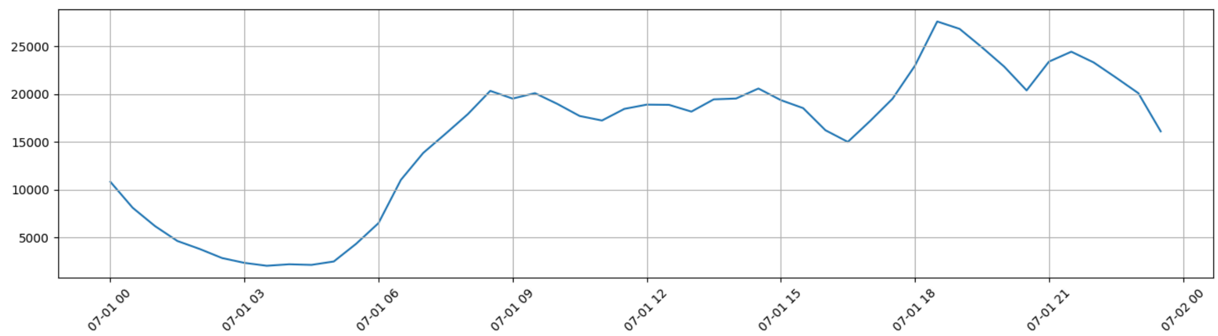
In [1]:
```
# ============================================================
# Notebook setup: run this before everything
# ============================================================

%load_ext autoreload
%autoreload 2

# Control figure size
figsize=(14, 4)

from sklearn.neighbors import KernelDensity
from sklearn.model_selection import GridSearchCV
from util import util
import numpy as np
from matplotlib import pyplot as plt
import pandas as pd
import os

# Load data
data_folder = os.path.join('..', 'data', 'nab')
file_name = os.path.join('realKnownCause', 'nyc_taxi.csv')
data, labels, windows = util.load_series(file_name, data_folder)

# Train and validation end
train_end = pd.to_datetime('2014-10-24 00:00:00')
val_end = pd.to_datetime('2014-12-10 00:00:00')

# Cost model parameters
c_alrm = 1 # Cost of investigating a false alarm
c_missed = 10 # Cost of missing an anomaly
c_late = 5 # Cost for late detection

# Separate the training data
data_tr = data[data.index < train_end]

# Build a cost model
cmodel = util.ADSimpleCostModel(c_alrm, c_missed, c_late)
```

Sliding Windows

Temporal Correlations

Let's have a closer look at our time series

In [2]:
```
util.plot_series(data.iloc[:48], figsize=figsize)
```



- Nearby points tend to have similar values
- ...Meaning they are *correlated*

Determine the Correlation Interval

How can we study such correlation?

A useful tool: autocorrelation plots

- Consider a range of possible *lags*
- For each lag value l:
  - Make a copy of the series and shift it by l time steps
  - Compute the Pearson Correlation Coefficient with the original series
- Plot the correlation coefficients over the lag values

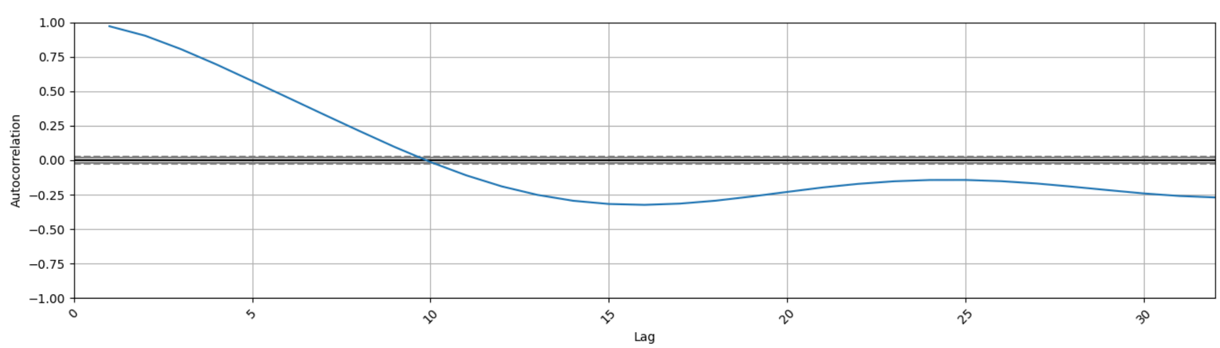
Then we look at the resulting plot:

- Where the curve is far from zero, there is a significant correlation
- Where it gets close to zero, no significant correlation exists

Temporal Correlations

Let's have a look at our plot

In [3]:
```
util.plot_autocorrelation(data, max_lag=32, figsize=figsize)
```



- The correlation is strong up to 4-5 lags

Temporal Correlations

These correlations are *a source of information*

- They could be exploited to improve our estimated probabilities
- ...But our models so far make *no use* of them

> **How can we take advantage of them?**

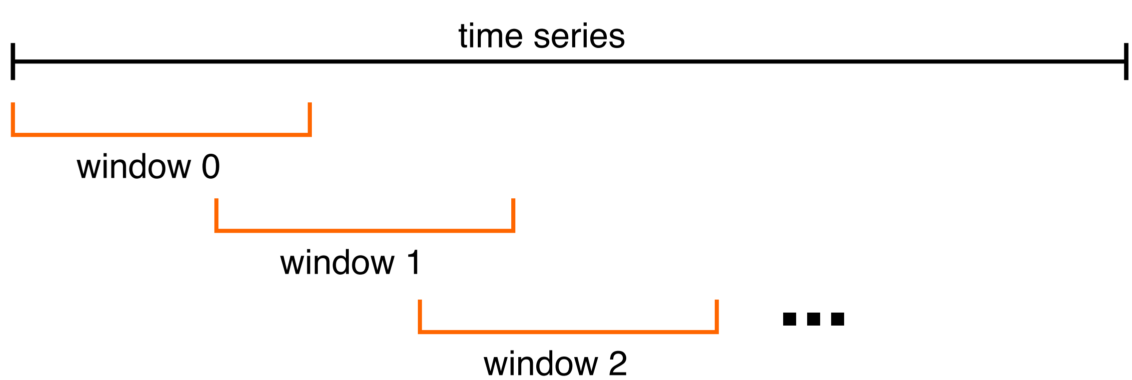
For example, rather then feeding our model with individual observations

We can use *sequences of observations* as input

- This is a *very common approach* in time series
- ...And in many cases it's a good idea

Sliding Window

A common approach consist in using a *sliding window*



- We choose a *window length* w, i.e. the length of each sub-sequence
- We place the "window" at the beginning of the series
- ...We extract the corresponding observations
- Then, we move the forward by a certain *stride* and we repeat

Sliding Window

The result is a table

Let m be the number of examples and w be the window length

	$\mathbf{s_0}$	$\mathbf{s_1}$	$\ldots$	$\mathbf{s_{w-1}}$
$\mathbf{t_{w-1}}$	x_0	x_1	$\ldots$	x_{w-1}
$\mathbf{t_w}$	x_1	x_2	$\ldots$	x_w
$\mathbf{t_{w+1}}$	x_2	x_3	$\ldots$	x_{w+1}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathbf{t_{m-1}}$	x_{m-w}	x_{m-w+1}	$\vdots$	x_{m-1}

- The first window includes observations from x_0 to x_{w-1}
- The second from x_1 to x_w and so on
- t_i is the *time window index* (where it was applied)
- s_j is the *position* of an observation *within a window*

Sliding Window in pandas

pandas provides a sliding window *iterator*

```
DataFrame.rolling(window, ...)
```

In [4]:
```
wlen = 10
for i, w in enumerate(data['value'].rolling(wlen)):
    print(w)
    if i == 2: break # We print the first three windows
```

```
timestamp
2014-07-01    10844.0
Name: value, dtype: float64
timestamp
2014-07-01 00:00:00    10844.0
2014-07-01 00:30:00     8127.0
Name: value, dtype: float64
timestamp
2014-07-01 00:00:00    10844.0
2014-07-01 00:30:00     8127.0
2014-07-01 01:00:00     6210.0
Name: value, dtype: float64
```

Notice how the first windows are not full (shorter than `wlen`)

Sliding Window in pandas

We can build our dataset using the `rolling` iterator

- We discard the first `wlen-1` (incomplete) applications
- Then we store each window in a list, and we wrap everything in a `DataFrame`

In [5]:
```
%%time
rows = []
for i, w in enumerate(data['value'].rolling(wlen)):
    if i >= wlen-1: rows.append(w.values)

wdata_index = data.index[wlen-1:]
wdata = pd.DataFrame(index=wdata_index, columns=range(wlen), data=rows)
```

```
CPU times: user 151 ms, sys: 2.22 ms, total: 153 ms
Wall time: 152 ms
```

- The `values` field allows access to the `Series` content as a numpy array
- We use it to *discard the index*
- ...Since the series for multiple iterations have inconsistent indexes

Sliding Window in pandas

This method works, but *it's a bit slow*

- We are building our table by rows...
- ...But it is usually *faster to do it by columns*!
- After all, there are usually *fewer columns than rows*

Let us look again at our table:

	$\mathbf{s_0}$	$\mathbf{s_1}$	$\ldots$	$\mathbf{s_{w-1}}$
$\mathbf{t_{w-1}}$	x_0	x_1	$\ldots$	x_{w-1}
$\mathbf{t_w}$	x_1	x_2	$\ldots$	x_w
$\mathbf{t_{w+1}}$	x_2	x_3	$\ldots$	x_{w+1}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathbf{t_{m-1}}$	x_{m-w}	x_{m-w+1}	$\vdots$	x_{m-1}

Sliding Window in pandas

We can build the columns by *slicing the original* `DataFrame`

In [6]:
```
m = len(data)
c0 = data.iloc[0:m-wlen+1]   # first column
c1 = data.iloc[1:m-wlen+1+1] # second column
print(c0.iloc[0:3])
print(c1.iloc[0:3])
```

```
                        value
timestamp
2014-07-01 00:00:00  10844.0
2014-07-01 00:30:00   8127.0
2014-07-01 01:00:00   6210.0
                       value
timestamp
2014-07-01 00:30:00  8127.0
2014-07-01 01:00:00  6210.0
2014-07-01 01:30:00  4656.0
```

- `iloc` in pandas allows to address a `DataFrame` by *position*

Sliding Window in pandas

Now we collect all columns in a list and we *stack them*

In [7]:
```
lc = [data.iloc[i:m-wlen+i+1].values for i in range(0, wlen)]
lc = np.hstack(lc)
wdata = pd.DataFrame(index=wdata_index, columns=range(wlen), data=lc)
wdata.head()
```

Out[7]:

timestamp	0	1	2	3	4	5	6	7	8
2014-07-01 04:30:00	10844.0	8127.0	6210.0	4656.0	3820.0	2873.0	2369.0	2064.0	2221.0
2014-07-01 05:00:00	8127.0	6210.0	4656.0	3820.0	2873.0	2369.0	2064.0	2221.0	2158.0
2014-07-01 05:30:00	6210.0	4656.0	3820.0	2873.0	2369.0	2064.0	2221.0	2158.0	2515.0
2014-07-01 06:00:00	4656.0	3820.0	2873.0	2369.0	2064.0	2221.0	2158.0	2515.0	4364.0
2014-07-01 06:30:00	3820.0	2873.0	2369.0	2064.0	2221.0	2158.0	2515.0	4364.0	6526.0

Sliding Window in pandas

We can wrap this approach in a function:

```
def sliding_window_1D(data, wlen):
    m = len(data)
    lc = [data.iloc[i:m-wlen+i+1] for i in range(0, wlen)]
```

```python
    wdata = np.hstack(lc)
    wdata = pd.DataFrame(index=data.index[wlen-1:], data=wdata,
columns=range(wlen))
    return wdata
```

In [8]:
```python
%%time
wdata = util.sliding_window_1D(data, wlen=wlen)
```

```
CPU times: user 1.03 ms, sys: 609 µs, total: 1.64 ms
Wall time: 1.11 ms
```

- This is available in the (updated)) `nab` module
- The function works for *univariate* data (but the approach is general)