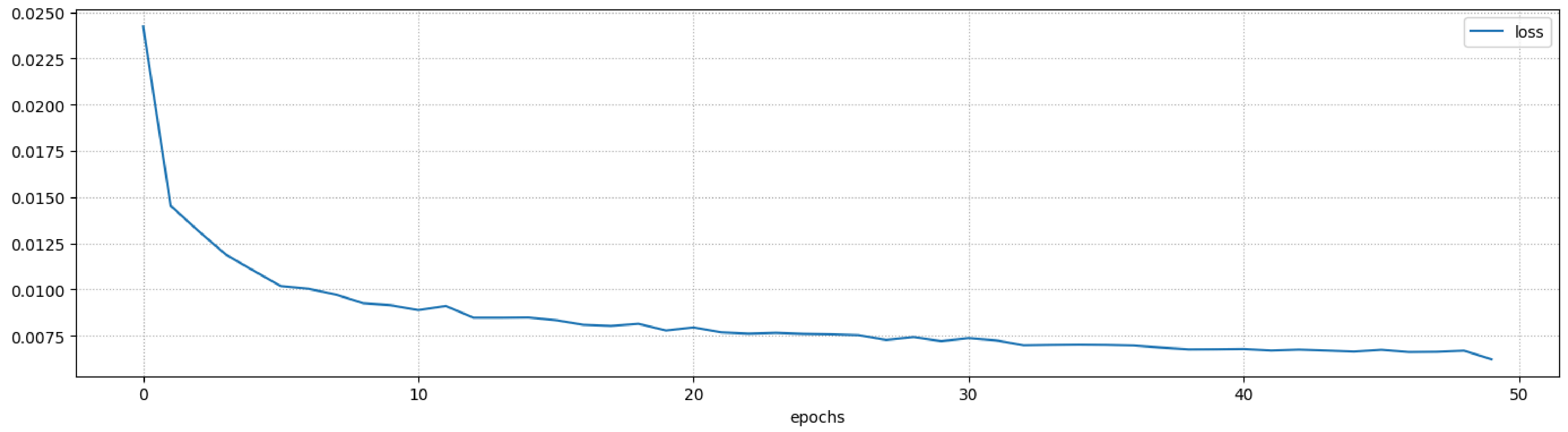
Therefore, we can discount censored samples in the distribution:

```
In [6]: n_zeros = (tr_lbl == False).sum()
        class_weights = np.array([(n_zeros - len(tru)) / n_zeros, 1.])
        sample_weight = np.choose(tr_lbl, class_weights)
```

Training the Hazard Estimator

Then we train the hazard estimator as any other classifier

```
In [7]: nnl = util.build_nn_model(input_shape=(len(dt_in), ), output_shape=1, hidden=[32], output_ac
        history = util.train_nn_model(nnl, tr_s[dt_in], tr_lbl, loss='binary_crossentropy', epochs=5
                verbose=0, patience=10, batch_size=32, validation_split=0.0, sample_weight=sample_we
        util.plot_training_history(history, figsize=figsize)
```



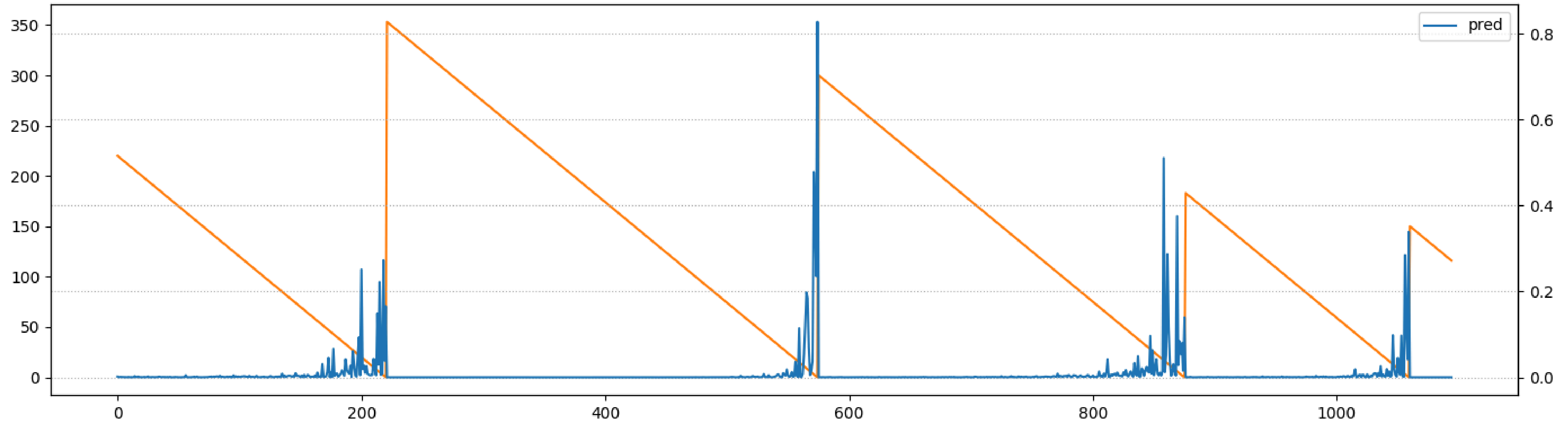
```
Final loss: 0.0062 (training)
```

Inspecting Hazards

We will start our evaluation by inspecting the hazard values

First for (part of) the training set:

```
In [8]: tr_pred = nnl.predict(tr_s[dt_in], verbose=0).ravel()
        stop = 1095
        util.plot_rul(pred=tr_pred[:stop], target=tr['rul'][:stop], same_scale=False, figsize=figsi
```

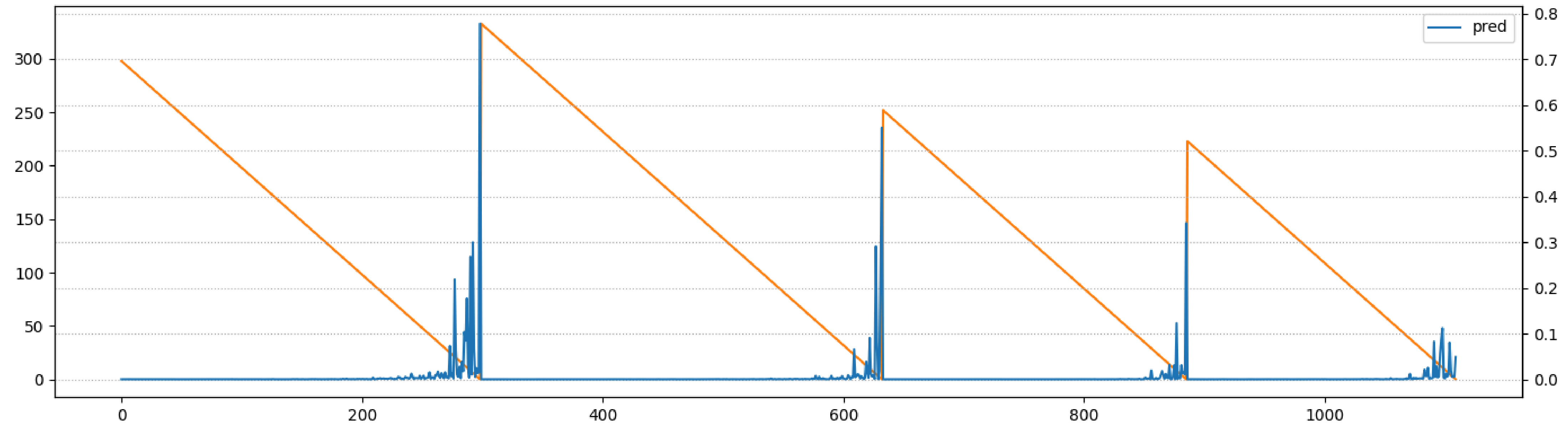


Inspecting Hazards

We will start our evaluation by inspecting the hazard values

...And here for (part of) the test set:

```
In [9]: ts_pred = nnl.predict(ts_s[dt_in], verbose=0).ravel()
        stop = 1110
        util.plot_rul(pred=ts_pred[:stop], target=ts['rul'][:stop], same_scale=False, figsize=figsi
```



Hazard-based Policies

We can define a policy based on the $\hat{\lambda}_\theta$ estimator as usual

Namely, we trigger maintenance when:

$$\hat{\lambda}_\theta(t, x_t) \geq \varepsilon$$

The threshold can be defined again based on some cost metric

Some comments

- The old classifier-based approach still makes sense
- ...Though reasoning in terms of hazard function can be more versatile
- This approach can be combined with a sliding window input
- ...And smoothing might be be a good idea to avoid accidental triggering

Using Hazards for Forecasting

Additionally, we can use $\hat{\lambda}_\theta$ to perform forecasting

In particular, we know the probability of surving n more steps is given by:

$$\frac{S(t+n)}{S(t)} = \prod_{h=0}^{n} (1 - \lambda(t+h, X_{t+h}))$$

...Which we can approximate (for a run k) as:

$$\frac{S(t+n)}{S(t)} \simeq \prod_{h=0}^{n} (1 - \hat{\lambda}_\theta(t+h, x_{k,t+h}))$$

- In theory, we can forecast survival probabilities arbitrarily far
- ...But in practice there is an issue

Using Hazards for Forecasting

The formula requires access to future values of the X_t variable

$$\frac{S(t+n)}{S(t)} \simeq \prod_{h=0}^{n} (1 - \hat{\lambda}_\theta(t+h, x_{k,t+h}))$$

- Unfortunately, we cannot access those in real life :-(
- We have two main options to deal with this

Using Hazards for Forecasting

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- We have two main options to deal with this

First, can ignore time-varying input in our estimator

Formally, this is the same as marginalizing out all time-varying factors

- $\hat{\lambda}_\theta(t, x_t)$ becomes $\hat{\lambda}_\theta(t, x)$, for a fixed x
- x represents some stable information, e.g. component type, genetics

In some cases, this is perfectly viable approach

Using Hazards for Forecasting

Second, we can attempt to predict future x_t values

This is viable as long as our predictions are good enough

- We can use a second ML estimator to predict x_t
- ...Or as a special case we can rely on the simple persistence model

In practice, we just assume x_t is stable for some time

With this simple assumption, we get:

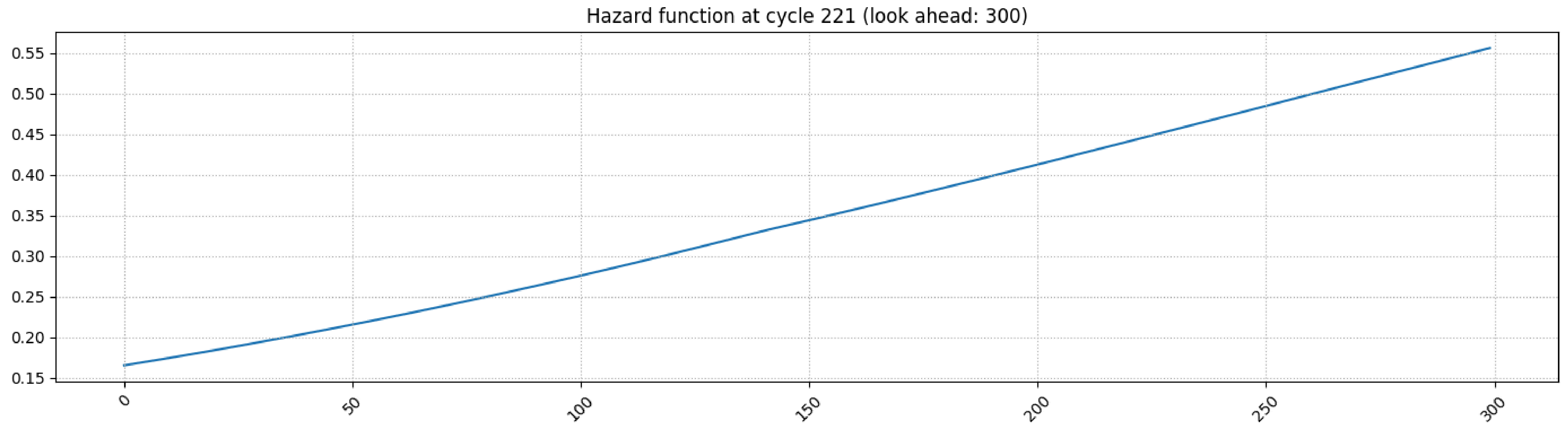
$$\frac{S(t+n)}{S(t)} \simeq \prod_{h=0}^{n} (1 - \hat{\lambda}_\theta(t+h, x_{k,t}))$$

- Unlike the original expression, this is easy to compute
- ...And it might be a reasonable approximation for shorter time horizons

Approximate Future Hazard

Let's check this approximate future hazard for one of our test runs

```
In [10]: ref_sample = tr_s.iloc[220]
         look_ahead = 300
         hazard = util.predict_cf(nnl, ref_sample[dt_in], columns='cycle',
                                  values=ref_sample['cycle'] + np.arange(look_ahead)/trmaxrul)
         util.plot_series(hazard, figsize=figsize, title=f'Hazard function at cycle {ref_sample["cyc
```

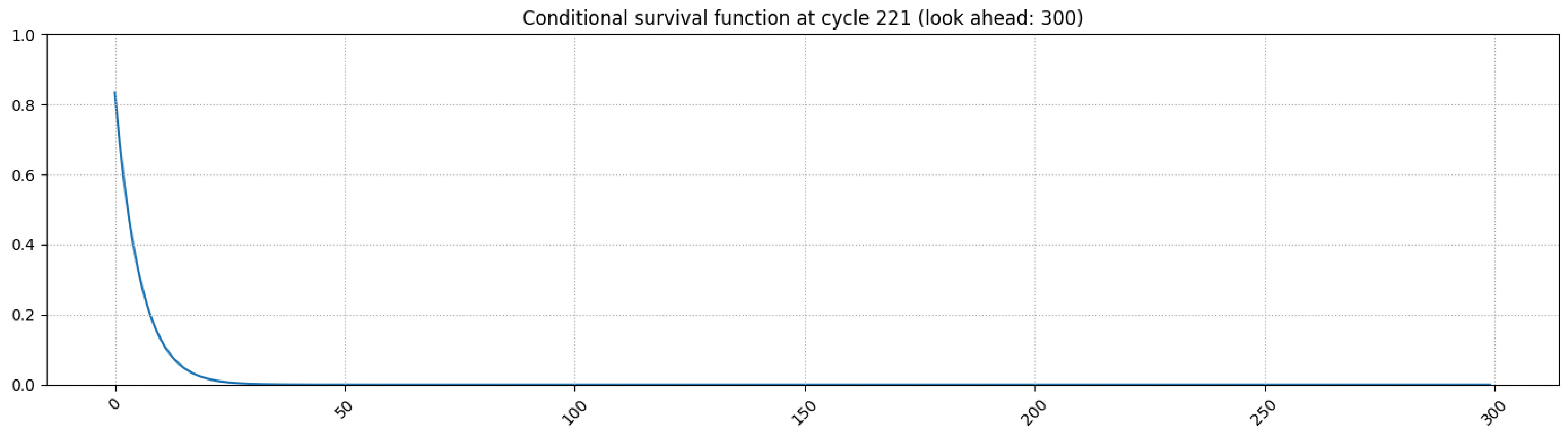


- The model has learned that time has an effect on λ

Approximate Conditional Survival

We can then estimate the conditional survival probability

```
In [11]: survival = pd.Series(data=np.cumprod(1-hazard))
         util.plot_series(survival, figsize=figsize, ylim=(0,1),
                          title=f'Conditional survival function at cycle {ref_sample["cycle"]*trmaxru
```



The chance of being still running is smaller even in a few tens of steps

Approximate Conditional Survival

We can continuously compute n-step ahead conditional survival

Here's an example for 30-steps ahead, on the first test set experiment

```
In [12]: ref_run = ts_s[ts_s['machine'] == ts_s.iloc[0]['machine']]
         look_up_window = np.arange(30)/trmaxrul
         rolling_survival = util.rolling_survival_cmapss(hazard_model=nnl, data=ref_run[dt_in], look_
         rolling_survival.columns = [f'S(t+{h})/S(t)' for h in range(30)]
         rolling_survival.head()
```

Out[12]:

	S(t+0)/S(t)	S(t+1)/S(t)	S(t+2)/S(t)	S(t+3)/S(t)	S(t+4)/S(t)	S(t+5)/S(t)	S(t+6)/S(t)	S(t+7)/S(t)	S(t+8)/S(t)	S(t+9)/S(t)	...	S(t+20)/S(t
321	0.999970	0.999940	0.999910	0.999879	0.999849	0.999818	0.999787	0.999756	0.999725	0.999693	...	0.999335
322	0.999916	0.999831	0.999746	0.999661	0.999575	0.999489	0.999404	0.999317	0.999231	0.999144	...	0.998173
323	0.999961	0.999921	0.999881	0.999841	0.999800	0.999760	0.999719	0.999678	0.999637	0.999595	...	0.999122
324	0.999990	0.999980	0.999971	0.999961	0.999951	0.999941	0.999931	0.999921	0.999910	0.999900	...	0.999783
325	0.999986	0.999972	0.999959	0.999945	0.999930	0.999916	0.999902	0.999888	0.999874	0.999859	...	0.999697

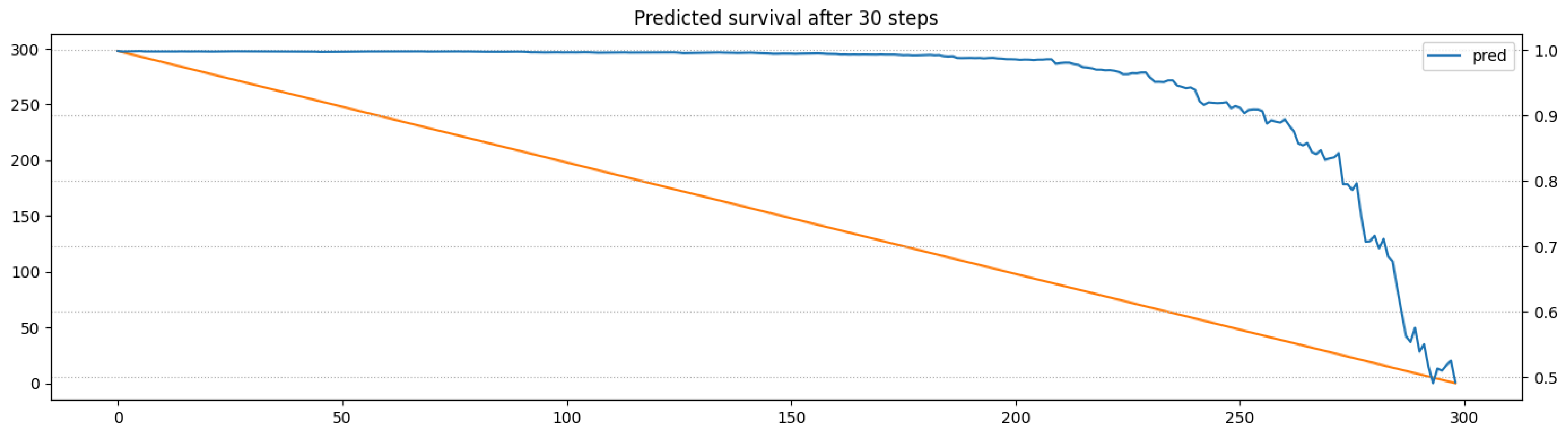
5 rows × 30 columns

- Each column contains the conditional survival h steps away

Approximate Conditional Survival

Here's a plot over time (after some smoothing)

In [13]:
```
rolling_survival_last = rolling_survival[rolling_survival.columns[-1]].ewm(15).mean()
util.plot_rul(pred=rolling_survival_last[:stop], target=ref_run['rul']*trmaxrul, same_scale=
              figsize=figsize, title='Predicted survival after 30 steps')
```



- Remember that this is a stochastic phenomenon
- So, even an 80% chance is quite dangerous to take!

In Hindsight...

This whole lecture block was about probabilistic models

- The techniques we covered are interesting per-se
- ...And way more useful in practice than you might think

...But what the core message I hope you glimpsed is another

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- The techniques we covered are interesting per-se
- ...And way more useful in practice than you might think

...But what the core message I hope you glimpsed is another

Machine Learning models are not inflexible tools

- If you spot a limit, or a piece of information you can use
- ...And you know what you are doing

Then you can dramatically change their behavior!