

$$\begin{array}{l} Z \\ X \times \\ Y \\ S = \\ \{z_1 = \\ (x_1, y_1), \dots, z_N = \\ (x_N, y_N)\} \\ N \\ X^Y \\ f: \\ X \rightarrow \\ Y \\ X \\ f_S \\ S \end{array}$$

$$R(f) :=_{z \sim_{XY}} [c(f,z)]$$

$$\begin{array}{l} f \\ S \\ R_N(f) := \frac{1}{N} \sum_{i=1}^N c(f,z_i) \end{array}$$

$$\begin{array}{l} R(f_S)- \\ R_N(f_S) \\ S = \\ \{z_1,\dots,z_{i-1},z_i,z_{i+1},\dots z_N\} \\ S_i := \\ \{z_1,\dots,z_{i-1},z'_i,z_{i+1},\dots,z_N\} \end{array}$$

$$\begin{array}{l} i^{th} \\ z_j \\ z_j \\ S \\ S_i \\ i^{th} \\ \beta \\ \|c(f_S,z)-c(f_{S_i},z)\|_\infty \leq \beta, \forall S \in Z^N, \forall z'_i, z \in Z \end{array}$$

$$\begin{array}{l} \beta \\ 1 \leq \\ i \leq \\ N \end{array}$$

$$s_{\sim_{XY}^N}[R(f_S)-R_N(f_S)]=_{S,z'_i\sim_{XY}^{N+1}}[c(f_S,z'_i)-c(f_{S_i},z'_i)]$$

$$s_{\sim_{XY}^N}[R_N(f_S)]=\frac{1}{N}\sum_{i=1}^Ns_{\sim_{XY}^N}[c(f_S,z_i)]=_{s\sim_{XY}^N}[c(f_S,z_i)],\,\forall i\in\{1,\ldots,N\}$$

$$\begin{array}{l} z_i \\ z'_i \end{array}$$

$$s_{\sim_{XY}^N}[R_N(f_S)]=_{s_i\sim_{XY}^N}[c(f_{S_i},z'_i)]$$

$$s_{\sim_{XY}^N}[R(f_S)]=_{S,z'_i\sim_{XY}^{N+1}}[c(f_S,z'_i)]$$

$$\beta$$

$$s_{\sim_{XY}^N}[R(f_S)-R_N(f_S)]=_{S,z'_i\sim_{XY}^{N+1}}[c(f_S,z'_i)-c(f_{S_i},z'_i)]\leq_{S,z'_i\sim_{XY}^{N+1}}[\beta]=\beta$$

$$\begin{array}{l} \beta \\ \beta \\ K \\ K \\ \kappa := \sup_{x \in X} \sqrt{K(x,x)} < \infty \end{array}$$

$$(1) \quad |f(x)| \leq \kappa \|f\|, \, \forall x \in X, \forall f \in$$

$$\begin{array}{l} \sigma \\ c(f(x),y) \\ \times_Y \\ \mathcal{C} \\ |c(y_1,y')-c(y_2,y')| \leq \sigma |y_1-y_2|, \, \forall y_1,y_2 \in \mathcal{D}, \forall y' \in Y, \end{array}$$

$$\begin{array}{l} \mathcal{D} = \\ \{y: \\ \exists f \in \\ , \exists x \in \\ X, f(x) = \end{array}$$