2st Assignment

CS330 Discrete Structures, Fall 2015

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Problems

1) **Show that each conditional statement in Exercise 10 is a tautology without using truth tables.**

   a) **[¬p ∧ (p ∨ q)] → q**

   ⇔ ¬ [¬p ^ (p v q)] V q
   ⇔ [p V ¬ (p V q)] V q
   ⇔ [p V (¬ p ^ ¬ q)] V q
   ⇔ [T^(p V ¬q)] V q
   ⇔ (p V ¬q) V q
   ⇔ p V (¬q V q)
   ⇔ p V T ⇔ T
   So this one is a tautology

   b) **[(p → q) ∧ (q → r)] → (p → r)**

   ⇔ [(¬p V q) ^ (¬q V r)] -> (¬p V r)
   ⇔ (¬[(¬p V q) ^ (¬q V r)])V (¬p V r)
   ⇔ (¬(¬p V q) V ¬ (¬q V r)) V (¬ p V r)
   ⇔ (¬¬p ^ ¬ q) V (¬¬q ^ ¬r) V (¬p V r)
   ⇔(p^¬q) V (q^¬r) V (¬p V r)
   ⇔(p^¬q)V(q^¬r)V(¬p)V(r)
   ⇔(¬p)V(p^¬q)V(r) V(¬r ^ q)
   ⇔ (T^(¬pV¬q)) V (T ^ (r V q))
   ⇔(¬p V ¬q) V (r V q)
   ⇔ (¬p V r) V T
   ⇔ T

c) $[p \wedge (p \rightarrow q)] \rightarrow q$

⇔[p ^ (¬p V q)] -> q
⇔[(p^¬p) V (p ^ q)] V q
⇔¬ [F V (p^q)] V q
⇔¬[p^q] v q
⇔(¬p)V(¬q)Vq
⇔(¬p)VT
⇔T

d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

⇔[(pVq) ^ (¬p V r)^ (¬qV r)] -> r
⇔¬ [(p V q) ^ (¬P v r) ^ (¬q V r)] V r
⇔ ¬(p V q) V ¬ (¬p V r) V ¬ (¬q V r) V r
⇔ (¬p ^ ¬q) V (¬¬p ^ ¬r) V (¬¬q ^ ¬r) V r
⇔(¬p ^ ¬q) V (p ^ ¬r) V (q ^ ¬r) V r
⇔ (¬p^ ¬q) V (p ^ ¬r) V (qV r) ^ (T)
⇔(¬p ^ ¬q) V (p ^ ¬r) V q V r
⇔(¬p ^ ¬q) V q V (p ^ ¬r) V r
⇔ [(¬p V q) ^ (T)]V [(p V r) ^ (T)]
⇔(¬p V q) V (p V r)
⇔ T V q v r
⇔ T

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2) Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty.

a) $\forall x(A \rightarrow P(x)) \equiv A \rightarrow \forall xP(x)$

If p(x) is ture for all x, then.

- ➔ A -> P(x) is true for all x
- ➔ Left side of this logical formula is true and ∀xP(x) is true
- ➔ A-> ∀xP(x) is true
- ➔ So left = right = true

If p(x) is false for all x, then.

- ➔ 1) A is true
- ➔ A → P(x) is false for some of x , so ∀x(A → P(x)) is false
- ➔ ∀x(A → P(x)) is false because A -> P(x) is always false
- ➔ So left = right = false
- ➔ 2)A is false
- ➔ A → P(x) is true for all x , left = false
- ➔ ∀xP(x) is false due to P(x) is false, so A-> **∀xP(x) is false, so right = false**
- ➔ Left = right = false

So $\forall x(A \rightarrow P(x)) \equiv A \rightarrow \forall xP(x)$

b) $\exists x(A \rightarrow P(x)) \equiv A \rightarrow \exists xP(x)$

If p(x) is true for some x

- ➔ Then A-> P(x) is true for some x
- ➔ Left = true
- ➔ ∃xP(x) is true
- ➔ **A -> ∃xP(x) is true**
- ➔ **Right = true**
- ➔ **Left = right = true**

If p(x) is false for all x

1# A is false

- ➔ Then A-> P(x) is true for all x
- ➔ Left = true
- ➔ **A (false)→ ∃xP(x) (false)**
- ➔ **So right = true**
- ➔ **Left = right = true**

2# A is true

- ➔ Then A-> P(x) is false for all x
- ➔ Left is false
- ➔ **∃xP(x) is false so A-> ∃xP(x) is false -> right = false**
- ➔ **So left = right = false**

3) Let P(x), Q(x), R(x), and S(x) be the statements “x is a duck,” “x is one of my poultry,” “x is an officer,” and “x is willing to waltz,” respectively. Express each of these statements using quantifiers; logical connectives; and P(x), Q(x), R(x), and S(x).

a) No ducks are willing to waltz.

∀x(P(x)-> ¬S(x))

b) No officers ever decline to waltz.

∀x(r(x)-> S(x))

c) All my poultry are ducks.

∀x (Q(x) -> p(x))

d) My poultry are not officers.

∀x (Q(x) ->¬ R(x))

∗e) Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?

yes

4) Let C(x, y) mean that student x is enrolled in class y, where the domain for x consists of all students in your school and the domain for y consists of all classes being given at your school. Express each of these statements by a simple English sentence.

a) Randy Goldberg is enrolled in class CS 252

b) someone is enrolled in class math695

c) Carol sitea is enrolled in some class

d)someone is enrolled in math222 and CS 252 at the same time

e)there are two kinds of distinct students, one kind of them enrolled in every course that the other kind of students enrolled in

f)there are two kinds of distinct students enrolled in exactly the same course

5) A discrete mathematics class contains 1 mathematics major who is a freshman, 12 mathematics majors who are
sophomores, 15 computer science majors who are sophomores,
2 mathematics majors who are juniors, 2 computer
science majors who are juniors, and 1 computer science
major who is a senior. Express each of these statements in
terms of quantifiers and then determine its truth value

let P(s, c, m) be the statement that student s academic year length level c and is majoring in m

a) $\exists s\, \exists m\, P(s, \text{junior}, m)$ true

b) $\forall s\, \exists c\, P(s, c, \text{computer science})$ false, due to there still some Math major student

c) $\exists s\, \exists c\, \exists m\, (P(s, c, m) \wedge (c \neq \text{junior}) \wedge (m \neq \text{mathematics}))$ true, since a sophomores in CS major

d) $\forall s(\exists c\, P(s, c, \text{computer science}) \vee \exists m\, P(s, \text{sophomore}, m))$. This is false. Freshman is in math major

e) $\exists s\, \forall c\, \exists m\, (s, c, m)$. false

6) Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

a) Let L(x, y) means that x has lost y bucks in playing lottery

Original $\neg\exists x\, \exists y\, ((y>100) \wedge L(x, y))$

Negation $\exists x\, \exists y\, ((y>100) \wedge L(x, y))$

Someone has lost more than 1000 bucks in playing lottery

b) Let L(x, y) means that x in class chatted with y

Original

$\exists x\, \exists y\, (y \neq x \wedge \forall z(z \neq x \rightarrow (z=y \Leftrightarrow C(x,z))))$

Negation

$\forall x\, \forall y\, (y \neq x \rightarrow \exists z(z \neq x \wedge \neg(z=y \Leftrightarrow C(x,z))))$

Everybody in the class has either chatted with no one else or has chatted with two or more others

c) Let E(x, y) means that student x has sent email to person y.

Original

$\neg\, \exists x\, \exists y\, \exists z\, (y \neq z \wedge x \neq y \wedge x \neq z \wedge \forall w(w \neq x \rightarrow (E(x, w) \Leftrightarrow (w=y \vee w = z))))$

Negation

$\exists x\, \exists y\, \exists z\, (y \neq z \wedge x \neq y \wedge x \neq z \wedge \forall w(w \neq x \rightarrow (E(x, w) \quad (w=y \vee w = z))))$

Some student in this class has sent email to exactly two other students in this class

d) Let S(x, y) means that student X has solved exercise y.
Original
$\forall x \forall y\, S(x, y)$
Negation
$\forall x \exists y \neg S(x, y)$
For every student in this class, there is some exercise that he or she has not solved

e) Let s(x, y) means that student x has solved exercise y, let B(y, z means that exercise y is in section z of the book.
Original
$\neg \exists x \exists y \forall z\, (B(y, z) \wedge S(x, y))$
Negation
$\exists x \exists y \forall z\, (B(y, z) \wedge S(x, y))$
Some student has solved at least one exercise in every section of this book.