

$$\begin{array}{l} Z \\ X \times \\ Y \\ S = \\ \{z_1 = \\ (x_1, y_1), \cdots, z_N = \\ (x_N, y_N)\} \\ N \\ XY \\ f: \\ X \rightarrow \\ Y \\ X \\ Y \\ f_S \\ S \\ R(f) :=_{z \sim_{XY}} [c(f, z)] \end{array}$$

$$\begin{array}{l} f \\ S \\ R_N(f) := \frac{1}{N} \sum_{i=1}^N c(f, z_i) \end{array}$$

$$\begin{array}{l} R(f_S)- \\ R_N(f_S) \\ S = \\ \{z_1, \ldots, z_{i-1}, z_i, z_{i+1}, \ldots z_N\} \\ S_i := \\ \{z_1, \ldots, z_{i-1}, z'_i, z_{i+1}, \ldots, z_N\} \\ i^{th} \\ z_i \\ z_i \\ S_i \\ S_i \\ i^{th} \\ \beta \\ \|c(f_S, z)-c(f_{S_i}, z)\|_\infty \leq \beta, \forall S \in Z^N, \forall z'_i, z \in Z, \end{array}$$

$$\begin{array}{l} c(\cdot,\cdot) \\ \beta \\ 1 \leq \\ i \leq \\ N \end{array}$$

$$_{S \sim_{XY}^N} [R(f_S) - R_N(f_S)] =_{S, z'_i \sim_{XY}^{N+1}} [c(f_S, z'_i) - c(f_{S_i}, z'_i)]$$

$$_{S \sim_{XY}^N} [R_N(f_S)] = \frac{1}{N} \sum_{i=1}^N \, _{S \sim_{XY}^N} [c(f_S, z_i)] =_{S \sim_{XY}^N} [c(f_S, z_i)], \, \forall i \in \{1, \ldots, N\}$$

$$\begin{array}{l} z_i \\ z_i \end{array}$$

$$_{S \sim_{XY}^N} [R_N(f_S)] =_{S_i \sim_{XY}^N} [c(f_{S_i}, z'_i)]$$

$$_{S \sim_{XY}^N} [R(f_S)] =_{S, z'_i \sim_{XY}^{N+1}} [c(f_S, z'_i)]$$

$$\beta$$

$$_{S \sim_{XY}^N} [R(f_S) - R_N(f_S)] =_{S, z'_i \sim_{XY}^{N+1}} [c(f_S, z'_i) - c(f_{S_i}, z'_i)] \leq_{S, z'_i \sim_{XY}^{N+1}} [\beta] = \beta$$

$$\begin{array}{l} \beta \\ \beta \\ K \\ K \end{array}$$

$$\kappa := \sup_{x \in X} \sqrt{K(x,x)} < \infty$$

$$(1) \quad |f(x)| \leq \kappa \|f\|, \forall x \in X, \forall f \in$$

$$\begin{array}{l} \sigma \\ c(f(x), y) \\ \times_Y \\ \mathcal{C} \\ |c(y_1, y') - c(y_2, y')| \leq \sigma |y_1 - y_2|, \, \forall y_1, y_2 \in \mathcal{D}, \forall y' \in Y, \end{array}$$

$$\begin{array}{l} \mathcal{D} = \\ \{y: \\ \exists f \in \\ , \exists x \in \\ X, f(x) = \\ y\} \end{array}$$