

XVA

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1. Analytical valuation (single product)

1. Value of equity swap. The valuation is conducted under risk neutral probability, $-0.0569 \times 100 = -5.69m$.
2. Expected exposure under physical expectation. The expected exposure at end of period (year 0,1,2,3,4,5): -5.69m, -1.92m, -0.83m, 0.31m, 1.49m, 2.72m.
3. Expected positive exposure. The expected positive exposure for each period is calculated as $EPE = (\mu - 0.5\sigma^2 - K) * CDF\left(\frac{\mu - 0.5\sigma^2 - K}{\sigma}\right) + \sigma PDF\left(\frac{\mu - 0.5\sigma^2 - K}{\sigma}\right)$, which is calculated under physical probability measure. After discounting, at period (year 0,1,2,3,4,5), EPE = 34.8m, 36.22m, 29.54m, 22.6m, 15.36m, 7.84m
4. Expected negative exposure. The expected negative exposure for each period is calculated as $ENE = (K - \mu + 0.5\sigma^2) * CDF\left(\frac{K - \mu + 0.5\sigma^2}{\sigma}\right) + \sigma PDF\left(\frac{K - \mu + 0.5\sigma^2}{\sigma}\right)$, which is calculated under physical probability measure. After discounting, at end of period (year 0, 1,2,3,4,5), ENE = 22.72m, 23.65m, 19.29m, 14.75m, 10.03m, 5.12m
5. Potential future exposure. The potential future exposure at 95% percentile positive tail is just $\mu - 0.5\sigma^2 - K + \sigma * 1.645$ for each period. After discounting, at end of period (year 0,1,2,3,4,5), PFE = 128.99, 134.25m, 109.5m, 83.75m, 56.94m, 29.04m.
6. CVA and DVA, under risk neutral measure. The default probability at each period is just $e^{-qt} - e^{-q(t+1)}$, multiply with EPE and 1-recovery rate, then discount back to today, the CVA is 0.95m. Similarly, the DVA is calculated as 1.16m.
7. Above calculation assume default happens just before mark to market at end of each period.

2. Analytical valuation (portfolio)

1. Independently, 5 equity swaps are valued as the same amount as $-0.0569 \times 20 = -1.19m$ for each.
2. From a portfolio perspective, a portfolio of 5 independent stock will follow the distribution $5\mu - 2.5\sigma^2 + \sigma(\epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 + \epsilon_5) \sim N(5\mu - 2.5\sigma^2, 5\sigma^2)$, under physical measure. As the mean doesn't change when multiply the notional, the expected exposure is the same with that in question 1.
3. The expected positive exposure for each period is calculated as $EPE = (5\mu - 2.5\sigma^2 - 5K) * CDF\left(\frac{5\mu - 2.5\sigma^2 - 5K}{\sqrt{5}\sigma}\right) + \sqrt{5}\sigma PDF\left(\frac{5\mu - 2.5\sigma^2 - 5K}{\sqrt{5}\sigma}\right)$, with a notional 20m, which is calculated under physical probability measure. After discounting, at end of period (year 0, 1,2,3,4,5), EPE = 18.77m, 19.53m, 15.93m, 12.18m, 8.28m, 4.23m
4. The expected negative exposure for each period is calculated as $ENE = (5K - 5\mu + 2.5\sigma^2) * CDF\left(\frac{5K - 5\mu + 2.5\sigma^2}{\sqrt{5}\sigma}\right) + \sqrt{5}\sigma PDF\left(\frac{5K - 5\mu + 2.5\sigma^2}{\sqrt{5}\sigma}\right)$, with a notional 20m, which is calculated under physical probability measure. After discounting, at end of period (year 0, 1,2,3,4,5), ENE = 6.68m, 6.96m, 5.68m, 4.34m, 2.95m, 1.51m.
5. Potential future exposure. The potential future exposure at 95% percentile positive tail is just $5\mu - 2.5\sigma^2 - 5K + \sqrt{5}\sigma * 1.645$ for each period under physical measure. After discounting, at end of period (year 0,1,2,3,4,5), PFE = 64.36m, 66.99m, 54.64m, 41.79m, 28.41m, 14.49m.

6. CVA and DVA under risk neutral measure. CVA=0.37m, DVA = 0.58m
7. From a risk management perspective, a company might prefer the 5 equity swaps position as the diversification benefits from netting and lower risks lead to less credit exposure.

3. Monte Carlo simulation(year time grid)

1. Value of equity swap. $V = -5.69m$ for the portfolio.
2. Expected exposure under physical expectation. EE = -5.69m, -1.9m, -0.85m, 0.34m, 1.5m, 2.74m at the end of year (0,1,2,3,4,5)
3. Expected positive exposure. EPE=30.31m, 31.55m, 25.76m, 19.73m, 13.42m, 6.85m at the end of each period (0,1,2,3,4,5) using physical measure.
4. Expected negative exposure, using physical measure. ENE = 21.44m, 22.32m , 18.2m, 13.92m, 9.47m ,4.82m
5. Potential future exposure, using physical measure. PFE = 64.39m, 67.02m, 54.68m, 41.88m, 28.49m, 14.52m
6. CVA = 0.87m, DVA = 1.02m under risk neutral measure.

4. Monte Carlo simulation(Month time grid)

1. Value of equity swap, $V = -5.82m$ for the portfolio.
2. Expected exposure at month end, expected positive exposure at month end, expected negative exposure at month end, potential future exposure is summarized in table below in million.

	EE	EPE	ENE	PFE
0	-5.83	94.11	85.12	196.79
1	-5.51	94.42	85.41	197.45
2	-5.43	92.99	84.12	194.46
3	-5.35	91.56	82.82	191.47
4	-5.27	90.13	81.52	188.47
5	-5.18	88.7	80.21	185.47
6	-5.09	87.25	78.91	182.45
7	-4.99	85.8	77.6	179.42
8	-4.91	84.35	76.29	176.38
9	-4.82	82.9	74.97	173.33
10	-4.73	81.43	73.64	170.27
11	-4.65	79.97	72.31	167.2
12	-4.55	78.5	70.99	164.13
13	-4.46	77.03	69.65	161.06
14	-4.36	75.56	68.31	157.98
15	-4.28	74.07	66.97	154.86
16	-4.18	72.58	65.62	151.76
17	-4.1	71.08	64.27	148.63
18	-4.01	69.59	62.91	145.51
19	-3.91	68.09	61.55	142.37
20	-3.81	66.57	60.18	139.2
21	-3.7	65.05	58.81	136.02

22	-3.62	63.53	57.44	132.83
23	-3.53	62	56.05	129.62
24	-3.42	60.46	54.66	126.41
25	-3.33	58.92	53.27	123.2
26	-3.23	57.38	51.88	119.97
27	-3.13	55.83	50.47	116.72
28	-3.04	54.27	49.06	113.47
29	-2.93	52.71	47.64	110.21
30	-2.85	51.14	46.23	106.93
31	-2.74	49.57	44.81	103.65
32	-2.66	48	43.39	100.36
33	-2.55	46.43	41.96	97.08
34	-2.45	44.85	40.53	93.76
35	-2.35	43.26	39.09	90.44
36	-2.25	41.67	37.65	87.11
37	-2.15	40.08	36.21	83.78
38	-2.05	38.48	34.75	80.43
39	-1.95	36.87	33.29	77.06
40	-1.85	35.26	31.83	73.69
41	-1.74	33.63	30.36	70.3
42	-1.64	32.01	28.9	66.89
43	-1.54	30.37	27.42	63.47
44	-1.45	28.72	25.94	60.04
45	-1.33	27.08	24.45	56.6
46	-1.22	25.43	22.97	53.15
47	-1.12	23.77	21.48	49.69
48	-1.02	22.11	19.98	46.21
49	-0.93	20.45	18.48	42.73
50	-0.84	18.77	16.97	39.23
51	-0.71	17.09	15.45	35.72
52	-0.61	15.4	13.93	32.19
53	-0.52	13.71	12.41	28.64
54	-0.42	12	10.88	25.1
55	-0.31	10.3	9.34	21.54
56	-0.21	8.6	7.8	17.98
57	-0.11	6.89	6.25	14.41
58	0	5.18	4.69	10.82
59	0.11	3.46	3.13	7.23
60	0.23	1.73	1.57	3.63

3. CVA = 2.67m, DVA = 2.8m. The calculations assume defaults happen at each month end.