In [1]:

```python
# ============================================================
# Notebook setup: run this before everything
# ============================================================

%load_ext autoreload
%autoreload 2

# Control figure size
figsize=(14, 4)

from sklearn.neighbors import KernelDensity
from sklearn.model_selection import GridSearchCV
from util import util
import numpy as np
from matplotlib import pyplot as plt
import pandas as pd
import os

# Load data
data_folder = os.path.join('..', 'data', 'nab')
file_name = os.path.join('realKnownCause', 'nyc_taxi.csv')
data, labels, windows = util.load_series(file_name, data_folder)

# Train and validation end
train_end = pd.to_datetime('2014-10-24 00:00:00')
val_end = pd.to_datetime('2014-12-10 00:00:00')

# Cost model parameters
c_alrm = 1 # Cost of investigating a false alarm
c_missed = 10 # Cost of missing an anomaly
c_late = 5 # Cost for late detection

# Build a cost model
cmodel = util.ADSimpleCostModel(c_alrm, c_missed, c_late)

# Compute the maximum over the training set
trmax = data[data.index < train_end]['value'].max()
# Normalize
data['value'] = data['value'] / trmax
# Separate the training data
data_tr = data[data.index < train_end]

# Apply a sliding window
wdata = util.sliding_window_1D(data, wlen=10)
```

Time Indexed Models

Exploiting Time

Let's consider how we dealt with time so far

- We learned an estimator for $f(t, x)$ and one for $f(t)$
- ...Which we used to compute $f(x \mid t) = f(t, x)/f(t)$

It worked well, but we had to introduce one additional dimension

> **What if we wanted to consider time *and* sequence input?**

Let's consider a second approach to handle time

- This consists in *learning many density estimators*:
- Each estimator is *specialized for a given time* (e.g. 00:00, 00:30, 01:00...)

We can then choose which estimator to use based on the current time

Exploiting Time

Formally, what we have is a first *ensemble model*

In particular, we obtain our estimated probabilities by evaluating:

$$f_{g(t)}(x)$$

- Each f_i function is an estimator
- The $g(t)$ retrieves the correct f_i based (in our case) on the time value

We'll call this general idea a "selection ensemble"

In terms of properties:

- Each f_i estimator works with *smaller amounts of data*
- ...But the individual problems are *easier*!

Learning an Estimator for one Time Value

Let us make a test by learning an estimator for a single time value

First, we separate the training data

In [2]:
```
wdata_tr = wdata[wdata.index < train_end]
wdata_tr.head()
```

Out[2]:

timestamp	0	1	2	3	4	5	6
2014-07-01 04:30:00	0.357028	0.267573	0.204458	0.153294	0.125770	0.094591	0.077997
2014-07-01 05:00:00	0.267573	0.204458	0.153294	0.125770	0.094591	0.077997	0.067955
2014-07-01 05:30:00	0.204458	0.153294	0.125770	0.094591	0.077997	0.067955	0.073124
2014-07-01 06:00:00	0.153294	0.125770	0.094591	0.077997	0.067955	0.073124	0.071050
2014-07-01 06:30:00	0.125770	0.094591	0.077997	0.067955	0.073124	0.071050	0.082804

- We'll use the normalized version
- ...So as to simplify our guesses for bandwidth selection

Learning an Estimator for one Time Value

Let us make a test by learning an estimator for a single time value

Then, we focus on the values for a single time value

In [3]:

```
wdata_tr_test = wdata_tr.iloc[0::48] # 48 is the step
wdata_tr_test.head()
```

Out[3]:

timestamp	0	1	2	3	4	5	6
2014-07-01 04:30:00	0.357028	0.267573	0.204458	0.153294	0.125770	0.094591	0.077997
2014-07-02 04:30:00	0.440194	0.327429	0.249267	0.194811	0.158694	0.119646	0.098541
2014-07-03 04:30:00	0.416357	0.347743	0.277088	0.233694	0.191815	0.144306	0.107661
2014-07-04 04:30:00	0.513318	0.473941	0.412702	0.373391	0.328581	0.276693	0.237053
2014-07-05 04:30:00	0.578672	0.533006	0.475455	0.412702	0.362361	0.301287	0.263721

Learning a 23:30 Estimator

Then we proceed as usual

We choose a bandwidth:

In [4]:
```
grid = GridSearchCV(KernelDensity(kernel='gaussian'), {'bandwidth': np.linsp
grid.fit(wdata_tr_test)
grid.best_params_
```

Out[4]: {'bandwidth': np.float64(0.019473684210526317)}

Then we store the bandwidth in a variable:

In [5]:
```
h = grid.best_params_['bandwidth']
```

- For sake of simplicity, we'll use the same bandwidth for all estimators
- Even if we should re-calibrate h for each estimator in principle

Learning the Ensemble

Now, we need to repeat the process for every unique time value

In [6]:
```
day_hours = data_tr.index.hour + data_tr.index.minute / 60
day_hours = day_hours.unique()
print(day_hours)
```

```
Index([ 0.0,  0.5,  1.0,  1.5,  2.0,  2.5,  3.0,  3.5,  4.0,  4.5,  5.0,  5.
5,
        6.0,  6.5,  7.0,  7.5,  8.0,  8.5,  9.0,  9.5, 10.0, 10.5, 11.0, 11.
5,
       12.0, 12.5, 13.0, 13.5, 14.0, 14.5, 15.0, 15.5, 16.0, 16.5, 17.0, 17.
5,
       18.0, 18.5, 19.0, 19.5, 20.0, 20.5, 21.0, 21.5, 22.0, 22.5, 23.0, 23.
5],
      dtype='float64', name='timestamp')
```

- `unique` in pandas returns a `Series` with all unique values
- We do not care about how time is measured
- ...We only care about having 48 discrete steps

Learning the Ensemble

Finally, we can learn 48 specialized estimators

In [7]:
```
kde = {}
for hidx, hour in enumerate(day_hours):
    tmp_data = wdata_tr.iloc[hidx::48]
    kde[hour] = KernelDensity(kernel='gaussian', bandwidth=h)
    kde[hour].fit(tmp_data)
```

- For each unique time value, we separate a subset of the *training* data
- Then we build and learn a KDE estimator

We chose to store everything in a dictionary:

In [8]:
```
print(str(kde)[:256], '...}')
```

```
{0.0: KernelDensity(bandwidth=np.float64(0.019473684210526317)), 0.5: Kernel
Density(bandwidth=np.float64(0.019473684210526317)), 1.0: KernelDensity(band
width=np.float64(0.019473684210526317)), 1.5: KernelDensity(bandwidth=np.flo
at64(0.019473684210526317)), ...}
```

Generating the Signal

The we can generate the alarm signal

- In a practical implementation we should do this step by step
- ...But for an evaluation purpose it is easier to do it all at once

In [9]:
```
ldens_list = []
for hidx, hour in enumerate(day_hours):
    tmp_data = wdata.iloc[hidx::48]
    tmp_ldens = kde[hour].score_samples(tmp_data)
    tmp_ldens = pd.Series(index=tmp_data.index, data=tmp_ldens)
    ldens_list.append(tmp_ldens)
```

- For each unique time value, we separate a subset of the *whole* data
- Then we obtain the estimated (log) probabilities

The process is even faster than before

- ...Because each KDE estimator is trained a smaller dataset

Generating the Signal

All signals are stored in a list

- We need to concatenate them all in single `DataFrame`
- Then we can sort all rows by timestamp (it's the index)

In [10]:
```
ldens = pd.concat(ldens_list, axis=0)
ldens = ldens.sort_index()
signal = -ldens
signal.head()
```

Out[10]:
```
timestamp
2014-07-01 04:30:00   -27.059255
2014-07-01 05:00:00   -27.505901
2014-07-01 05:30:00   -27.741645
2014-07-01 06:00:00   -27.925602
2014-07-01 06:30:00   -27.657585
dtype: float64
```

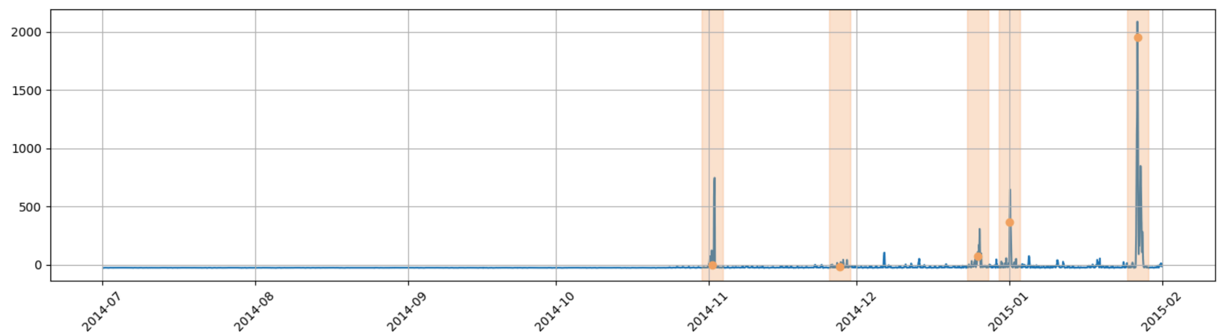
A suggestion: always do concatenations in a single step in `pandas`

It's way faster than appending `DataFrame` objects one by one

Generating the Signal

Now we can plot out signal:

In [11]:
```
util.plot_series(signal, labels=labels, windows=windows, figsize=figsize)
```



- It's very similar to that of the other time-based model
- ...But also a bit smoother, like that of the sequence-based model

Threshold Optimization and Evaluation

Now we can optimize the threshold and evaluate the results

In [12]:
```
signal_opt = signal[signal.index < val_end]
labels_opt = labels[labels < val_end]
windows_opt = windows[windows['end'] < val_end]
thr_range = np.linspace(10, 200, 100)

best_thr, best_cost = util.opt_thr(signal_opt, labels_opt, windows_opt, cmod
print(f'Best threshold: {best_thr}, corresponding cost: {best_cost}')
```

```
Best threshold: 104.04040404040404, corresponding cost: 10
```

Let us see the cost on the whole dataset:

In [13]:
```
ctst = cmodel.cost(signal, labels, windows, best_thr)
print(f'Cost on the whole dataset {ctst}')
```

```
Cost on the whole dataset 10
```

This is the best result we have achieved so far!

What if we used this approach for the second period?