Jinyang Li A20317851 hw09

7.1-16. What is the probability that a five-card poker hand contains a flush, that is, five cards of the same suit (spade, diamond, heart or clover)?

One suit can be selected from 4 suits in C(4, 1) ways.
Five cards can be selected from a suit of 13 cards in C(13, 5) ways.
Five cards can be drawn from 52 cards in C(52,5) ways

So the probability that a poker hand contains a flash = C(4,1) * C(13,5) / C(52,5) = 0.198%

7.1-17. What is the probability that a five-card poker hand contains a straight, that is, five cards that have consecutive kinds? (Note that an ace can be considered either the lowest card of an A-2-3-4-5 straight or the highest card of a 10-J-Q-K-A straight.)

Five cards can be drawn from 52 cards in C(52,5) ways.
There are four cards of each kind.
First - Fifth kind card can be select in C(4,1) ways.

Possible number of straights are A-2-3-4-5, 2-3-4-5 ….. 10-J-Q-K-A

So the probability = 10 (C(4,1) ^ 5) / C(52,5) = 10240 / 2598960 = 0.394%

7.1-23. What is the probability that a positive integer not exceeding 1000 selected at random is divisible by 5 or 7?

Let 'S' be the set of all positive integer not exceeding 100.
-> S = { 1, 2, 3 …… 100}
Let 'E1' be the event that the integer selected is divisible by 5.
-> E1 = {5, 10, 15 ….. 100}
Let 'E2' be the event that the integer selected is divisible by 7.
-> E2= {7, 14, …..98}

E1 U E2 is the event that it is divisible by either 5 or 7.

P(E1 U E2) = P(E1) + P (E2) - P (E1 n E2)) = 20/100 + 14/100 - 2/100 = 8/25

7.2-31. Find the probability that a family with five children does not have a boy, if the sexes of children are independent and if
a) a boy and a girl are equally likely.

being a boy and being a girl are both 50%.

So the answer is C(5,0) (1/2)^0 (1/2) ^5 = 1/ 32

b) the probability of a boy is 0.51.

be a girl is 0.49

So the answer is C(5,0) (0.51)^0 (0.49) ^5 = 0.02824

c) the probability that the i – th child is a boy is 0.51 – i/100

the probability be i th chid a girl is 0.49 + i /100
so the answer is (0.5)(0.51)(0.52)(0.53)(0.54) = 0.03795012

7.4-8. What is the expected sum of the numbers that appear when three fair dice are rolled?

let x1 (I,J,K) = i X2 (i,j,k) =j x3(i,j,k)=k
E(X1) = E(X2) = E(X3) = 7/2
E(X1+X2+X3) = 21/2

7.4-12. Suppose that we roll a fair dice until a 6 comes up.
a) What is the probability that we roll the dice n times?

if x is the number of times we roll the dice,
P (X = N) = (1-P) ^(n-1) p
= (1/6)(5/6) ^ n-1
= 5^(n-1) / 6^n

b) What is the expected number of times we roll the dice?

E(X) = 1/P
E(X) = 6

3 Problem not on the book
Assume that there are n boxes B1, B2, , Bn and 2n balls. Each ball is uni- formly and randomly placed into some box. Let the random variable Xi be the number of balls placed in the box Bi. Note that 0 ≤ Xi ≤ 2n. If Xi is 0, we call the box Bi is empty. Let the random variable Y be the number of boxes that are empty. Solve the following problems.

a) Compute the expected value E(X1).

E(X1) = 2

b) Compute the variance Var(X1).

Var(X1) = 2(n-1)/n

c) Compute the probability P r(Y = k) for a given constant k.

P(Y = k) = [nCk][(2n-1)C(n+k)]/n^(2n), there the expression nCm is defined as n!/(m!(n-m)!) for n>=m and 0 otherwise.

d) Compute the expected value E(Y).

E(Y) = n((n-1)/n)^(2n)