Challenge 4 – ε-NFAs

ε-NFAs are very useful. For instance, when we have two or more regular languages (RLs), each one represented by an FA[1] (DFA, NFA, or even ε-NFA), one can easily obtain the FA representing the union or the concatenation of those languages. When considering an FA representing a language L it is also easy with the use of ε-transitions to obtain the FA representing L*.

(a) Given the languages defined by $A_1 = \{0^k \mid k \equiv 0 \bmod 2\}$[2] and $A_2 = \{0^k \mid k \equiv 0 \bmod 3\}$, draw a DFA for each one and then an FA for $A_1 \cup A_2$, i.e., the language $\{0^k \mid k \equiv 0 \bmod 2 \text{ or } k \equiv 0 \bmod 3\}$.

(b) Given the FAs for each of the languages L_1, L_2,..., L_N, explain how you can obtain an FA representing: (1) $L_1 \cup L_2 \cup \ldots \cup L_N$, (2) $L_1.L_2 \ldots L_N$; and (3) L_1^*. Note: try to explain visually the three operations and considering the main elements of the individual FAs needed for applying these operations.

[1] Finite Automata.
[2] I.e., the language of the strings of 0s with length mutiple of 2.