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CS330-HW-03

Problems

1.5-22.

Use predicates, quantifiers, logical connectives and mathematical operators to express the statement that there is a positive integer that is not the sum of three squares.

∃x ∀a ∀b ∀c ((x > 0)^x ≠ (a^3) + (b^3) + (c^3))

1.5-28.

Determine the truth value of each of these statements if the domain of each variable consists of all real numbers. Then, translate the h)-j) to English sentences.

a)∀x∃y(x2 = y)

true

b)∀x∃y(x = y2)

false, since if x <0, then you can never find a real number y that y^2 is negative

c)∃x∀y(xy = 0)

true

d)∃x∃y(x + y≠ y + x)

false, it's obvious false.

e)∀x(x≠ 0 → ∃y(xy = 1))

true

f)∃x∀y(y≠ 0 → xy = 1)

true

g)∀x∃y(x + y = 1)

true

h)$\exists x \exists y(x+2y = 2 \wedge 2x+4y = 5)$

$x + 2y = 2 \iff 2x + 4y = 4$

$2x + 4y = 5$

since we can never find real number of x and y to make the two statement to be true same time, so the statement is **false**.

Translate to EN: There were one bag of some kinds of apples and two bags of some kinds of oranges, the total weight of them is 2kg, and there were two bags of the same kinds of apples and four bags of the same kinds oranges, the total weight of them is 5kg.

i)$\forall x \exists y(x+y = 2 \wedge 2x-y = 1)$

Since when solve $\{ x+y =2 , 2x - y = 1 \}$ we get $x=1, y=1$

but when we let $x = 0$, we get $y =2$; the equation $2x - y =1$ given out that $y= -1$. we get two y values when $x = 0$. So the statement is **false**.

Translate to EN: For all real numbers x and some real numbers y, x plus y equal to 2 and the double of x minus y equal to 1 are true.

j)$\forall x \forall y \exists z(z = (x + y)/2)$

It's true. Since the sum of x and y is a real number and when divide by a non zero number '2', the final result is belong to real number set.

Translate to EN: For all real numbers x, y there are some z that make the equation z equal to the sum of x plus y and then divide the sum by two .

1.6-8.

What rules of inference are used in this argument? "No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."

modus peon and universal instantiation were used in this argument.

1.6-24.

Identify the error or errors in this argument that supposedly shows that if ∀x(P (x) ∨ Q(x)) is true then ∀xP (x) ∨ ∀xQ(x) is true.

1.$\forall x(P(x) \vee Q(x))$	Premise
2.$P(c) \vee Q(c)$	Universal instantiation from (1)
3.$P(c)$	Simplification from (2)
4.$\forall x P(x)$	Universal generalization from (3)
5.$Q(c)$	Simplification from (2)
6.$\forall x Q(x)$	Universal generalization from (5)
7.$\forall x(P(x) \vee xQ(x))$	Conjunction from (4) and (6)

1 and 2 is correct.

3 is not correct due to wether P(c) is true of false, when the Q(c) is true the P(c) V Q(c) is always true.

since 3 is not correct, whatever obtained from 3 is also incorrect. So the 4 is not correct.

for step5 , it is also not correct. the reason is the same class of step 3.

the step 6 is just like 4.

since 4 and 6 is not valid, so the step 7 is not possible to be valid. Also, the conjunction is not V.

1.7-8.

Prove that if n is a perfect square, then n + 2 is not a perfect square.

Using contradiction.

Assume n+2 is a perfect square when n is a perfect square.

then, n = x^2 x is a positive integer

then, (n+2) = y^2 y is also a positive integer.

x^2 + 2 = y^2

y^2 - x^2 = 2

(y-x)(y+x) = 2 since 2 only have factory 1 and 2, so the { y+x =2, y-x =1} -> {y=3/2}

y is not a integer, which is a contradiction. so assume is wrong. so the origin statement is true.

1.7-16.

Prove that if m and n are integers and mn is even, then m is even or n is even.

Using contradiction.

Assume that m is not even and n is not even, then mn is even.

then $m = 2x + 1$, $n = 2y+1$

$mn = (2x+1)(2y + 1)$

$mn = 4xy + 2x + 2y + 1$

$mn = 2(2xy + x + y) + 1$

the mn is not even since 2 * (a real integer) + 1 never be even.

which is a contradiction.

so our assume is not true, the original statement is true.