

Jinyang Li - A20317851

Problem 1 (5 points) Exercise 4.70 from Textbook

$$g(x) = \begin{cases} 8/x^3 & (x > 2) \\ 0 & \text{elsewhere} \end{cases}$$

$$h(y) = \begin{cases} 2y, & (0 < y < 1) \\ 0, & \text{elsewhere} \end{cases}$$

$u_X =$

$$E(X) = \int_2^{\infty} x g(x) dx = 8 \int_2^{\infty} x \cdot \frac{1}{x^3} dx = -8 \cdot \left[0 - \frac{1}{2} \right] = 4$$

Problem 2 (10 points: 5 pts/question) Exercise 4.73 from Textbook

a) $u_X = E(X) =$

$$\int_0^5 x f(x) dx = \frac{1}{5} \int_0^5 x dx = \frac{1}{10} \cdot (5^2 - 0) = 2.5$$

$$\sigma^2 = E[(X - \mu)^2] = E[(X - 2.5)^2] = E(X^2) - 5 E(X) + 2.5^2 = \int_0^5 x^2 f(x) dx - 12.5 + 6.25 = 5 \cdot 5 \cdot \frac{1}{3} - 6.25 = 2.08$$

b)

$$P(\mu - 2\sigma < X < \mu + 2\sigma) = P[-1.66 < x < 6.66] = \int_{-1.66}^0 0 dx + \int_0^5 \frac{1}{5} dx + \int_5^{6.66} 1 dx = 1$$

insert $k=2$ into Chebyshev Theorem, above is true since $1 - 1/2^2 = 0.75$

insert $k=3$ into Chebyshev Theorem, above is also true since $1 - 1/3^2 = 8/9$

Problem 3 (5 points) Exercise 4.74 from Textbook

$$P = 50 I^2$$

$$u_I = E[I] = 15$$

$$\sigma_I^2 = 0.03$$

let $g(I) = 50 I^2$ then

$$P = g(I)$$

$$u_P = E[P] = E[g(I)] = g(u_I) + \frac{\partial^2}{\partial I^2} (g(I)) \cdot I \cdot \sigma_I^2 / 2 = 11251.5$$

$$\text{var}(P) = \text{var}[g(I)] = 67500$$

Problem 4 (10 Points: 5 pts/question) Exercise 5.6 from Textbook

$$\text{a) } P(2 \leq X \leq 5) = P(X \leq 5) - P(X < 2) = 0.9844 - 0.1094 = 0.875$$

$$\text{b) } P(X < 3) = P(X \leq 2) = 0.3438$$

Problem 5 (5 points) Exercise 5.19 from Textbook

$P(G) = 0.35$ let the number of G happened be x

$P(Y) = 0.05$ y

$P(R) = 0.6$ z

$$\text{Joint distribution } f(G, Y, R) = \binom{n}{g, y, r} (0.35)^x (0.05)^y (0.6)^z$$