4 Mathematical Formulation under Conditions

4.1 Background

- problem state in this section

To restore VRPD mathematically, this chapter explains in detailed the definition, formulation and implementation: tasks of each kind of nodes in the express network, and dynamic activation of vehicles with the nodes of the express network in a minutia view.

4.1.1 Objects and Tasks with Use Case Diagram

- Objects in the problem, vehicles and graph nodes

Overall, there are two categories of objects, namely the vehicles V and graphic nodes $G = (N, Rou, W)$, which contains the stations N of the express network, route Rou from a node to another, and weight W of this routine for the vehicles to cover through. Nodes in the graphic G contains depot stations N_{dep}, docking hub stations N_{doc}, and customers nodes N_{cus}. The vehicle combination V_{set} as described in section 3.1 consists of a van V_{van} and two drones V_{dro} in this thesis. This section interprets at first the definition of objects during distribution in the express network, then illustrates tasks of each and activities with each other.

- tasks of depot and docking hub stations

Generally, depot and docking hub stations support the vehicles through recharging, refueling, and reloading. Firstly, both depot and docking hub stations have the inventory of energy cells and parcels. There are electricity cells E_{dep} and E_{doc}, which support drones to change. Meanwhile, they can provide the possibility for the van to refuell or recharge under different charger power level.[56] Comparing to the traditional delivery truck, used vans and drones have moderately limited packages capacity. Thus, the van must visit stations (from a N_{dep} and many N_{doc} on the way) and reload packages regularly. The drones deliver only the "last mile," accordingly, they reload packages from both N_{doc} or V_{van}.

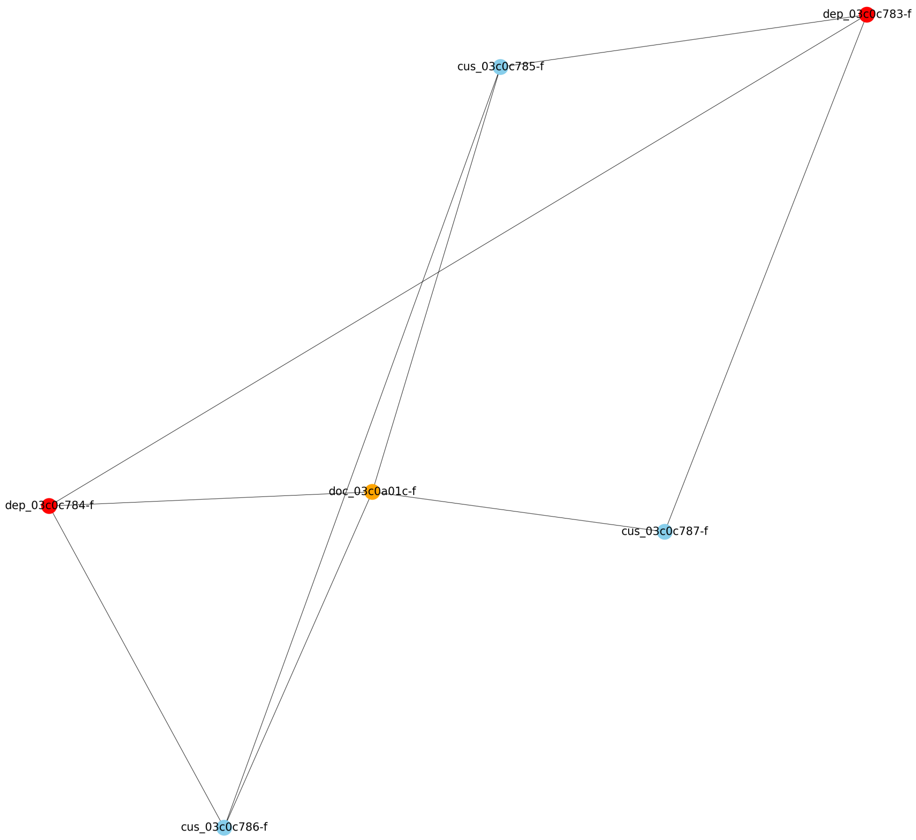

Figure 4.1: Graph Example with six Nodes

- demand and external constraints symbols

Customer nodes contain more information about whether they live in a “no-fly area” or not (if in a ”no-fly” area, $A_{N_i}^{no-fly} = 1$). The route $Rou_{(N_i,N_j)}$ between N_i and N_j (meanwhile, N_i, $N_j \in N$) is measured by the distance of nodes in a form of $W_{(N_i,N_j)}$. During covering of the $Rou_{(N_i,N_j)}$, other conditions like wind ratio with direction (positive ratio R_{wind} for tailwind to decrease the drone’s delivery costs, negative for headwind to increase the costs) and the distance for the drones to detour $Det_{(N_i,N_j)}$ in the air.

- vehicles covering route formulation

V_{van} and two V_{dro} (namely, V_{dro_1} and V_{dro_2}, and the number of drones $Num_{dro} = 2$) launch together from the depot with full energy and packages, and cover routes that mark with

$$Rou_{N_i,N_j}^{dro}, Rou_{N_i,N_j}^{van}, \{N_i, N_j \in N = N_{dep} \cup N_{doc} \cup N_{cus} \mid n \in Num_{dro}\}$$

in the network to satisfy the demand of every customer $D_{N_i}, N_i \in N_{cus}$.

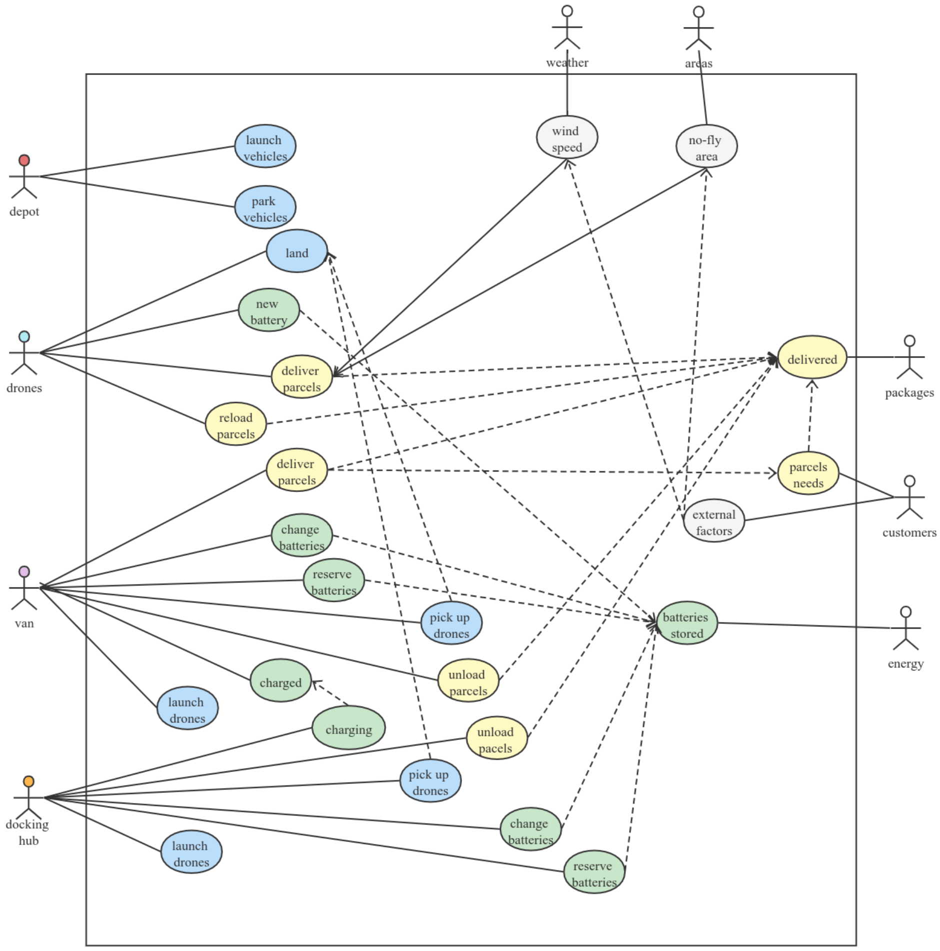

Figure 4.2: Objects and their Functions

- co-actions of drones with van and depot or docking hubs

As the process described in section 3.1, the V_{set} launch together from a depot and turn back to one as functions. During the delivery, drones can either land on a van or any docking hub nodes, thus, both classes of vans and docking hub nodes have the activities with drones of picking up and launching. All these actions are in blue in fig. 4.2. The main activities between vehicles and graph nodes are managements of parcels, and energy cells under external factors. The utilization of use case chart[57] presents a vivid explanation for these activations. The roles in the following picture are related classes of vehicles and graph nodes in fig. 4.2. The activities for packages management, namely load ing, unloading, and delivering parcels, are shown as the bubbles with yellow background. When the energy on the drone runs out, it can either change or refuel on the van or at a docking hub, which are in green in the chart. As for the energy management for van, namely charging and charged activities, are included in green bubbles too. Other factors of weather and area types decide whether distribution with drones is allowed or not firstly, secondly affets the their cost by increasing (such as customer in a no-fly area, risk area, or headwind) or decreasing (such as weather that is tailwind).

- summarizing this and next section

With the help of use case diagram, the functions of each object is vivid and easy to understand the overall relationships between them. However, more minutiae of working process should is more significant for formulation and implementation, which is the main work of the next section.

4.1.2 Distribution Process with Flow Chart

- necessary of using flow chart

The use case chart let demonstrating static relationships of objects and their functions precedence over minutiae of dynamic process. The flow chart can display exact working process and cooperation between objects with each other,[58] thus, is usually necessary to make up its deficiency.

- explaining the symbol of flow chart: flow line, terminal, process, decision.

Symbols in the flowchart have well-defined meanings, namely the flow line, terminal, process, decision, etc. So that the charts are understandable without extra explanation. Shown as in fig. 4.3, the depots in oval boxes indicate the beginning and ending of a process. Logically, the van and drones leave from the depot and start to distribute parcels, which demonstrates itself with arrowhead flow line. When the next station is a docking hub, the van needs to decide whether there is necessary to reload packages for following delivery tasks. Thus the decision block with rhombus box is essential. If any values require modification, actions present in a rectangular box. When there are multiple roles and different sub-processes, multiple swim pools are inevitable, as the four pools in this section.

- dynamic process conditions

Dynamic activities increases the complexity of explaining in detailed textually, thus, flow chart is vivid and convenient to clarify visually. Accordingly, complicated interactions in this thesis is in fig. 4.3, which starts from launching of a van with two drones from a depot, and ends with its arrival at a depot. If it heads to a docking hub node, it can either reload parcels or charge there when it is necessary; if it arrives at a customer node, who has specific parcel needs, the van with drones can either serve by itself or by drones on its top. Vehicles can continue to deliver if their left energy is more than covering the way back. As described in section 3.1, energy cells for the drones are ready on vans and docking hub nodes for exhausted or nearly exhausted drones. Quickly charging the van is possible at any docking hub nodes. As the tasks description of the drones in section 3.1, they can deliver multiple parcels in each trip, accordingly, they could fly to both the van and any docking hub node to reload packages. Similar mechanism lies in the case of van, but their reloading is available only at docking hub nodes. In a precondition that there are enough energy cells for drones in the express network, so that they can choose to turn by themselves, but not on the top of the van peremptorily. Accordingly, the Vehicle Routing Problem for Multi-trip Logistics Delivery with Considering External Conditions (MVRPDE) is actually not an Open Vehicle Routing Problem (OVRP), while all vehicles return to depots (the start and end nodes of depot could be different).

- summarizing this section

Static functions in section 4.1.1 contains three for different aspects, namely energy management, parcels management, drones management, and other external factors. First three in green, yellow, and blue in fig. 4.2 are regulated the same in fig. 4.3. Namely, activities related to energy management are in green symbols, like decisions of "change battery," "change battery in truck,"enough left energy" for the van, etc. Activities of packages management, like value modification operations of "unload package to drones," "deliver the i. package," and "receive the packages," and drones management, for instance decision operators of "deliver with drone," "return to the van," and "completes the tasks," are also included in yellow and blue.

- the main purpose of this section

The definition of detailed process affects the mathematical formulation and ways to implement. For instance, the definition of activities for energy management, like where the drones can obtain a new battery and required parcels and whether the vehicles turn back to the same depot or not, is the foundation of the following sections. To be exact, the next following sections interprets minutiae for the mathematical formulation of the decision variables, objective function, and the constraints.

4.1.3 Decision Variables and Objective Function

- decision variables

This thesis searches for legal solution of routes that vehicles cover, namely a list of stations names which vehicles will visit in order. It depends on used vehicles - a van and two drones, which could be presented as adjacency matrix of nodes, or simply a list of nodes in order. However, the adjacency of each routes are either connected or not. In another words, the program must assign a value for each route $Rou_{(N_i,N_j)}$. If it is connected, route's value is assigned with 1, otherwise, with 0.

$$Rou^{v}_{(Ni,Nj)} \in \{0,1\}, \{Ni, Nj \in N \mid v \in V_{set} = V_{dro_n} \cup V_{van} \mid n \in Num_{dro}\}$$

To be specific, decision variables index the ways that vehicles take to complete delivery tasks. The chosen nodes in each searching are nodes vehicles visited $P_v = \{N_1, ..., N_i\}, N_1, ..., N_i \in N$ and $v \in V_{set}$, namely routes have a value of $Rou^{v}_{(N_i,N_j)} = 1$.

- objective function

This work aims to minimize the delivery costs, which consists of fixed and variable cost, namely fixed cost for hiring the van and drivers C^{van}_{fix}, energy cells of the drones C^{dro}_{fix}, and other accidental costs C_{othr}, and specific cost ratios of van and drones for covering each in an unit of distance, namely R_{dro} and R_{van} that spend for tasks delivery. Thus, the fix cost and objective function in this problem are:

$$C_{fix} = C^{van}_{fix} + C^{dro}_{fix} + C_{othr} \tag{4.1}$$

$$\begin{aligned} \min C_{min} = C_{fix} + \sum_{N_i,N_j \in N} (\sum_{n}^{Num_{dro}} (Rou^{dro_n}_{(N_i,N_j)} \times (W_{(N_i,N_j)} + Det_{(Ni,Nj)}) \times R_{dro} \times \\ (1 + R_{wind})) + Rou^{van}_{(N_i,N_j)} \times W_{(N_i,N_j)} \times R_{van}) \end{aligned} \tag{4.2}$$

- components and explanation of objective function

Based on the definition in section 4.1.1 and decision variable before, the variable cost for drones is the sum of each routes' muliplication. Namely, the connectivity of a drone times its weight with extra factor for drones to detour in an unit of distance firstly, namely $W_{(N_i,N_j)} + Det_{(N_i,N_j)}$. Furtherly, the weighted distance of drones converts into the cost in a ratio of R_{dro}. The wind affects this cost in a specific ratio R_{wind}. Thus, if the wind is headwind that help drones to fly, negative R_{wind} decreases the costs in percentage of $1 + R_{wind}$. Meanwhile, this thesis tests with two drones, thus, two drones' variable cost operate with van's variable cost. Variable cost of van has similar distance measurement through decision variable connectivity of the van's each route $Rou^{van}_{(N_i,N_j}$ and its weight, then, transforms itself to the cost in a ratio of R_{van}.

- summarizing decision variables and objective function

Legal adjacency matrixes for the van and two drones input into a optimized searching algorithm, which satisfy all constraints listed in the following sections. The index used to optimize of heuristics algorithms is the overall cost (fixed cost C_{fix} pluses the variable cost of drones and van).

4.2 Constraints

- not implemented relationship of drones' left energy for flying and its left packages

Packages that vehicles can deliver are coupled with their left energy, which requires further development based on hardware according to the drone type with specialized weights of battery and it own, which is not easy to simulate so that not implemented in this thesis. In addition, its implementation requires far more computational power to search a legel decision variable, which requires fine optimization of the implementation so that not implemented in this thesis. Thus, constraints for the relationship of drones' left energy for flying and left packages are not included.

4.2.1 The Packages Constraints

- potential trends of design

The vehicles' different operations at the docking hub nodes affect the minutiae of the mathematical formulation, like whether they have enough energy to achieve the next docking hub node or the depot node. Hence, two possible methods are able to avoid the fatal situation, namely refuel and recharge the hybrid cells and batteries of the van and drones whenever they arrive at a docking hub node, or return more frequently to docking hub nodes to refuel and recharge involved in left packages they need to distribute.

- theoretical packages capacity constraint

Packages capacity of the drone and the van decide maximum packages they can load once, namely B_{dro}^{pac} and B_{van}^{pac}. The van and drones can not deliver more packages than they can afford. Thus, this section compares the overall maximum packages that vehicles can achieve with the packages that they delivered according to customer demand $Dem_{N_{cus}}$. Firstly, the overall maximum packages constraint is related to the number of intersection of the drones with the van I_{van}^{dro} and the docking hub nodes I_{doc}^{dro}, and of the van with the docking hub nodes. Each arrival at these intersections demonstrates a possibility of loading packages fully. Packages that are distributed by drones refers to their intersection with customers $I_{cus-N_{cus}}^{dro}$, namely the customers they have served. With similar mechanism, intersections of the van's adjacency matrix with docking hub node I^{van}_{doc} and customers nodes $I^{van}_{cus-N_{cus}}$ help us to verify its packages capacity. Generally, intersections of decision variables with related nodes are main parameter to verify their packages capacity (Con^{Mpac}_{dro} and Con^{Mpac}_{van}) in this section, as shown in the following itemization.

- I^{dro}_{van}: The intersection of the drones with the van
- I^{dro}_{doc}: The intersection of the drones with the docking hub nodes
- I^{van}_{doc}: The intersection of the van with the docking hub nodes
- $I^{dro}_{cus-N_{cus}}$: The intersection of the drones with the customer nodes
- $I^{van}_{cus-N_{cus}}$: The intersection of the van with the customer nodes
- Con^{Mpac}_{dro}: The overall maximum packages capacity constraint of the drones
- Con^{Mpac}_{van}: The overall maximum package capacity constraint of the van

$$I^{v}_{van} = \sum B_{N_i,N_j}, \{B_{N_i,N_j} \in \{0,1\} \mid v \in V_{dro} \mid B_{N_i,N_j} = 1, whenRou^{v}_{N_i,N_j} = Rou^{van}_{(N_i,N_j)} = 1\} \tag{4.3}$$

$$\begin{aligned} I^{v}_{doc} = \sum B_{N_i,N_j}, \{B_{N_i,N_j} \in \{0,1\} \mid v \in V_{dro} \mid N_i \in N_{doc} or N_j \in N_{doc} \\ \mid B_{N_i,N_j} = 1, whenRou^{v}_{(N_i,N_j)} = 1\} \end{aligned} \tag{4.4}$$

$$\begin{aligned} I^{v}_{doc} = \sum B_{N_i,N_j}, \{B_{N_i,N_j} \in \{0,1\} \mid v \in V_{van} \mid N_i \in N_{doc} or N_j \in N_{doc} \\ \mid B_{N_i,N_j} = 1, whenRou^{v}_{(N_i,N_j)} = 1\} \end{aligned} \tag{4.5}$$

$$I^{v}_{cus-N_i} = B_{N_i,N_j}, \{B_{N_i,N_j} \in \{0,1\} \mid v \in V_{dro} \mid N_i \in N_{cus} \mid B_{N_i,N_j} = 1, whenRou^{v}_{(N_i,N_j)} = 1\} \tag{4.6}$$

$$I^{v}_{cus-N_i} = B_{N_i,N_j}, \{B_{N_i,N_j} \in \{0,1\} \mid v \in V_{van} \mid N_i \in N_{cus} \mid B_{N_i,N_j} = 1, whenRou^{v}_{(N_i,N_j)} = 1\} \tag{4.7}$$

$$Con^{Mpac}_{dro_n} : (I^{dro_n}_{doc} + I^{dro_n}_{van} + 1) \times B^{pac}_{dro} >= \sum I^{dro_n}_{cus-N_i} \times Dem_{N_i}, \{n \in Num_{dro} \mid N_i \in N_{cus}\} \tag{4.8}$$

$$Con_{van}^{Mpac} : (I_{doc}^{van} + 1) \times B_{van}^{pac} >= \sum I_{cus-N_i}^{van} \times Dem_{N_i}, \{N_i \in N_{cus}\} \tag{4.9}$$

Because the van and drones launch from a depot that load with full packages. Thus in eq. (4.8) and eq. (4.9), intersections add with a extra one, namely $(I_{doc}^{van} + 1)$ and $(I_{doc}^{dro_n} + I_{van}^{dro_n} + 1)$. It is a pity that because shortage of multi-core CPU programming experience and detailed research on realistic drones, this section formulates without checking packages capacity contraints in every trip. It demonstrates still a clear design of overall packages capacity checking through searching intersections of decision variables with relevant nodes in the express network.

4.2.2 The Energy Constraints

The constraints of MVRPDE share similar principles with that of the packages, because drones retain their batteries at the same place, like in the packages constraints (on the van V_{van} and at any docking hub nodes N_{doc}). The case is also same to the van. Thus, the limited number of energy cells for the drones on a van Ra_{van}^{bat} and at a docking hub Ra_{doc}^{bat} restricts their energy capacity.

- $Ra_{V_{set}}$: the maximum range that vehicles (the van and drones) can achieve.
- $Con_{V_{set}}^{Mbat}$: the overall energy constraint for the vehicles, that energy units they used are no more than the maximum they can get in the graph $G = (N, W)$.
- $Con_{V_{set}}^{Lbat}$: the energy limitation of the vehicles during each segment.

$$\begin{gathered} SR_a^{van} = \{(N_{D_a+1}, N_{D_a+2}), ..., N_{D_{a+1}-2}, N_{D_{a+1}-1})\}, \\ \{(N_{D_a+1}, N_{D_a+2}), ..., N_{D_{a+1}-2}, N_{D_{a+1}-1}) \in Rou_{(N_i,N_j)}^{van} \mid N_i, N_j \in N\} \end{gathered} \tag{4.10}$$

$$\begin{gathered} SR_a^{dro_n} = \{(N_{D_a+1}, N_{D_a+2}), ..., N_{D_{a+1}-2}, N_{D_{a+1}-1})\}, \\ \{(N_{D_a+1}, N_{D_a+2}), ..., N_{D_{a+1}-2}, N_{D_{a+1}-1}) \in Rou_{(N_i,N_j)}^{dro_n} \mid N_i, N_j \in N \mid n \in Num_{dro}\} \end{gathered} \tag{4.11}$$

$$Con_{van}^{Mbat} : (I_{doc}^{van} + 1) \times Ra_{van} >= \sum Rou_{(N_i,N_j)}^{van} \times W_{(N_i,N_j)}, N_i, N_j \in N \tag{4.12}$$

$$Con_{dro_n}^{Mbat} : (I_{doc}^{dro_n} + I_{van}^{dro_n} + 1) \times Ra_{dro_n} >= \sum Rou_{(N_i,N_j)}^{dro_n} \times W_{(N_i,N_j)}, \{N_i, N_j \in N \mid n \in Num_{dro}\} \tag{4.13}$$

$$Con_{van}^{Lbat} : E_i = 0, \{E_i\{0,1\} \mid E_i = 1, when \sum W_{(N_a,N_b)} > Ra_{van} \mid N_a, N_b \in S_i^{van}\} \quad (4.14)$$

$$Con_{dro_n}^{Lbat} : E_i = 0, \{E_i\{0,1\} \mid E_i = 1, when \sum W_{(N_a,N_b)} > Ra_{dro_n} \mid N_a, N_b \in S_i^{dro_n} \mid n \in Num_{dro}\} \quad (4.15)$$

4.2.3 The Customer Demand Constraints

The packages required by the customers must be fulfilled, which will be carried out by either the van or drones. Thus, they must visit every customer nodes at least once. This section lists the tasks' constraint with assessment indicator of packages delivered comparing to the required. Namely,

$$Con^{task} : \sum(Dem_{N_a} \times Rou_{N_a,N_b}^{van}) + \sum_{dro_n}^{Num_{dro}} (Dem_{N_c}^{dro_n} \times Rou_{N_c,N_d}) = \sum Dem_{N_d}, \quad (4.16)$$
$$\{N_a, N_b \in P_{van} \cap N_{cus} \mid N_c, N_d \in P_{dro_n} \cap N_{cus} \mid N_d \in N_{cus} \mid n \in Num_{dro}\}$$

4.2.4 No-fly Zone Constraints

Currently, the most important issue that affects the realistic application of VRPD is the society and politics. Many areas might be running security missions that allow no commercial drones to fly over, or exist a swam of rare animals that would be affected severely by drones' flight. Therefore, the related constraints is necessary to be included during formulating, like in a form of "no-fly area". That is, customers, $N_a \in N_{cus}$ who live in a no-fly area that allows no drones to fly over. Namely, the attribute of the no-fly for the N_a exists, viz. $A_{N_a}^{no-fly} = 1$. Thus, the routes that connect with the node can not be covered by drones, but by the van. I.e., the no-fly zone constraint of the MVRPDE formulates as eq. (4.17).

$$Con_{dro}^{nf} : Rou_{N_i,N_j}^{dro_n} = 0, Rou_{N_i,N_j}^{van} = 1, \{n \in Num_{dro} \mid N_i, N_j \in N \mid A_{N_i}^{no-fly} = 1 or A_{N_j}^{no-fly} = 1\} \quad (4.17)$$

4.2.5 The Legal Routes Constraints

MVRPDE is not an OVRP, that requires vehicles to return to a depot. The drones can choose to return with a van or by themselves. Thus, the routes that the vehicles should cover are not only ordered, but also needed to satisfy more conditions. Firstly, the start and end of the vehicles' path $P_{V_{set}}$ as described in section 4.1.1 must be depot nodes N_{dep}. Meanwhile, when the van with drones leaves the depot and heads to a docking hub node, drones stay with the van, but not act by their own. Thus, no matter in the beginning or at the end of the delivery process, the drones cover no routes that connect depot and docking hub nodes, as in the following formulation:

$$\begin{aligned} Rou^{van}_{N_a,N_b} \in \{0,1\}, \{N_a, N_b \in P_{van} \mid N_a \notin N_{doc} \cup N_{dep}, when N_b \in N_{doc} \cup N_{dep} \mid \\ N_b \notin N_{doc} \cup N_{dep}, when N_a \in N_{doc} \cup N_{dep}\}. \end{aligned} \tag{4.18}$$

$$\begin{gathered} Rou^{dro_n}_{N_a,N_b} \in \{0,1\}, \{N_a, N_b \in P_{dro_n} \mid n \in Num_{dro} \mid N_a \notin N_{doc} \cup N_{dep}, \\ when N_b \in N_{doc} \cup N_{dep} \mid N_b \notin N_{doc} \cup N_{dep}, when N_a \in N_{doc} \cup N_{dep}\}. \end{gathered} \tag{4.19}$$