$P_0 =$ Initial Loan Size (\$)
$R =$ Annual interest rate (APR) described as an annual number (% 0-1)
$r = R/12$ Interest rate per month (% 0-1)
$M =$ Monthly Payment (\$)
$n =$ Number of payments (months)

Each month there is an interest payment on the principal. Anything extra beyond the interest payment will deduct from the principal. For month 1:

$$\text{Interest owed} = P_0 * r \tag{1}$$

Since the principal is reduced by payment beyond the interest, the principal is reduced by

$$M - P_0 r \tag{2}$$

So the principal after the first payment will be

$$\begin{aligned} P_1 &= P_0 - (M - P_0 r) \\ &= P_0(1+r) - M \end{aligned} \tag{3}$$

And the principal after the second payment will be

$$\begin{aligned} P_2 &= P_1 - (M - P_1 r) \\ &= P_1(1+r) - M \end{aligned} \tag{4}$$

And the third payment will be

$$\begin{aligned} P_3 &= P_2 - (M - P_2 r) \\ &= P_2(1+r) - M \end{aligned} \tag{5}$$

Clearly, there is a pattern, the following recursive formula is true:

$$P_{n+1} = P_n(1+r) - M \tag{6}$$

Right now, this is not useful. If we substitute (4) into (5) we get

$$\begin{aligned} P_3 &= (P_1(1+r) - M)(1+r) - M \\ &= P_1(1+r)^2 - M(1+r) - M \end{aligned} \tag{7}$$

And substituting (3) into (7) gives

$$\begin{aligned} P_3 &= (P_0(1+r) - M)(1+r)^2 - M(1+r) - M \\ &= P_0(1+r)^3 - M(1+r)^2 - M(1+r) - M \end{aligned} \tag{8}$$

The pattern here is

$$P_n = P_0(1+r)^n - [M + M(1+r) + M(1+r)^2 + ... + M(1+r)^{n-1}] \tag{9}$$

The M terms are going through a geometric series. The factor by which M is multiplied is $(1+r)$. For a standard geometric series:

$$\sum_{n=0}^{\infty} ar^n = a\left(\frac{1-r^n}{1-r}\right) \tag{10}$$

and substituting values from (9) gives

$$\begin{aligned} P_n &= P_0(1+r)^n - \sum_{n=0}^{\infty} M(1+r)^n \\ &= P_0(1+r)^n - \left[M\left(\frac{1-(1+r)^n}{1-(1+r)}\right)\right] \end{aligned} \tag{11}$$

We are interested in when $P_n = 0$ because this is when the amount owed is 0. Now we just need to solve for M when $P_n = 0$. Rearranging (11),

$$M\left(\frac{(1+r)^n - 1}{r}\right) = P_0(1+r)^n \tag{12}$$

so

$$\boxed{M = \frac{rP_0(1+r)^n}{(1+r)^n - 1}} \tag{13}$$