Polynomial Expression

In mathematics, a polynomial expression contains multiple variables (also called indeterminates) and coefficients, involving only the operations of addition, subtraction, multiplication.

An example of polynomial expression in two variables:

$$f(x, y) = x^3 + 2xy^2 - xy + 1 \tag{1}$$

Single-variable Polynomial

For a polynomial expression in one variable of maximum degree n:

$$f(x) = \sum_{i=0}^{n} c_i x^i \tag{2}$$

Polynomial Regression

Task:

1. Define a sing-variable polynomial function and visualize the line in a 2D space
2. Prepare a training set by sampling points from the single-variable polynomial function, and visualize the set in a D space.
3. Polymonial Regression: Find a polymonial function to best fit the training set. Solving by scikit-learn.

1. Polynomial function definition

Here we define the function of degree 4 as:

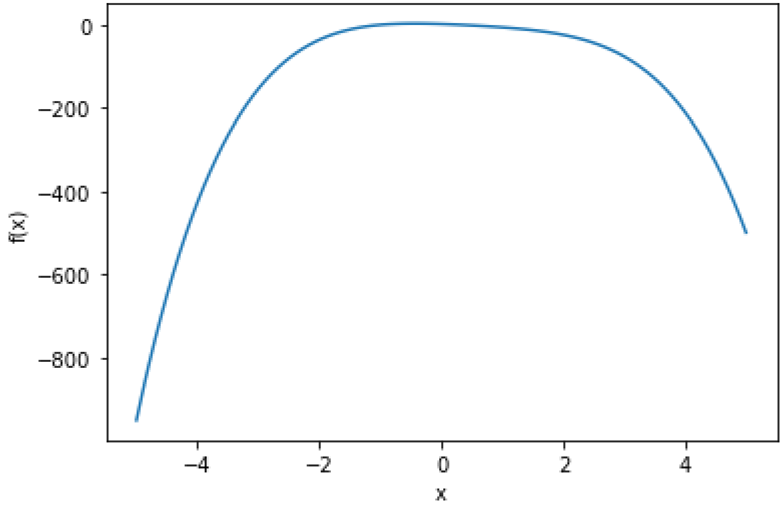
$$f(x) = 1 - 5x - 4x^2 + 2x^3 - x^4 \tag{3}$$

In [1]:

```
coefficients=[1., -5., -4., 2., -1]
# Solution 1: Using numpy.polynomial lib
# The returned p is function that maps an input to a value according to the polynomial coffecients.
import numpy as np
p = np.polynomial.Polynomial(coefficients)


# Solution 2: define the function manually
p = lambda x: sum([coefficients[i] * x**i for i in range(len(coefficients))])
```

In [2]:

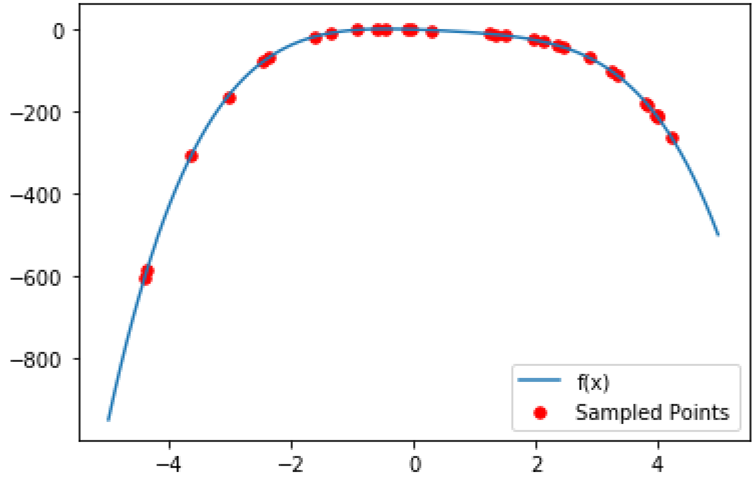
```
import matplotlib.pyplot as plt
%matplotlib inline

x = np.linspace(-5., 5., 100)
y = p(x)

plt.plot(x, y)
plt.xlabel('x')
plt.ylabel('f(x)')
plt.show()
```



2. Sampling and visualize the data from the defined function

In [3]:
```python
# Sample 30 points, value in range [-5, 5]
n_samples = 30
X = np.random.uniform(low=-5., high=5., size=n_samples)
Y = p(X)

plt.plot(x, y)
plt.scatter(X, Y, color='red', marker='o', s=30)
plt.legend(['f(x)', 'Sampled Points'])
```

Out[3]: <matplotlib.legend.Legend at 0x1304ea1d0>



3. Use Scikit-learn to solve the regression problem

Scikit-learn: Simple and efficient tools for predictive data analysis (url:[https://scikit-learn.org/stable/ (https://scikit-learn.org/stable/)](https://scikit-learn.org/stable/))

Task:

Step 1: Convert the training data to a polymonial space.

E.g.

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1^0 & 1^1 & 1^2 & 1^3 \\ 2^0 & 2^1 & 2^2 & 2^3 \\ 3^0 & 3^1 & 3^2 & 3^3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \end{bmatrix} \tag{4}$$

Step 2: Use **Linear Regression** in Scikit-Learn to solve the fit the training set.

In [4]:
```
from sklearn.linear_model import Ridge, LinearRegression
from sklearn.preprocessing import PolynomialFeatures
from sklearn.pipeline import make_pipeline

# Transform to Polymonial space of a specific degree
ploy_fea = PolynomialFeatures(degree=3)
X_transform = ploy_fea.fit_transform(X[:, np.newaxis])

# Linear Regression
regression_model = LinearRegression(fit_intercept=False)
regression_model.fit(X_transform, Y[:, np.newaxis])
```

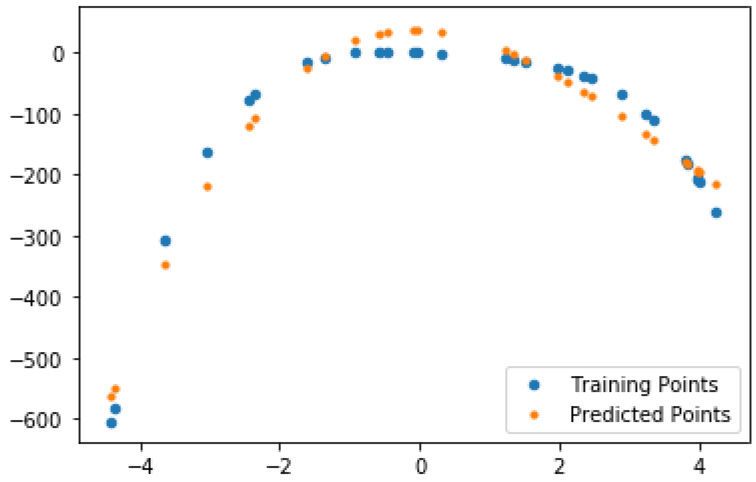
Out[4]:
```
LinearRegression(copy_X=True, fit_intercept=False, n_jobs=None, normalize=False)
```

In [5]:
```
# Visualize the coefficients of the regression model
print('Coefficients')
print('Ground Truth:', np.array(coefficients))
print('Prediction  :', regression_model.coef_[0])
```

```
Coefficients
Ground Truth: [ 1. -5. -4.  2. -1.]
Prediction  : [ 37.64693356  -3.07727116 -22.22794197   2.10781008]
```

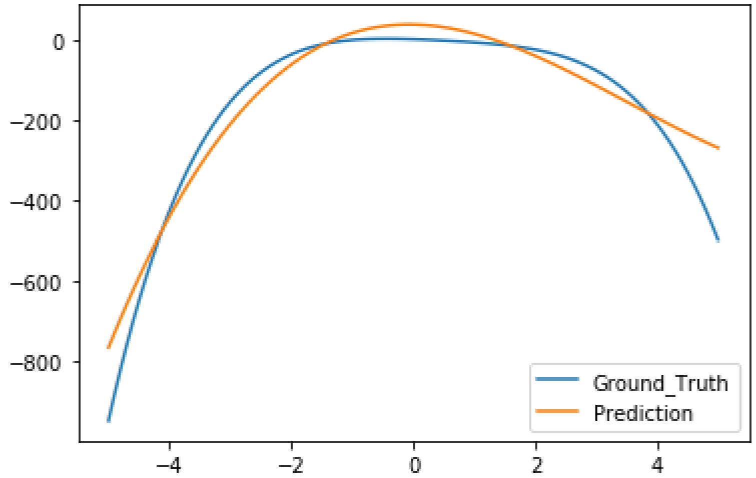
In [6]:
```
# Visualize the training points and predicted points
predicted_Y = regression_model.predict(X_transform)

plt.scatter(X, Y, s=20)
plt.scatter(X, predicted_Y, s=10)
plt.legend(['Training Points', 'Predicted Points'])
```

Out[6]: <matplotlib.legend.Legend at 0x13050bfd0>



In [7]:
```
# Visualize the training points and predicted points
x = np.linspace(-5., 5., 100)
y = p(x)
x_transformed = ploy_fea.fit_transform(x[:, np.newaxis])
y_predicted = regression_model.predict(x_transformed)

plt.plot(x, y)
plt.plot(x, y_predicted)
plt.legend(['Ground_Truth', 'Prediction'])
```

Out[7]: <matplotlib.legend.Legend at 0x142d010b8>

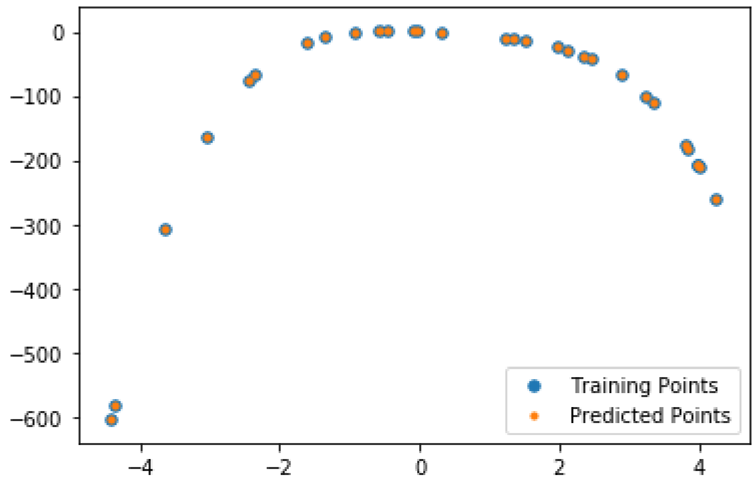


Altertively, change degree to 4

In [8]:
```
# Transform to Polynomial space of a specific degree
ploy_fea = PolynomialFeatures(degree=4)
X_transform = ploy_fea.fit_transform(X[:, np.newaxis])

# Linear Regression
regression_model = LinearRegression(fit_intercept=False)
regression_model.fit(X_transform, Y[:, np.newaxis])

# Visualize the coefficients of the regression model
print('Coefficients')
print('Ground Truth:', np.array(coefficients))
print('Prediction  :', regression_model.coef_[0])

# Visualize the training points and predicted points
predicted_Y = regression_model.predict(X_transform)

plt.scatter(X, Y, s=30)
plt.scatter(X, predicted_Y, s=10)
plt.legend(['Training Points', 'Predicted Points'])
```

```
Coefficients
Ground Truth: [ 1. -5. -4.  2. -1.]
Prediction  : [ 1. -5. -4.  2. -1.]
```

Out[8]: <matplotlib.legend.Legend at 0x142dde278>

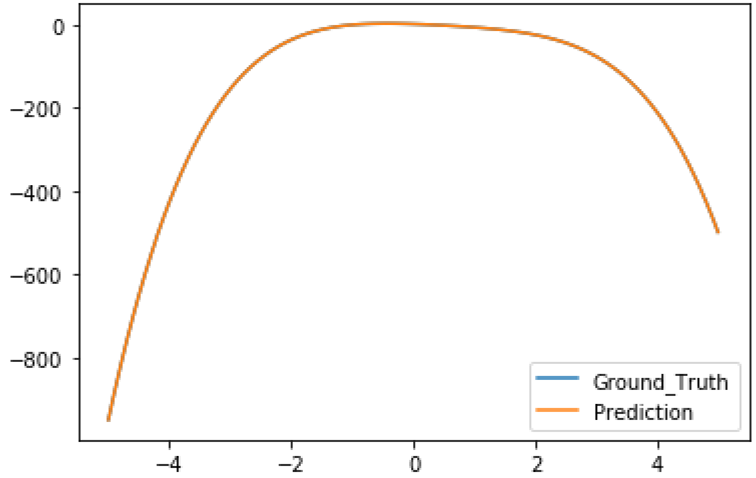


In [9]:

```
# Visualize the training points and predicted points
x = np.linspace(-5., 5., 100)
y = p(x)
x_transformed = ploy_fea.fit_transform(x[:, np.newaxis])
y_predicted = regression_model.predict(x_transformed)

plt.plot(x, y)
plt.plot(x, y_predicted)
plt.legend(['Ground_Truth', 'Prediction'])
```

Out[9]:

```
<matplotlib.legend.Legend at 0x142e422b0>
```



In [ ]: