?
$Z =$
$X \times$
Y
$S =$
$\{z_1 =$
$(x_1, y_1), \cdots, z_N =$
$(x_N, y_N)\}$
N
XY
$f :$
$X \to$
Y
X
Y
f_S
S
$R(f) :=_{z \sim XY} [c(f, z)]$

f
S

$$R_N(f) := \frac{1}{N} \sum_{i=1}^{N} c(f, z_i)$$

$R(f_S) -$
$R_N(f_S)$
$S =$
$\{z_1, \ldots, z_{i-1}, z_i, z_{i+1}, \ldots z_N\}$
$S_i :=$
$\{z_1, \ldots, z_{i-1}, z'_i, z_{i+1}, \ldots, z_N\}$
i^{th}
z_i
z'_i
S
S_i
i^{th}
β

$$\|c(f_S, z) - c(f_{S_i}, z)\|_\infty \leq \beta, \forall S \in Z^N, \forall z'_i, z \in Z,$$

$c(\cdot, \cdot)$
β
$1 \leq$
$i \leq$
N

$$_{S \sim^N_{XY}} [R(f_S) - R_N(f_S)] =_{S, z'_i \sim^{N+1}_{XY}} [c(f_S, z'_i) - c(f_{S_i}, z'_i)]$$

$$_{S \sim^N_{XY}} [R_N(f_S)] = \frac{1}{N} \sum_{i=1}^{N} {}_{S \sim^N_{XY}} [c(f_S, z_i)] =_{S \sim^N_{XY}} [c(f_S, z_i)], \forall i \in \{1, \ldots, N\}$$

z_i
z'_i

$$_{S \sim^N_{XY}} [R_N(f_S)] =_{S_i \sim^N_{XY}} [c(f_{S_i}, z'_i)]$$

$$_{S \sim^N_{XY}} [R(f_S)] =_{S, z'_i \sim^{N+1}_{XY}} [c(f_S, z'_i)]$$

β

$$_{S \sim^N_{XY}} [R(f_S) - R_N(f_S)] =_{S, z'_i \sim^{N+1}_{XY}} [c(f_S, z'_i) - c(f_{S_i}, z'_i)] \leq_{S, z'_i \sim^{N+1}_{XY}} [\beta] = \beta$$

β
β
K
K

$$\kappa := \sup_{x \in X} \sqrt{K(x, x)} < \infty$$

$$|f(x)| \leq \kappa \|f\|, \forall x \in X, \forall f \in \qquad (1)$$

σ
$c(f(x), y)$
$\times Y$
c

$$|c(y_1, y') - c(y_2, y')| \leq \sigma |y_1 - y_2|, \forall y_1, y_2 \in \mathcal{D}, \forall y' \in Y,$$

$\mathcal{D} =$
$\{y :$
$\exists f \in$
$, \exists x \in$
$X, f(x) =$
$y\}$