Dependence

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	Gaussian copula (ρ)			Student t-copula (ρ)		
	0%	20%	40%	0%	20%	40%
$P(D \geq 9, \tau \leq 1y)$	0.12%	5.50%	9.30%	0.12%	5.90%	10.30%
$P(D \geq 11, \tau \leq 1y)$	0.01%	2.49%	6.04%	0.00%	3.29%	7.24%
$P(D \geq 14, \tau \leq 1y)$	0.00%	0.74%	3.24%	0.00%	1.26%	4.82%
$P(D = 0, \tau > 1.5y)$	1.14%	11.26%	25.03%	1.01%	9.48%	19.17%
$P(D = 0, \tau > 2.5y)$	0.06%	4.02%	13.08%	0.06%	3.76%	10.33%
$P(D = 0, \tau > 3.5y)$	0.01%	1.49%	7.80%	0.00%	1.71%	6.36%

Discussion: **Firstly**, look at the default situations. For each type of copula, the higher default numbers, the lower probabilities at each correlation category. Increase the correlation will increase the default probabilities. Compare with student t-copula and gaussian copula, the default probabilities following student t-copula have higher probabilities than gaussian copula in general, it can be attributed to the fact that more tail risks will happen for student t distribution and tail dependence for student t-copula. **Secondly**, regarding the survive situations, the probabilities of zero default decrease as time spans increase at each correlation category for each type of copula. Increase the correlation will increases the joint survival probabilities. Compare with student t-copula and gaussian copula, the survival probabilities following student t-copula are lower for each correlation level and time span in general. Since tail risk such as default could happen more frequently and tail risk dependence for student t-copula distribution, the observations here are not unexpected.

To value n to default, the first to default probability within 1 year could be similar with $P(D \geq 1, \tau \leq 1y)$ as the same way calculated in the table with corresponding correlation, depends on whether the distribution follows gaussian copula or student t-copula. Hence n to default is just $P(D \geq n, \tau)$. As can be seen in the table, the first to default could be more valuable when correlation is 0, since the lower probability of no default happen within certain period, the higher the probability of 1 or more default happen. On the contrary, the higher the correlation the more valuable for n to default for n more than 1. With the probability, the value could be calculated as expected portfolio payoff given default multiplies by the default probability then discount back to today

To value CDOs, the above techniques could be used to model the impacts of joint defaults of companies on principle and the contagious effect between tranches. The equity tranche could be similar with a first to default CDS and the senior tranches could be similar with n to default, their probabilities can be obtained as above. With the default probability, the expected tranche principal after taking correlation into account could be modelled, with a known zero coupon yield curve, the tranche spreads could be calculated.