Titel der Seminarausarbeitung

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Abstract— **Zusammenfassender Kurztext, nicht mehr als 150 Worte.**

Keywords— **Stichwörter**

I. Introduction

Temperature dependent material properties and Joule heat losses in dissipative media imply coupled thermal and electromagnetic processes. Microwave heating, the self heating of semiconductor devices and the induction-heat treatment of metals are a few examples of applications, where such a coupling occurs. For the design and optimization of these applications, an accurate, simultaneous solution of Maxwell's equation and of the heat conduction equation is required.

Several **FEM** algorithms for the solution of the coupled thermal and electromagnetic equations have been suggested (cf. [?]). However, it remains difficult to achieve satisfactory convergence conditions and consistent numerical results [?]. This is mainly due to the different nature of the two sets of equations, implying very distinct time scales t_{EM} and t_{T}, characteristic for the electromagnetic and thermal precesses, respectively.

The Finite Integration (**FI**) Method has been successfully employed for the solution of Maxwell's equations [?]. The resulting Maxwell-Grid-Equations (**MGE**) allow for strict conservation of charge, momentum and energy, and thus guarantee for the long-time-stability of the numerical solution. This feature is especially attractive for the simulation of the slowly changing thermal fields, which are typically characterized by long transient states. The **FI** Method has also been used in the calculation of stationary thermal and electromagnetic field distributions [?].

The present work extends the previous study [?] to the calculation of transient fields. The time dependent heat conduction equation for isotropic media reads

$$\varrho(\boldsymbol{r},T)c(\boldsymbol{r},T)\frac{\partial T(\boldsymbol{r},t)}{\partial t} = -\,\mathrm{div}\,\boldsymbol{J}_w(\boldsymbol{r},t) + Q_w(\boldsymbol{r},t)\ , \quad (1)$$

where ϱ and c denote the generally temperature dependent density and volumetric heat capacity of the medium. The thermal current density $\boldsymbol{J}_w$ is given by the Fourier law,

$$\boldsymbol{J}_w(\boldsymbol{r},t) = -\lambda\,\mathrm{grad}\,T(\boldsymbol{r},t) \quad (2)$$

with $\lambda = \lambda(\boldsymbol{r},T)$ the thermal conductivity. Except for the Joule heat excitation term Q_w appearing in the thermal equation, also the radiant and convective boundary conditions are accommodated in the model. The algorithm for the integration in the time domain makes explicitly use of the different time scales t_{EM} and t_{T}. Thus, unnecessary iterations in the electromagnetic computation are excluded and the solution speed and convergence rate are improved. Currently, only temperature dependent material properties are supported, whereas nonlinear properties of, e.g., ferromagnetic materials are not yet implemented.

II. The FI Method

The **FI** discretization scheme is based on a dual grid-doublet $\{G,\widetilde{G}\}$, which decomposes the computation domain into two sets of dual cells [?]. Integral quantities $\mathbf{q}$, $\widehat{\mathbf{e}}$ and $\widehat{\widehat{\mathbf{b}}}$ are defined on the grid G, corresponding to the total charge in the cell volumes, to the electric voltage along the cell edges and to the magnetic induction flux on the cell facets, respectively. Analogously, $\widehat{\widehat{\mathbf{j}}}$, $\widehat{\widehat{\mathbf{d}}}$ and $\widehat{\mathbf{h}}$ are the vectors of charge current, electric displacement flux and magnetic voltage defined on the facets and edges of the dual grid $\widetilde{G}$. Fig. **??** illustrates the allocation of fluxes and voltages in the case of rectangular dual grids G and $\widetilde{G}$.

Using these integral quantities, Maxwell's equations in discrete form, the so-called Maxwell-Grid-Equations (**MGE**) are obtained:

$$\begin{cases} \mathbf{C}\widehat{\mathbf{e}} = -\dfrac{\mathrm{d}}{\mathrm{d}t}\widehat{\widehat{\mathbf{b}}}\ , & \widetilde{\mathbf{C}}\widehat{\mathbf{h}} = \dfrac{\mathrm{d}}{\mathrm{d}t}\widehat{\widehat{\mathbf{d}}} + \widehat{\widehat{\mathbf{j}}}\ , \\ \mathbf{S}\widehat{\widehat{\mathbf{b}}} = \mathbf{0}\ , & \widetilde{\mathbf{S}}\widehat{\widehat{\mathbf{d}}} = \mathbf{q}\ . \end{cases} \quad (3)$$

The support matrix operators $(\mathbf{C},\mathbf{S})$ and $(\widetilde{\mathbf{C}},\widetilde{\mathbf{S}})$ defined on G and $\widetilde{G}$ are discrete mappings of the differential "curl" and "div". It follows from the topology of the dual grid-doublet [?], that the operators $\mathbf{C}$, $\mathbf{S}$, $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{S}}$ fulfill the identities $\mathbf{SC} = \mathbf{C}\widetilde{\mathbf{S}}^{\mathrm{T}} = 0$ and $\widetilde{\mathbf{S}}\widetilde{\mathbf{C}} = \widetilde{\mathbf{C}}\mathbf{S}^{\mathrm{T}} = 0$, which obviously correspond to the continuum relations $\mathrm{div}\,\mathrm{curl} = 0$ and $\mathrm{curl}\,\mathrm{grad} = 0$. Note, that equations (**??**) hold true exactly, thus exact conservation laws in discrete form for charge, momentum and energy may be derived [?].

The discretization approximation enters the **FI** Method through the constitutive material equations

$$\widehat{\widehat{\mathbf{d}}} = \mathbf{M}_\varepsilon\widehat{\mathbf{e}} + \mathbf{p}, \quad \widehat{\widehat{\mathbf{j}}} = \mathbf{M}_\sigma\widehat{\mathbf{e}} \quad \text{and} \quad \widehat{\widehat{\mathbf{b}}} = \mathbf{M}_\mu\widehat{\mathbf{h}} + \mathbf{m}\ , \quad (4)$$

which close the system of **MGE** (**??**) and relate vectors defined on G and $\widetilde{G}$. Here, $\mathbf{M}_\varepsilon$, $\mathbf{M}_\sigma$ and $\mathbf{M}_\mu$ are matrix operators taking into account the average effects of linear polarization, electric conductivity and linear magnetization of the material medium. Details on the material and geometry averaging techniques used with the **FI** Method are found in [?]. For dual orthogonal grids G and $\widetilde{G}$ the global discretization error of this approximation is of second order accuracy in the discrete solutions of (**??**) and (**??**).

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Fig. 1. **(a)** Two cells of the rectangular dual grids G and $\widetilde{G}$ with given indices (i, j, k) and the allocation of charge $\mathbf{q}$ are shown. **(b)** Allocation of electric voltage $\widehat{\mathbf{e}}$ and of magnetic induction flux $\widehat{\widehat{\mathbf{b}}}$ on the direct grid G.

The discretization of the heat conduction equation is carried out analogously on the same dual grid-doublet $\{G, \widetilde{G}\}$ as above. Integrating equation (**??**) over the cell volumes of the grid G and equation (**??**) over the cell facets of the dual grid $\widetilde{G}$ one obtains the set of discrete equations,

$$\begin{cases} \mathbf{M}_{\mathrm{m}} \dfrac{\mathrm{d}}{\mathrm{d}t} \mathbf{T} = -\widetilde{\mathbf{S}} \widehat{\widehat{\mathbf{j}}}_w + \mathbf{q}_w \\ \widehat{\widehat{\mathbf{j}}}_w = \mathbf{M}_\lambda \widetilde{\mathbf{S}}^{\mathrm{T}} \mathbf{T} \, . \end{cases} \tag{5}$$

The vector $\widehat{\widehat{\mathbf{j}}}_w$ of thermal currents is defined on the facets of the dual grid $\widetilde{G}$, $\mathbf{T}$ is the vector of all temperature values allocated on the nodes of the grid G. Similarly, the vector $\mathbf{q}_w$ of Joule heat losses is allocated on the nodes of G (see Fig. **??** for the allocation in the case of rectangular dual grids). The matrixes $\mathbf{M}_{\mathrm{m}}$ and $\mathbf{M}_\lambda$ contain information on the (cell-averaged) heat capacity and thermal conductivity of the material medium. Combining equations (**??**) and (**??**) yields

$$\mathbf{M}_{\mathrm{m}} \frac{\mathrm{d}}{\mathrm{d}t} \mathbf{T} = -\widetilde{\mathbf{S}} \mathbf{M}_\lambda \widetilde{\mathbf{S}}^{\mathrm{T}} \mathbf{T} + \mathbf{q}_w \, , \tag{6}$$

which is the discrete thermal equation in the **FI** formulation. Note, that this formulation implies energy conservation at any given time t. Since the thermal current $\widehat{\widehat{\mathbf{j}}}_w$ has the same value and direction on the common facet of two neighboring cells, no energy loss occurs while heat is transferred from one cell to the other.

Fig. 2. **(a)** Allocation of temperature T, heat source $\mathbf{q}_w$ and thermal current $\widehat{\widehat{\mathbf{j}}}_w$ on a rectangular dual grid-doublet $\{G, \widetilde{G}\}$. **(b)** Allocation of thermal current on $\widetilde{G}$.

III. Integration in the time domain

The time domain equivalent to the **FI** Method is the well known **FDTD** scheme of leapfrog integration. Applied to the time dependent **MGE** (**??**) this procedure is restricted by the Courant stability criterion on the time step length: $\Delta t \le \Delta t_{\mathrm{EM}}^{\max}$, where

$$\Delta t_{\mathrm{EM}}^{\max} = \min_G \left\{ \sqrt{\varepsilon_i \mu_i} \left(\frac{1}{\Delta x_i^2} + \frac{1}{\Delta y_i^2} + \frac{1}{\Delta z_i^2} \right)^{-\frac{1}{2}} \right\} \, . \tag{7}$$

For the time integration of equation (**??**) an explicit forward time difference scheme is used. The corresponding update relation is

$$\mathbf{T}^{n+1} = (\mathbb{1} - \Delta t \mathbf{M}_{\mathrm{m}}^{-1} \widetilde{\mathbf{S}} \mathbf{M}_\lambda \widetilde{\mathbf{S}}^{\mathrm{T}}) \mathbf{T}^n + \Delta t \mathbf{M}_{\mathrm{m}}^{-1} \mathbf{q}_w^n \, , \tag{8}$$

with the maximal stable time step given by

$$\Delta t_{\mathrm{T}}^{\max} = \min_G \left\{ \frac{\rho_i c_i}{2\lambda_i} \left(\frac{1}{\Delta x_i^2} + \frac{1}{\Delta y_i^2} + \frac{1}{\Delta z_i^2} \right)^{-1} \right\} \, . \tag{9}$$

Equation (**??**) is slightly modified, if radiant and convective boundary conditions are considered (see [**?**] for details). For homogeneous material media and equidistant grid points the integration scheme (**??**) reduces to Richardson's explicit method for linear diffusion equations.

Since the stability criterion (**??**) may become restrictive, an implicit time integration scheme is used, alternatively. The update relation for the heat equation in this case reads

$$\mathbf{T}^{n+1} = (2\cdot\mathbb{1}+\mathbf{\Gamma})^{-1}(2\cdot\mathbb{1}-\mathbf{\Gamma})\mathbf{T}^{n} + \Delta t\cdot\mathbf{M}_{\mathrm{m}}^{-1}(\mathbf{q}_{w}^{n+1}+\mathbf{q}_{w}^{n})\,, \tag{10}$$

with $\mathbf{\Gamma} = \Delta t\cdot\mathbf{M}_{\mathrm{m}}^{-1}\widetilde{\mathbf{S}}\mathbf{M}_{\lambda}\widetilde{\mathbf{S}}^{\mathrm{T}}$. If the material medium is homogeneous and the grid points equidistant, this procedure corresponds to the well known Crank-Nicolson implicit time integration scheme. For the solution of the implicit recursion in (**??**) a Preconditioned Conjugate Gradient (**PCG**) method is used.

The calculation effort for the coupled electromagnetic and thermal fields may be significantly reduced, if the time scales t_{EM} and t_{T} are taken into account. The electromagnetic time scale t_{EM} describes, e.g., the time interval until the steady state distribution of the electromagnetic fields is established. The thermal time scale t_{T} describes the velocity of diffusion of the temperature field in absence of external heat sources. If the same dual grid-doublet $\{G,\widetilde{G}\}$ is used for both discretizations in (**??**) and (**??**), then the maximal stable time steps given in (**??**) and (**??**) are appropriate estimations of the time scales t_{EM} and t_{T}, respectively. For many material media, it is generally observed, that $\Delta t_{\mathrm{EM}}^{\max} \ll \Delta t_{\mathrm{T}}^{\max}$ (e.g., for a copper material block discretized with $\Delta x = \Delta y = \Delta z = 1\,\mathrm{cm}$ one obtains $\Delta t_{\mathrm{EM}}^{\max} = 1.93\cdot 10^{-11}\,\mathrm{s}$ and $\Delta t_{\mathrm{T}}^{\max} = 0.15\,\mathrm{s}$). Therefore, a stationary electromagnetic field distribution is established long before significant modifications of the temperature distribution are observed. For such a weak coupling the algorithm shown in Fig. **??** is applicable. Here, the electromagnetic computation is initiated only if the change in the temperature distribution is significant, i.e., if the electromagnetic material properties $\varepsilon(\boldsymbol{r},T)$, $\mu(\boldsymbol{r},T)$ change.

IV. Validation

The validity of the method is demonstrated in the calculation of the transient temperature distribution of a lossy dielectric rod heated by microwaves. The sample is aligned along the axes of a cylindrical cavity as shown in Fig. **??**a. Hybrid heating, due to microwave absorption as well as to thermal radiation from the sample surface is considered.

Jackson et al. [**?**], [**?**] developed an analytical method for calculating the transient temperature profile in this model. In particular, in the case of a single driving mode of the type $\mathbf{TM}_{010}$ the spatial variation of both, thermal and electromagnetic fields is on the radial direction alone and analytical results are easier to obtain. The sample discussed in [**?**], [**?**] was an alumina rod of radius 1.87 cm, length 6.63 cm and thermal emissivity 0.31. The radius of the cavity was 4.69 cm and the cavity walls were considered as perfectly absorbing at a constant temperature of 25 ℃. The amplitude of the driving mode $\mathbf{TM}_{010}$ corresponded

Fig. 3. Transient solution algorithm for the coupled thermal and electromagnetic equations.

to a constant input power of 400 W. The temperature dependent values of ε and λ for nominal alumina materials refer to the experimental data given in [**?**].

Figures **??**-**??** show the results of the numerical simulation with the **FI** Method. Fig. **??** shows the transient temperature profile on the rod axes until the stationary state is established. The relative deviation from the analytical results (see Fig. **??**) remains well below 1%. Fig. **??** shows the electric field strength inside the cavity at different times. Also in this calculation the deviation from the reference solution is negligible. Additional results related to the microwave heating of the cylindrical sample, including runaway heating effects, are given in [**?**] and [**?**].

V. Transient microwave heating of a jelly block

As a second application of the method, the heating of a jelly block in a PTFE container inside a rectangular microwave cavity is considered (see Fig. **??**). Convective boundary conditions for a constant air temperature of 30 ℃ and a convective heat exchange parameter of $10\,\mathrm{W}/(\mathrm{m}^2\ \mathrm{K})$ are applied on all faces of the block. The incoming mode, $\mathbf{TE}_{01}$ is exited at 2.45 GHz, providing an input power of 600 W. Experimental data on the temperature dependent thermal and electromagnetic material properties of jelly and PTFE are available in [**?**].

Fig. 4. **(a)** Lossy dielectric rod in the microwave cavity. **(b)** Magnetic flux density created by the ground mode $\mathbf{TM}_{010}$ at $T = 25\,°\mathrm{C}$ (the resonance frequency is $\omega_\mathrm{R} = 1.09\,\mathrm{GHz}$).

The application of the method implies several steps. The discretization according to the **FI** technique is realized with a total of ca. 300.000 grid points. Then, the higher modes exited in the cavity are calculated in the time domain. Because of the energy loss in the jelly block, the steady state is established after $\approx 10\,\mathrm{ns}$. Finally, the coupled thermal and electromagnetic calculation is performed according to the algorithm in Fig. **??**.

The calculated temperature distribution inside the jelly block after 180 s of microwave heating is shown in Fig. **??**b. Fig. **??**a shows the temperature distribution taken by a thermal camera in an experimental setup. Both figures show good agreement, especially concerning the location of hot spots. Discrepancies arise mainly because of the inevitable material cooling before the measurement takes place.

The temperature as a function of time at a fixed point inside the block is shown in Fig. **??**. In the same Figure, the temperature at this point for an uncoupled calculation is also shown. In the latter, the jelly material retains its thermal and electromagnetic properties at initial temperature. The final temperature in the uncoupled model is almost 20% higher than in the coupled simulation.

Fig. 5. Temperature profile on the rod axes as a function of time.

VI. Conclusion

In the present study a consistent formulation for the calculation of coupled transient thermal and electromagnetic fields using the **FI** Method is presented. Due to the weak coupling between both fields, an iterative algorithm was implemented, which considerably reduces the calculation effort. The validity of the model was demonstrated in two microwave heating examples, showing good agreement to analytical and experimental data, respectively.

References

[1] J. Driesen, R. Belmans, K. Hameyer, "Adaptive relaxation algorithms for thermo-electromagnetic FEM problems", IEEE Trans. on Magnetics, Vol. 35, No. 3, 1999, pp. 1622-1625.

[2] L. Jänicke, A. Kost, "Convergence properties of of the Newton-Raphson Method for nonlinear problems", in Proc. XI$^{\text{th}}$ IEEE COMPUMAG, Rio de Janeiro, Brazil, PF3-6, 1997, pp. 585-586.

[3] T. Weiland, "Time domain electromagnetic field computation with Finite Difference Methods", Int. J. Num. Mod., Vol. 9, 1996, pp. 259-319.

[4] U. van Rienen, P. Pinder, and T. Weiland, "Consistent finite integration approach for coupled calculations of electromagnetic fields and stationary temperature distributions", in Proc. VII$^{\text{th}}$ IEEE CEFC, Okayama, 1996, pp. 294.

[5] P. Thoma, T. Weiland, "Numerical stability of Finite Difference Time Domain Methods", IEEE Trans. on Magnetics, Vol. 34, No. 5, 1998, pp. 2740-2743.

[6] P. Pinder, "Zur numerischen Berechnung gekoppelter elektromagnetischer und thermischer Felder", PhD Thesis, Darmstadt, 1998.

[7] H. Jackson, H. Barmatz, P. Wagner, "Transient temperature distribution in a cylinder heated by microwaves", Mat. Res. Soc., Spring Meeting, San Francisco, California, April 8, 1996.

[8] H. Jackson H. Barmatz, P. Wagner, priv. comm., 1998.

[9] L. Fukushima, T. Yamanaka and M. Matsui, J. of Japan Soc. of Prec. Eng. **53**, 1987, pp. 743-748.

[10] L. Ma, D. Paul, N. Pothecary, C. Railton, "Experimental validation of a combined electromagnetic and thermal FDTD model of a microwave heating process", IEEE Trans. on Microwave Theory and Techniques, Vol. 43, No. 11, 1995, pp. 2565-2572.

Fig. 6. Relative deviation $\Delta T/T$ from the reference solution.

Fig. 7. Electric field strength $|E|$ inside the cavity as a function of the radial coordinate r.

Fig. 8. Geometry of the jelly block and microwave cavity. **(a)** Vertical cut. **(b)** Horizontal cut.

Fig. 9. Measurement and simulation of the temperature distribution inside the jelly block (horizontal cut).

Fig. 10. Temperature profile for coupled und uncoupled calculations.