In [1]:
```python
# ============================================================
# Notebook setup: run this before everything
# ============================================================

%load_ext autoreload
%autoreload 2

# Control figure size
figsize=(14, 4)


from sklearn.neighbors import KernelDensity
from sklearn.model_selection import GridSearchCV
from sklearn.preprocessing import MinMaxScaler
from util import util
import numpy as np
from matplotlib import pyplot as plt
import pandas as pd
import os

# Load data
data_folder = os.path.join('..', 'data', 'nab')
file_name = os.path.join('realKnownCause', 'nyc_taxi.csv')
data, labels, windows = util.load_series(file_name, data_folder)

# Train and validation end
train_end = pd.to_datetime('2014-10-24 00:00:00')
val_end = pd.to_datetime('2014-12-10 00:00:00')

# Cost model parameters
c_alrm = 1 # Cost of investigating a false alarm
c_missed = 10 # Cost of missing an anomaly
c_late = 5 # Cost for late detection

# Build a cost model
cmodel = util.ADSimpleCostModel(c_alrm, c_missed, c_late)

# Separate the training data
data_tr = data[data.index < train_end]

# Apply a sliding window
wdata = util.sliding_window_1D(data, wlen=48)
```

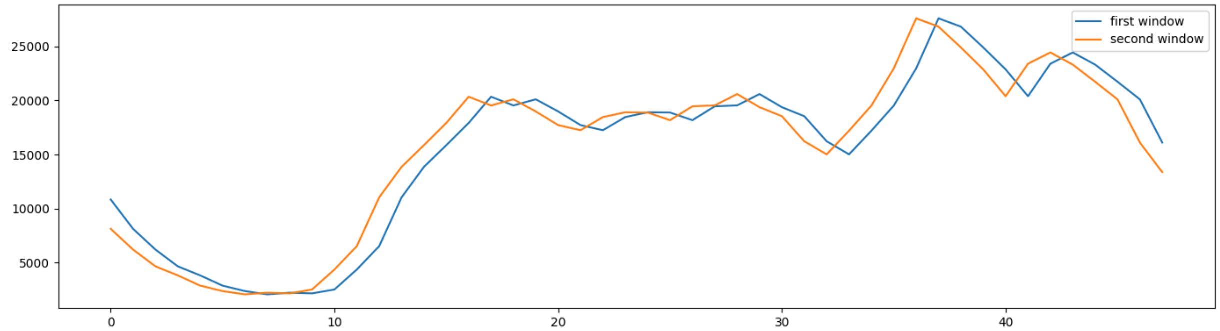
A Time-Dependent Estimator

Spotting the Problem

The sequence-based estimator we built learns from all the training data

This means it will learn from both these series, for example:

In [2]:
```
plt.figure(figsize=figsize)
plt.plot(wdata.iloc[0], label='first window')
plt.plot(wdata.iloc[1], label='second window')
plt.legend()
plt.tight_layout()
```



Spotting the Problem

Let us consider the first two window applications

- In the first window, the observations are x_0, x_1 and so on
- In the second window, the observations are x_1, x_2 and so on

x_0 is number of taxis as 00:00, x_1 at 00:30, and so on

- Hence, the first observation in the first window corresponds to 00:00
- ...But in the second window corresponds to 00:30

Our estimator learns a distribution for the *observations*:

- Moving the window forward changes "who is who"
- We learn the distribution of x_0 (and its correlations) *multiple times*!

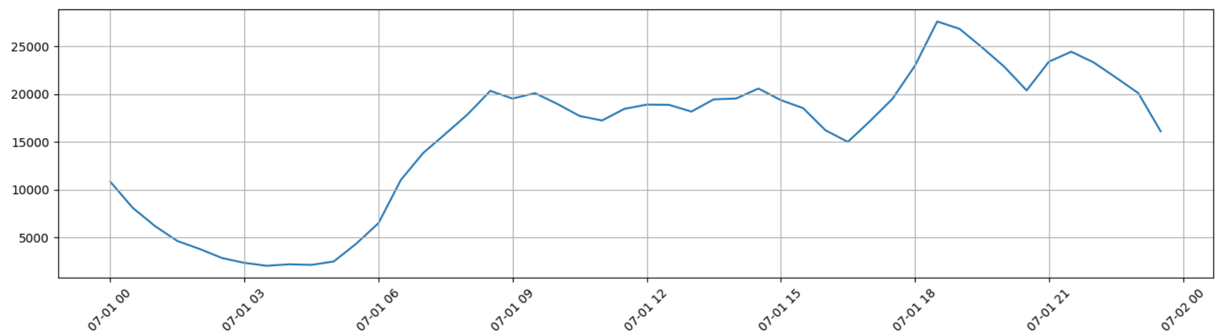
The learning problem is still well defined, but also *very complex*

This is the reason for (most of) the noise in the alarm signal

Rewind a Little

Remember *why* we introduced the sequence based estimator?

In [3]:
```
util.plot_series(data.iloc[:48], figsize=figsize)
```

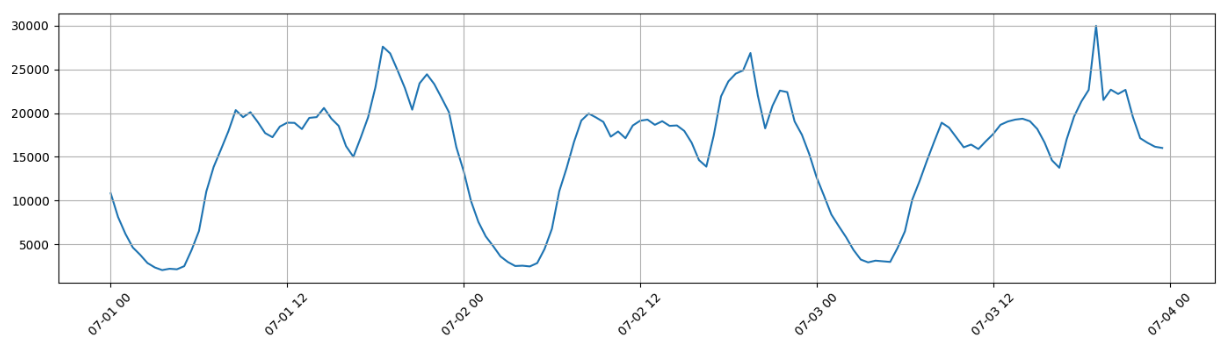


- We wanted to take advantage of correlation between nearby points

...Then Forward Again

But there is more! Let's look just a little bit further

In [4]:
```
util.plot_series(data.iloc[:3*48], figsize=figsize)
```

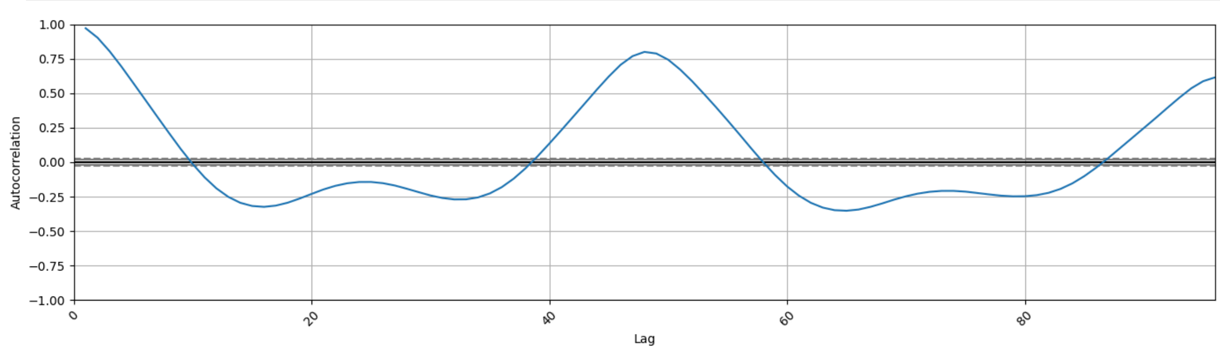


- There is recurring pattern!
- I.e. the series is *approximately periodic*

Determine the Period

This is even clearer in the autocorrelation plot

In [5]:
```
util.plot_autocorrelation(data, max_lag=96, figsize=figsize)
```



* There is *strong peak at 48* time steps (a time step is 30 minutes)
* This is consistent with a period of 24 hours

Reevaluate

Let's recap our situation

Our sequence-based estimator

* ...Is solving a uselessly complicated problem
* ...And it's not using all the available knowledge

> **These are both *very serious drawbacks***

In any problem:

* *Never* introduce complications unless they are worth it
* *Never* willingly throw away information

Can we do something to tackle both problems?

Time as an Additional Input

One way to look at that

...Is that the distribution *depends on the time of the day*

* Therefore, we should consider the number of taxi calls x
* ...And the time of the day t together

Let us extract (from the index) the time information information:

In [6]:
```
dayhour = (data.index.hour + data.index.minute / 60)
```

We can then add it as a separate column to the data:

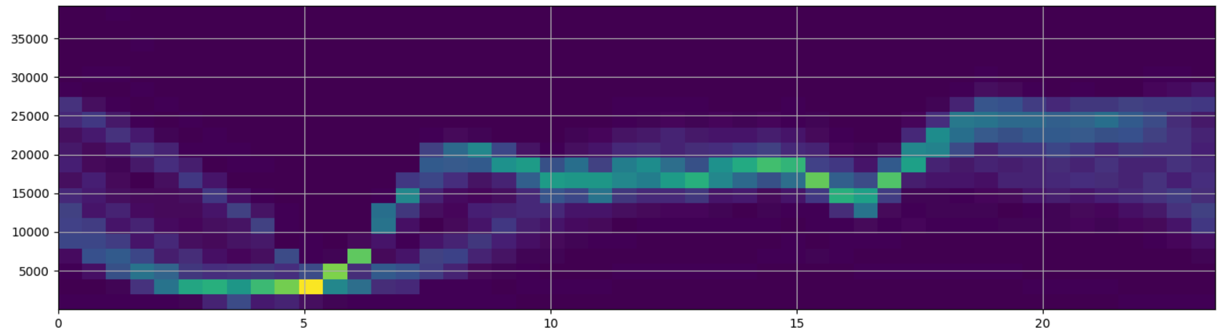
In [7]:
```
data2 = data.copy()
data2['dayhour'] = dayhour
```

Multivariate Distribution

Let us examine the resulting multivariate distribution

We can use a 2D histogram:

```
In [8]: util.plot_histogram2d(data2['dayhour'], data2['value'], bins=(48, 20), figsi
```



* x = time, y = value, color = frequency of occurrence

Anomaly Detection with Controlled Variables

If we feed this information to KDE

...We learn an estimator for the *joint* PDF:

$$f(t, x)$$

...Which is not exactly what we were looking for

Assume we flag an anomaly when $f(t, x) \leq \theta$

* This may happen when x (the number of cars) takes an unlikely value
* ...Or when t (the time) does

Except that the *time is completely predictable*

* Any different in its estimated density is only due to sampling choices
* In practice, it's a *controlled variable*

Anomaly Detection with Controlled Variables

What we really care about is the *conditional density*, i.e.

$$f(x \mid t)$$

* I.e. the density value of the observed value of x
* Assuming that the time t is *known*

Our true anomaly detection conditions should then be:

$$f(x \mid t) \leq \varepsilon$$

...We know how to approximate only to the *joint* density function $f(t, x)$

> **How to handle the conditioning variable?**

Anomaly Detection with Controlled Variables

There's more than one way to do it

...The one we'll see starts with the definition of conditional probability:

$$f(t, x) = f(x \mid t) f(t)$$

Meaning that we can detect anomalies by evaluating:

$$\frac{f(t, x)}{f(t)} \leq \varepsilon$$

In order to pull this off, we need

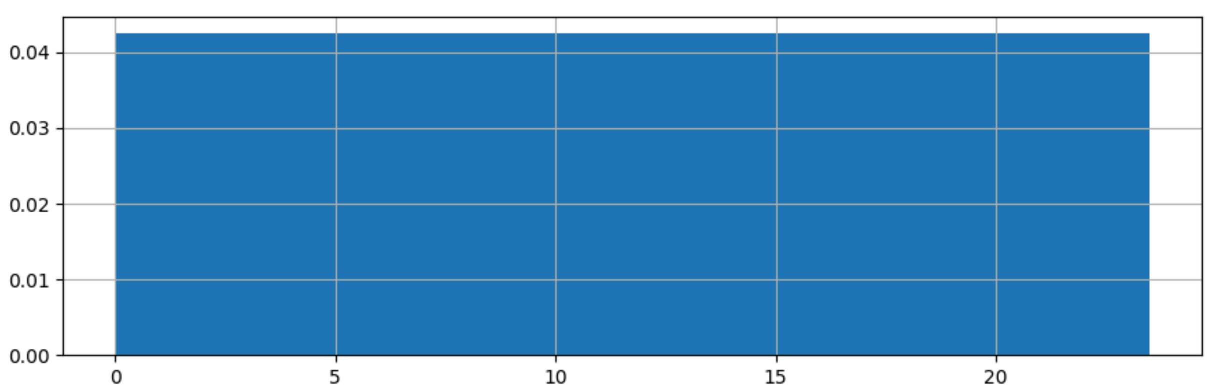
- An estimator for $f(t, x)$, which we already have
- An estimator for $f(t)$, which we can easily obtain (e.g. using KDE again)

...But in our specific case, things are *even simpler*

Time Distribution

In particular, the distribution of time values is uniform:

In [9]:
```
util.plot_histogram(data2['dayhour'], bins=48)
```



Our Time-Dependent Estimator

In non-degenerate cases, our condition can always be rewritten:

$$\frac{f(t, x)}{f(t)} \leq \varepsilon \quad \longrightarrow \quad f(t, x) \leq \varepsilon f(t)$$

- But since $f(t)$ is constant this is equivalent to checking the joint probability
- ...With a modified threshold

$$f(t, x) \leq \varepsilon'$$

- The threshold ε' now represents $\varepsilon f(t)$
- ...But since we still need to choose it value, it make little difference to us

Hence, for this problem we can use $f(t, x)$ for anomaly detection

Choosing a Bandwidth

We now need to pick a threshold

- We can use grid search and cross-validation again
- ...But this time we need to make sure to *normalize the data*

In [10]:
```
scaler = MinMaxScaler()
data2_n_tr = data2[data2.index < train_end].copy()
data2_n_tr[:] = scaler.fit_transform(data2_n_tr)
data2_n = data2.copy()
data2_n[:] = scaler.transform(data2)
```

This is due to a low-level technical detail:

- scikit-learn uses a very efficient KDE implementation
- ...But it requires using *the same bandwidth* for all input dimensions

Normalization makes this drawback less impactful

Choosing a Bandwidth

We can then optimize the bandwidth as usual

We'll use cross-validation, since we have vector input

In [11]:
```
from sklearn.model_selection import GridSearchCV
params = {'bandwidth': np.linspace(0.001, 0.01, 10)}
opt = GridSearchCV(KernelDensity(kernel='gaussian'), params, cv=5)
opt.fit(data2_n_tr);
opt.best_params_
```

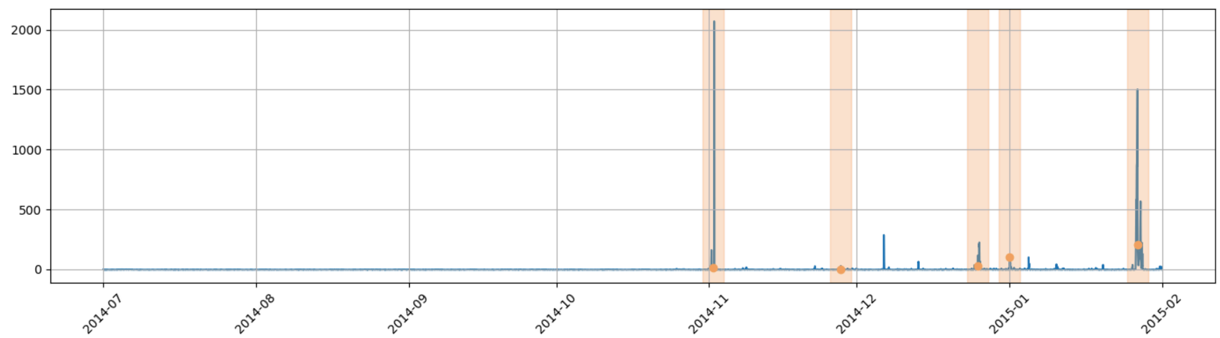
Out[11]:
```
{'bandwidth': np.float64(0.006)}
```

- As another small advantage of normalization
- ...Choosing the grid search range becomes a bit easier

Alarm Signal

Let us obtain the alarm signal

In [12]:
```
ldens2 = opt.score_samples(data2_n)
signal2 = pd.Series(index=data2.index, data=-ldens2)
util.plot_series(signal2, labels=labels, windows=windows, figsize=figsize)
```



Threshold Optimization

Now, let us optimize our threshold:

In [13]:
```
signal2_opt = signal2[signal2.index < val_end]
labels_opt = labels[labels < val_end]
windows_opt = windows[windows['end'] < val_end]
thr2_range = np.linspace(10, 100, 100)
best_thr2, best_cost2 = util.opt_thr(signal2_opt, labels_opt, windows_opt, c
print(f'Best threshold: {best_thr2:.3f}, corresponding cost: {best_cost2:.3f
```

```
Best threshold: 27.273, corresponding cost: 9.000
```

On the whole dataset:

In [14]:
```
c2tst = cmodel.cost(signal2, labels, windows, best_thr2)
print(f'Cost on the whole dataset {c2tst}')
```

```
Cost on the whole dataset 18
```

- It was 45 for the first approach and 30 for the second

There is a second period in the data!
Can you guess which one?