

Challenge 8 – Context-Free Grammars (CFGs) and Push Down Automata (PDAs)

We need to translate regular expressions (assume that they can be formed with symbols: $(,), +, *, \varepsilon$ and with $\Sigma = \{a, b, c\}$) to context-free grammars (CFGs).

- a) One possible technique is to think about an algorithm that receives a syntax tree (also known as analysis tree) corresponding to the input regular expression and generates a CFG. Considering the CFG₁ below representing the language of regular expressions, describe an algorithm to translate the syntax tree to a CFG and show the main steps of the algorithm and the resultant CFG for the following regular expression: $(ba^* + b^*)^*(c + \varepsilon)ab^*$.

CFG₁: $E \rightarrow E + E \mid EE \mid E^* \mid (E) \mid a \mid b \mid c \mid \varepsilon$

- b) Another technique is to translate each input regular expression to a DFA and then to translate the DFA to a CFG. Show a possible CFG obtained using this technique and considering the regular expression $(ba^* + b^*)^*(c + \varepsilon)ab^*$ as input.
- c) Show a non-deterministic PDA for the language $L = \{ (ba^n + b^k)^m (c + \varepsilon) ab^t \mid n, k, m, t \geq 0 \}$.