

Jinyang Li -A20317851-HW05

Problem 1 (5 points) Exercise 4.17 from Textbook

x	-3	6	9
f(x)	1/6	1/2	1/3

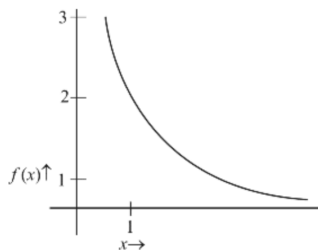
To find $u_g(X)$, $g(X) = (2X + 1)^2$

$$\begin{aligned} E[g(X)] &= \sum_{x=-3,6,9} (2x+1)^2 f(x) \\ &= [1/6 \cdot (2(-3)+1)^2] + [1/2 \cdot (2 \cdot 6+1)^2] + [1/3 \cdot (2 \cdot 9+1)^2] \\ &= 209 \end{aligned}$$

Problem 2 (10 points: 5 pts/question) Exercise 4.29

$$f(x) = \begin{cases} 3x^{-4}, & x > 1 \\ 0, & \text{elsewhere} \end{cases}$$

a)



$$\text{b) } E(X) = 3 \int_1^{\infty} x x^{-4} dx$$

$$= 3/2$$

Problem 3 (5 points) Exercise 4.41 from Textbook

To find $u(g(X))$, $g(X) = (2X + 1)^2$

$$u[g(X)] = \sum_{x=-3,6,9} (2x+1)^2 f(x)$$

$$= [1/6 \cdot (2(-3)+1)^2] + [1/2 \cdot (2 \cdot 6+1)^2] + [1/3 \cdot (2 \cdot 9+1)^2]$$

$$= 209$$

$$E[g(X)]^2 = \sum (2x+1)^4 f(x)$$

$$= 57825$$

$$\sigma_{g(X)}^2 = E[g(X)]^2 - \{E[g(X)]\}^2$$

$$= 57825 - (209)^2$$

$$= 14144$$

Problem 4 (5 Points) Exercise 4.50 from Textbook

The mean of X

$$E(X) = \int_0^1 xf(x)dx$$

$$= 1/3$$

the value of $E(X^2)$

$$E(X^2) = \int_0^1 x^2 f(x)dx$$

$$= 1/6$$

Variance of the working of equipment

$$\sigma^2 = \text{Var}(X) = E(X^2) - [E(X)]^2 = 1/18$$

So the standard deviation of x is 0.2357

Problem 5 (5 points) Exercise 4.56 from Textbook

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_0^1 xx dx + \int_1^2 x(2-x)dx = 1$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_0^1 x^2 x dx + \int_1^2 x^2(2-x)dx = \frac{14}{12}$$

The mean of the random variable $y = 60x^2 + 39x$

$$E(Y) = E(60X^2 + 39X) = 60E(X^2) + 39E(X) = \left(60 \cdot \frac{14}{12}\right) + 39 = 109$$