Exam 1 Solution

CS 536: Science of Programming, Spring 2018

Multiple choice questions: A D C A C A D B A C D B A C B

Short Answer

16. [3 points] Write the definition of a predicate function `Run(b, m, k)` that yields true iff `b[m]` = `b[m+1]` = … `b[m+k-1]`. E.g., in {`b` = (1, 0, 0, 0, 8)}, `Run(b, 1, 3)` is true (since 0 = 0 = 0), `Run(b, 0, 2)` is false (since 1 ≠ 0) and `Run(b, 4, 1)` is true (since 8 = itself). You can assume without testing that `k` ≥ 1 and that all the indexes `m`, `m+1`, …, `m+k-1` are within the bounds of `b`. (Remember, this has to be a predicate **function**, not a boolean-returning function.)

```
Run(b, m, k) ≡ ∀ 0 ≤ j < k . b[m] = b[m+j]
```

17. [4 points] Calculate $\neg((\exists \mathtt{y} . r) \to (\forall \mathtt{x} . (p \wedge (\neg q))))$. Show your work.

$\neg((\exists \mathtt{y} . r) \to (\forall \mathtt{x} . (p \wedge (\neg q))))$

$\Leftrightarrow (\exists \mathtt{y} . r) \wedge \neg(\forall \mathtt{x} . (p \wedge \neg q))$

$\Leftrightarrow (\exists \mathtt{y} . r) \wedge (\exists \mathtt{x} . \neg(p \wedge \neg q))$

$\Leftrightarrow (\exists \mathtt{y} . r) \wedge (\exists \mathtt{x} . ((\neg p) \vee q))$

18. [4 points] Translate the C code fragment `x = 0; m = 0; while (x++ < n) { y = x * (++m); }` to our language:

```
x := 0; m := 0;
while x < n do
   x := x+1;
   m := m+1;
   y := x*m
od; x := x+1
```

19. [4 points] Let $W \equiv$ **while** `x < y` **do** S **od** where $S \equiv$ `x := x+1; y := y-1`, and let $\sigma_0 = \{\mathtt{x} = 5, \mathtt{y} = 9\}$. Show the operational semantics of executing $\langle W, \sigma_0 \rangle \to \ldots$ to completion. You can use multistep arrows ($\to^2$) to shorten your writing, but give the configuration for each loop test (i.e., show $\langle W, \ldots \rangle$ every time it occurs).

Let $\sigma_0 = \{\mathtt{x} = 5, \mathtt{y} = 9\}$, and for each $k \geq 1$, $\sigma_k = \{\mathtt{x} = 5 + k, \mathtt{y} = 9 - k\}$.

(Explicit values: $\sigma_0 = \{\mathtt{x} = 5, \mathtt{y} = 9\}$, $\sigma_1 = \{\mathtt{x} = 6, \mathtt{y} = 8\}$, $\sigma_2 = \{\mathtt{x} = 7, \mathtt{y} = 7\}$)

Then $\langle W, \sigma_0 \rangle \to \langle S \,;\, W, \sigma_0 \rangle \to^2 \langle W, \sigma_1 \rangle \to^3 \langle W, \sigma_2 \rangle \to \langle E, \sigma_3 \rangle$

CS 536 Exam 1 scores (full credit for Problem 8)

Question		# Students	51	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Obj	16	17	18	19
Answer		Answer		a	d	c	a	c	a	d	-	a	c	d	b	a	c	b					
Max pts		Max Pts	60	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	45	3	4	4	4
Avg score	47.5	Average		73	90	88	92	96	61	80	100	71	67	80	94	65	73	67	76	42	74	87	97
79.2%																							

Exam 1 Scores
60 60 60
59 59 58 58 57 57 56 56 56 56 56 55 55
54 53 52 52 51 51 51 50 50
49 48 48 48 47 46 45 45 45
44 44 43 42 41 41
38 38 37 37 37 36 36
33
29
25
17

