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CS330-HW04

1. Prove that either 2·10^500 +15 or 2·10^500 +16 is not a perfect square. Is your proof constructive or nonconstructive?

Asume 2*10^500 + 15 is a perfect square.

so, n^2 = 2* 10^500 + 15. Here the n is a natural number.

the difference between two perfect square is | n^2 - (n+a) ^ 2 | = a ^2 + 2an

since a is an integer and the n is also a integer that must bigger than 0, the difference of two perfect squares can never be 1. So, either 2 ·10^500 +15 or 2 ·10^500 +16 is not a perfect square.

Since I have given out the example —— a^2 + 2an, my proof is constructive.

2. Prove that there are infinitely many solutions in positive integers x,y and z to the equation x2+y2 =z2. [Hint: Let x=m2−n2,y= 2mn and z = m2 + n2, where m, n are integers.]

Let assume the same as hint

then, the left =(m^2 - n^2)^2 + (2mn)^2 = m^4 - 2*m^2* n^2 + n^4 + 4 M^2*N^2

= m^4 + 2*m^2*n^2 + n^4

= (m^2 + n^2) ^ 2

the right = (m^2 + n^2) ^ 2….. the left = the right forever. Since x and y consist of integers which are infinity, so, the amount of solutions are also infinity.

3. Show that if a,b,c, and m are integers such that $m \geq 2$, $c > 0$,and $a \equiv b \pmod{m}$, then $ac \equiv bc \pmod{mc}$.

Since $a \equiv b \pmod{m}$, the $(a - b) = m\,q$ for integer q.

multiply both sides by c, $c(a-b) = c\,m\,q \Rightarrow ac - bc = cmq \Rightarrow ac \equiv bc \pmod{mc}$

4. Show that a positive integer is divisible by 11 if and only if the difference of the sum of its decimal digits in even-numbered positions and the sum of its decimal digits in odd-numbered positions is divisible by 11.

Let $n = n_1\, n_2 \ldots n_k$ be the given integer.

Then $n = n_1 \times 10^k + n_2 \times 10^{k-1} + \ldots + n_k$

If n is divisible by 11 if $n \equiv 0 \pmod{11}$.

That is $n_1 \times 10^k + n_2 \times 10^{k-1} + \ldots + n_k \equiv 0 \pmod{11}$.

$n_1 \times 10^k \pmod{11} + n_2 \times 10^{k-1} \pmod{11} \ldots + n_k \pmod{11}. \equiv 0 \pmod{11}$.

Now we have

$10 \equiv -1 \pmod{11}$,

$10^2 = 100 \equiv 1 \pmod{11}$,

$10^3 = 10.100 \equiv -1 \pmod{11}$,

$10^4 = 100.100 \equiv 1 \pmod{11}$,

That is $10^i \equiv -1 \pmod{11}$, if i is odd positive integer

and

$10^j \equiv -1 \pmod{11}$, if j is even positive integer.

Then we have

$n_i \times 10^k \equiv n_i .1 \pmod{11} \equiv n_i \pmod{11}$, if i is odd and

$n_i \times 10^k \equiv n_i .{-1} \pmod{11} \equiv -n_i \pmod{11}$,

So we have

$n \equiv n_1 x 10^k \pmod{11} + n_2 x 10^{k-1} \pmod{11} \ldots . + n_k \pmod{11} \equiv n_1 - n_2 + n_3 - \ldots + n_k \pmod{11}$.

if k is even

$n \equiv (n_1+n_3+ n_k) - (n_2+n_4+\ldots+n_{k-1}) \pmod{11}$.
If k is odd

we have $n \equiv (n_2+n_4+\ldots+n_{k-1}) - (n_1+n_3+ n_k) \pmod{11}$.

So if n is divisible by 11 then $n \equiv 0 \pmod{11}$ and hence$(n_1+n_3+ n_k) -(n_2+n_4+\ldots+n_{k-1}) \pmod{11} = 0 \pmod{11}$ in either case. That is 11 divides$(n_1+n_3+ n_k) - (n_2+n_4+\ldots+n_{k-1})$.

Hence the difference of the sum of its decimal digits in even-numbered positions and the sum of its decimal digits in odd-numbered positions is divisible by 11.

5. Use the extended Euclidean algorithm to express gcd(252, 356) as a linear combination of 252 and 356.

gcd(252,356)

=gcd(356,252)

=gcd(252,104) 356 = 252 * 1 + 104

=gcd(104,44) 252 = 104 * 2 + 44

=gcd(44,16) 104 = 44 * 2 +16

=gcd(16,12) 44 = 16 * 2 + 12

=gcd(12,4) 16 = 12 * 1 + 4

=gcd(4,0) 4 = 4 * 1 + 0

so gcd(252,356) = 4,

4 = s * 356+ t * 252

252 = 0 * 356 + 1 * 252

104 = 1 * 356 + (-1) * 252

44 = (-2) * 356 +3 *252

16 = 5 *356 + (-7)* 252

4 = 17 * 356 + (-24) * 252

so gcd(252,356) is a linear combination of 252 and 356