HL Physics Internal Assessment

Amplitude of ripples caused by free-falling objects on water bodies of constant depth

Candidate number: gbc594

Research question

What is the relationship between the height from which a 200 gram brass mass is dropped and the maximum amplitude of ripples created by the impact of the mass, before reaching terminal velocity, with a water body of constant depth measured at a constant distance from the center of impact?

Introduction

<u>Background and relevance</u>

The research question is developed from my experience of jumping into a swimming pool from different levels of a diving platform when I was little. I noticed that jumping into the pool from greater heights will cause greater disturbances on the water surface. The research question is formulated to investigate this phenomenon, where the maximum amplitude of ripples caused by the impact between free-falling masses and a still water surface is a measure of said disturbance.

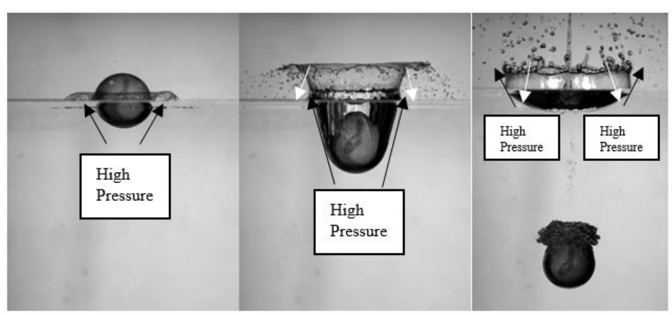
This investigation is relevant in real-life, as its results can provide insight into meteorite tsunami, which is theoretically caused by meteorites falling into the ocean and creating ripples with extremely large amplitudes upon impact.

<u>Theory</u>

Ripples are more formally known as "gravity-capillary waves". The following line of reasoning demonstrates how ripples are produced by free-falling objects upon impact with water.

An object falling into water will displace an amount of water equal to its volume. Since water has a much higher fluid resistance than air, the velocity of the free-falling object will be drastically reduced upon its impact with the water surface, where a significant portion of the object's kinetic energy will be transferred into the displaced volume of water. As illustrated on *Figure-1* (*Popular Mechanics*), the transfer of kinetic energy into water creates a region of relative high pressure underneath the object, forcing water in these regions into an upward motion towards the surface and away from the center of impact where the object has landed; the black arrows on the figure indicate this motion. However, as the displaced water breaks the surface and continues to travel upwards, gravity and surface tension will begin to pull it back towards the surface, as indicated by the white arrows on the figure. Water falling back to the surface are also free-falling objects, and hence this subsequently creates an oscillating cycle of upward and downward motion in water that propagates away from the center of impact.

Figure-1: Slow motion photograph of the impact between a spherical free-falling object and water. *(Popular Mechanics)*



Ripples can be considered as a type of harmonic motion when regarded with respect to the vertical axis alone, where the forces of water pressure, surface tension, and gravity are always in the direction opposite to the motion of the volume of displaced water and towards the surface as an equilibrium position, satisfying the relation:

$$F \propto -x$$

, where *F* is the vertical component force acting on the displaced water and *x* is the vertical component displacement of water. Therefore, by putting the vertical harmonic motion of ripples into perspective with the ripples' horizontal propagation away from the center of impact, it can be easily deduced that ripples are transverse waves, where the harmonic displacement of water is perpendicular to the direction of propagation as illustrated on *Figure-2*.

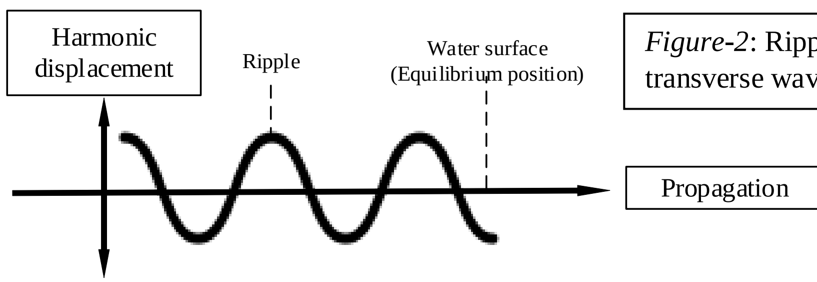


Figure-2: Ripples as a transverse wave

Hypothesis

Given that ripples are the result of kinetic energy being transferred from the falling mass into water, we can relate the amount of transferred kinetic energy and the amplitude of ripples by considering the relation:

$$I \propto A^2$$

, where *I* is the intensity of ripples, and *A* is the amplitude of the ripple.

The kinetic energy that an object falling from height *H* from the water surface possesses at the exact instant before its impact with water will be equal to its potential energy E_p at height *H*, which has the formula of:

$$E_p = m\ g\ H$$

, where *m* is the mass of the object, and *g* is the gravitational acceleration, g = 9.8 m s^{-2}, assuming terminal velocity has not been reached.

As intensity *I* is a representation of the amount of energy carried by the wave, it is a logical assumption that:

$$E_p \propto I$$

Hence, we can anticipate the relationship:

$$\boxed{H \propto A^2}$$

The anticipated shape of the line of best-fit for this investigation is illustrated on *Figure-3*, which has a decreasing rate of change and passes through the origin.

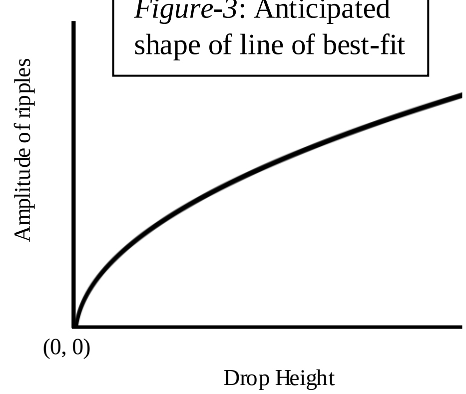


Figure-3: Anticipated shape of line of best-fit

Experiment variables

Independent Variable	Symbol	Definition
	H (m)	The height from which the 200g brass mass is released for free-fall, measured from the bottom of the mass to the surface of water by a meter ruler as seen on *Figure-4*.

<table>
<tr><td rowspan="5">Dependent Variable</td><td>Symbol</td><td>Definition</td><td colspan="2">Proximate dependent variables</td></tr>
<tr><td rowspan="4">A
(m)</td><td rowspan="4">The maximum amplitude of ripples created upon impact between the brass mass and water surface.

From Figure-5 it can be deduced that:
$A = h - h_0$</td><td>Symbol</td><td>Definition</td></tr>
<tr><td>h_0
(m)</td><td>Equilibrium position of the water surface, measured by the height of the initial stain mark on a folded unabsorbant Kraft paper stripe as specified in the method.</td></tr>
<tr><td>Symbol</td><td>Definition</td></tr>
<tr><td>h
(m)</td><td>Height of the stain mark on the folded paper stripe after the ripples have passed through, measured by the second stain mark on a folded unabsorbant Kraft paper stripe as specified in the method.</td></tr>
</table>

Figure-4: Variable *H*

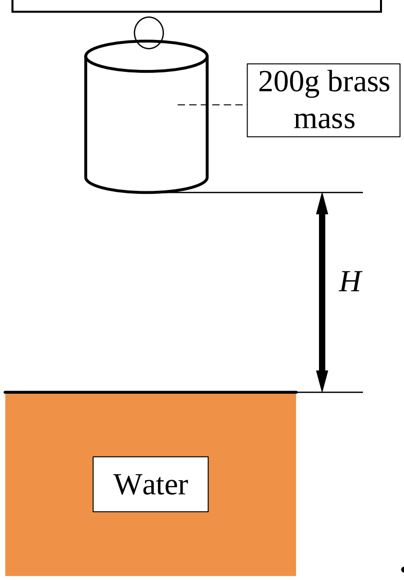
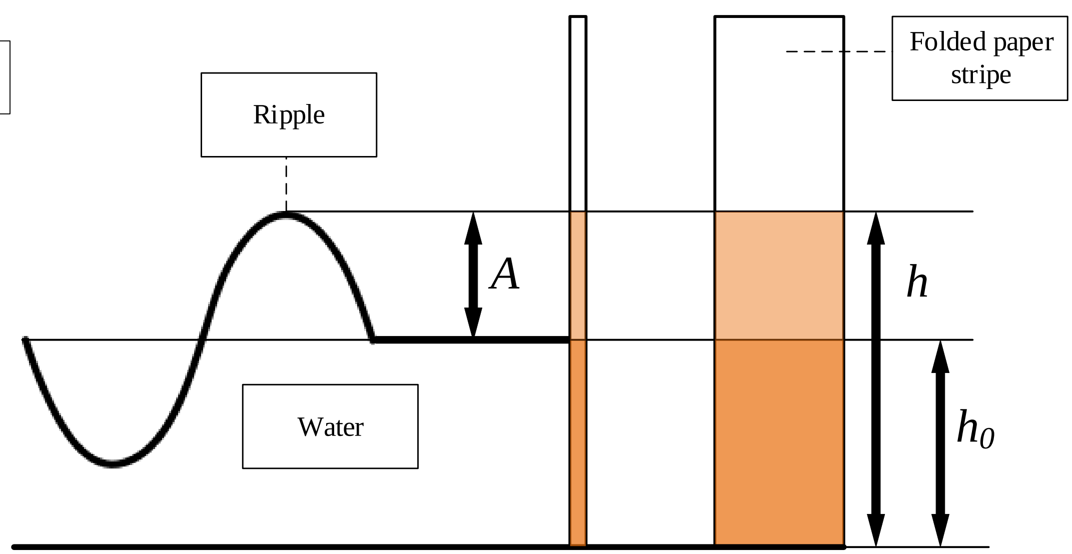


Figure-5: Variable *A*



Controlled Variables	Symbol	Definition	Controlled Value/quality	Reason for control	Control measures
	D	Radial distance between the center of impact and the measuring device (folded paper stripe)	0.240 ± 0.005 m (Uncertainty is the sum of the radius of the small metal loop and total instrument errors)	The amplitude of waves decrease following the inverse square law as distance increases, hence distance influences *A* independent of *H*.	A nylon string is attached to the brass mass, where it is passed through a small metal loop attached to a clamp stand to form a pulley system that controls where the brass mass lands after free-fall. (see *Figure-6)*
	m	Mass of the brass mass used	200 g	With reference to the theory, the mass of the brass mass used will influence the rate of energy transfer between the brass mass and the body of water during impact, hence influencing *A* independent of *H*.	The same 200 gram cylindrical brass is used throughout the experiment.
	-	Dimensions of the brass mass used	Diameter: 2.87 ± 0.005 cm Height: 2.71 ± 0.005 cm (Measured by a Vernier caliper)	The surface area of impact also influences the amount of energy transferred during impact, hence influencing *A* independent of *H*.	
	h_0 *	Equilibrium position of the water surface, measured by the height of the initial stain mark on the folded paper stripe. (see method)	4.0 ± 0.1 cm	The amplitude of water waves change when the depth of water changes, hence depth influences *A* independent of *H*.	See method.
	-	Material that the paper stripes are made out of.	Unabsorbent Kraft paper	Capillary activity in fibrous material will cause water to be absorbed upward along the paper stripe, making measurements of *h* and h_0 inaccurate.	All paper stripes used in the experiment are constructed from the same unabsorbent Kraft paper.

* Note that h_0 is both a proximate dependent variable and a controlled variable because it is impractical to control the variable at the level of precision consistent with the measuring instrument, given that water is constantly and uncontrollably leaving its container through means such as evaporation and splashes.

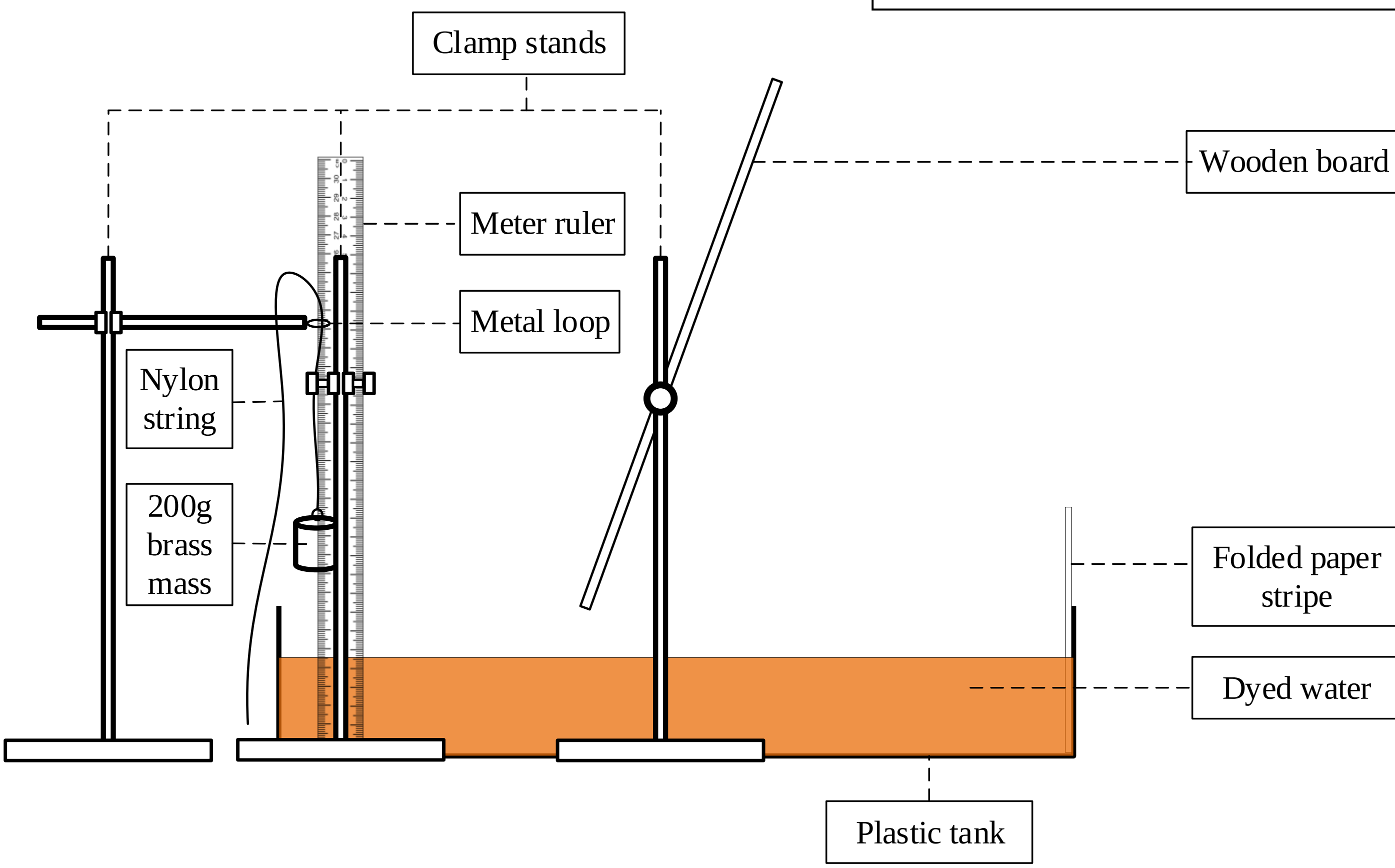
Method

Equipment list

Equipment	Quantity	Instrument error	Dimensions
Tap water	Constant supply	-	Depth in the transparent plastic tank: 4 ± 0.1 cm (Controlled by a meter ruler)
Orange color food dye	50 ml	-	-
200 gram cylindrical brass mass	1	-	Diameter: 2.87 ± 0.005 cm Height: 2.71 ± 0.005 cm (Measured by a Vernier caliper)
Transparent plastic tank	1	-	Length: 28 ± 0.05 cm Width: 19 ± 0.05 cm Height: 8 ± 0.05 cm (Measured by a meter ruler)
Wooden board	1	-	Side length: 34 ± 0.05 cm
Meter ruler	1	± 0.0005 m	Length: 1 m
Vernier caliper	1	± 0.00005 m	-
Folded paper stripe **(**Unabsorbent Kraft paper**)**	Constant supply		Folded twice, 4 layers thick
Metal loop	1	-	Diameter: 0.97 ± 0.05 cm
Nylon string	2 meters	-	-
Clamp stand and clamps	3	-	-
Dropper	1	-	-

Equipment setup

Figure-6: Equipment setup for experiment



The following details regarding the setup of the experiment should be noted with respect to *Figure-6*:

- The meter ruler is inserted perpendicularly into the tank reaching its bottom, where it serves both as a measure to control h_0 and vary H.

- Orange food dye is added to water in the plastic tank to make stain marks on the unabsorbent Kraft paper more visible.

- The purpose of the wooden board is to prevent splashes from interfering with the measurement of h and h_0 by staining the folded paper stripes prematurely.

<u>Data collection</u>

As a safety precaution, heavy instruments used in this experiment, such as clamp stands and brass masses, should be handled with care to prevent injury.

On the setup as illustrated on *Figure-6*, three trials of the following steps are conducted for $0.010\text{ m} \leq H \leq 0.200\text{ m}$:

1. Check on the meter ruler that the depth of water in the plastic tank is within 4.0 ± 0.5 cm, if not, add or remove appropriate volumes of water by using dropper until the condition is satisfied.

2. Perpendicularly insert a fresh piece of folded paper stripe into the tank reaching its bottom from the end opposite to where the brass mass is suspended.

3. Remove the paper stripe from the tank. Measure and record the height of the stain mark on the paper by using a Vernier caliper as the equilibrium position h_0, if this value is not within 4.0 ± 0.1 cm, add or remove appropriate volumes of water by using dropper until this condition is satisfied.

4. Suspend the 200g brass mass above the surface by H meters through the pulley system created by the metal loop and the nylon string. The measurement of H is made with the meter ruler.

5. While the brass mass is suspended, repeat step 2 with the same piece of folded paper stripe, this time with the same piece of folded paper stripe.

6. Allow the brass mass to go into free-fall by releasing the nylon string.

7. Remove the paper stripe from the tank after the water settles. Measure and record the height of the new stain mark on the paper by using a Vernier caliper as h.

Treatment of errors and uncertainties

To simultaneously account for both the discrepancy of measured values in three repeated trials of measurement and the instrument error in each individual measurement as uncertainties, an aggregate uncertainty is calculated for every trial average as follows:

$$\text{Aggregate error} = \frac{\text{Trial maximum} - \text{Trial minimum}}{2} + \sum \text{Instrument error}$$

, where each measured value's deviation from the average is added to the sum of the instrument error.

Aggregate error sample calculation: *Table-1*, average amplitude, H = 0.010	
Trial maximum = 0.0005 m Trial minimum = 0.0003 m	$\text{Aggregate error} = \pm\left(\frac{0.0005 - 0.0003}{2} + 2 \times 0.00005\right)$
Total instrument error = ± 2 × 0.00005 m (the Vernier caliper is used twice, once for measuring h_0 and once for measuring h)	$\text{Aggregate error} = \pm\, 0.0002$

When uncertainties are propagated down into calculated values, the sum of the relative uncertainties associated with each value used in the calculation is taken, and then multiplied by the value of the result to give the propagated absolute error.

$$\text{Propagated error} = \sum \frac{\text{Associated error}}{\text{Value used in calculation}} \times \text{Calculated value}$$

Propagated error sample calculation: *Table-1*, Drop height squared, H^2 = 0.00010	
Values used in calculation:	$\text{Propagated error} = \pm\left(\frac{0.005}{0.010}\right) \times 0.010^2$
H = 0.010 ± 0.005 m	$\text{Propagated error} = \pm\, 0.00005$

Data and analysis

Experiment data

Table-1: Relationship between the drop height H of a 200g brass mass into water of constant depth and the maximum amplitude A of ripples created by the impact measured at a constant distance

Drop height H (m) ± 0.005 m instrument error	Maximum amplitude of ripples A (m)									
	Trial 1			Trial 2			Trial 3			Average amplitude
	± 0.00005 m instrument error									
	Raw measurements		Calculated amplitude	Raw measurements		Calculated amplitude	Raw measurements		Calculated amplitude	with aggregate error
	h_0	h		h_0	h		h_0	h		
0.010	0.0402	0.0407	0.0005	0.0407	0.0410	0.0003	0.0396	0.0401	0.0005	0.0004 ± 0.0002
0.020	0.0403	0.0405	0.0002	0.0405	0.0411	0.0006	0.0402	0.0411	0.0009	0.0006 ± 0.0004
0.030	0.0401	0.0412	0.0011	0.0402	0.0408	0.0006	0.0408	0.0416	0.0008	0.0008 ± 0.0003
0.040	0.0398	0.0406	0.0008	0.0398	0.0407	0.0009	0.0395	0.0405	0.0010	0.0009 ± 0.0002
0.050	0.0396	0.0409	0.0013	0.0405	0.0416	0.0011	0.0403	0.0415	0.0012	0.0012 ± 0.0002
0.060	0.0402	0.0428	0.0026	0.0401	0.0421	0.0020	0.0406	0.0419	0.0013	0.0020 ± 0.0007
0.070	0.0408	0.0427	0.0019	0.0394	0.0417	0.0023	0.0405	0.0424	0.0019	0.0020 ± 0.0003
0.080	0.0397	0.0423	0.0026	0.0408	0.0435	0.0027	0.0406	0.0428	0.0022	0.0025 ± 0.0003
0.090	0.0401	0.0433	0.0032	0.0401	0.0439	0.0038	0.0402	0.0429	0.0027	0.0032 ± 0.0006
0.100	0.0405	0.0451	0.0046	0.0406	0.0441	0.0035	0.0399	0.0437	0.0038	0.0040 ± 0.0006
0.110	0.0394	0.0427	0.0033	0.0397	0.0443	0.0046	0.0393	0.0442	0.0049	0.0043 ± 0.0008
0.120	0.0408	0.0457	0.0049	0.0404	0.0451	0.0047	0.0406	0.0451	0.0045	0.0047 ± 0.0002
0.130	0.0397	0.0454	0.0057	0.0406	0.0453	0.0047	0.0403	0.0457	0.0054	0.0053 ± 0.0006
0.140	0.0403	0.0464	0.0061	0.0397	0.0454	0.0057	0.0404	0.0452	0.0048	0.0055 ± 0.0007
0.150	0.0391	0.0471	0.0080	0.0401	0.0468	0.0067	0.0401	0.0472	0.0071	0.0073 ± 0.0007
0.160	0.0400	0.0477	0.0077	0.0402	0.0481	0.0079	0.0397	0.0474	0.0077	0.0078 ± 0.0002
0.170	0.0391	0.0472	0.0081	0.0397	0.0484	0.0087	0.0392	0.0482	0.0090	0.0086 ± 0.0005
0.180	0.0394	0.0492	0.0098	0.0405	0.0506	0.0101	0.0407	0.0487	0.0080	0.009 ± 0.001
0.190	0.0393	0.0499	0.0106	0.0403	0.0512	0.0109	0.0403	0.0514	0.0111	0.0109 ± 0.0003
0.200	0.0402	0.0521	0.0119	0.0395	0.0506	0.0111	0.0398	0.0527	0.0129	0.0120 ± 0.0009

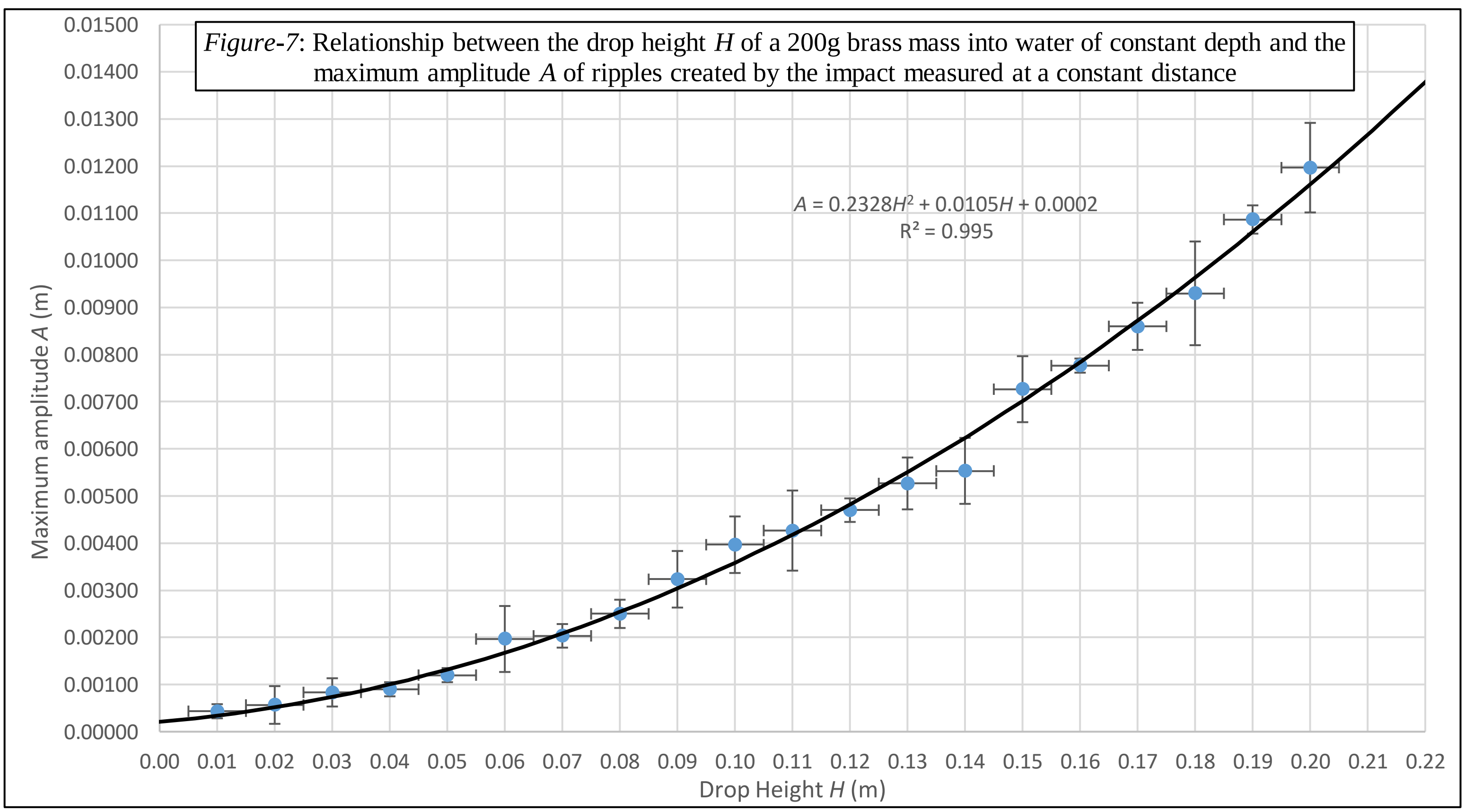


Figure-7: Relationship between the drop height H of a 200g brass mass into water of constant depth and the maximum amplitude A of ripples created by the impact measured at a constant distance

The quadratic line of best-fit on *Figure-7* with the equation of "$A = 0.2328H^2 + 0.0105H + 0.0002$" has a coefficient of determination of $R^2 = 0.995$ with the dataset on *Table-1*, while it agrees with all data points plotted within the acceptable margin of aggregate error, showing that it is a valid model for all measured values of A across the tested domain of independent variable.

However, the equation for the line of best-fit on *Figure-7* does not pass through the origin, suggesting that there could be a systematic error in the method of this experiment. Linear analysis of the experiment results is conducted to determine whether if this deviation from the origin is within the acceptable margin of error.

Linear analysis

The dataset on *Table-1* is linearized as a quadratic with respect to the format of "$A = \mathrm{m}\, H^2$", where H^2 is plotted on the x-axis against A on the y-axis, while coefficient m is a real constant represented by the gradient of the linear expression. As the linearized quadratic relation does not have a constant term, the range of acceptable y-intercepts produced by the linearized expression should include the origin if there is no systematic error.

Table-2: Relationship between H^2 and A

Drop height squared H^2 (m^2) ± propagated error	Maximum amplitude of ripples A (m) ± aggregate error
0.00010 ± 0.00005	0.0004 ± 0.0002
0.0004 ± 0.0001	0.0006 ± 0.0004
0.0009 ± 0.0002	0.0008 ± 0.0003
0.0016 ± 0.0002	0.0009 ± 0.0002
0.0025 ± 0.0003	0.0012 ± 0.0002
0.0036 ± 0.0003	0.0020 ± 0.0007
0.0049 ± 0.0004	0.0020 ± 0.0003
0.0064 ± 0.0004	0.0025 ± 0.0003
0.0081 ± 0.0005	0.0032 ± 0.0006
0.0100 ± 0.0005	0.0040 ± 0.0006
0.0121 ± 0.0006	0.0043 ± 0.0008
0.0144 ± 0.0006	0.0047 ± 0.0002
0.0169 ± 0.0007	0.0053 ± 0.0006
0.0196 ± 0.0007	0.0055 ± 0.0007
0.0225 ± 0.0008	0.0073 ± 0.0007
0.0256 ± 0.0008	0.0078 ± 0.0002
0.0289 ± 0.0009	0.0086 ± 0.0005
0.0324 ± 0.0009	0.009 ± 0.001
0.036 ± 0.001	0.0109 ± 0.0003
0.040 ± 0.001	0.0120 ± 0.0009

Figure-8: Relationship between H^2 and A

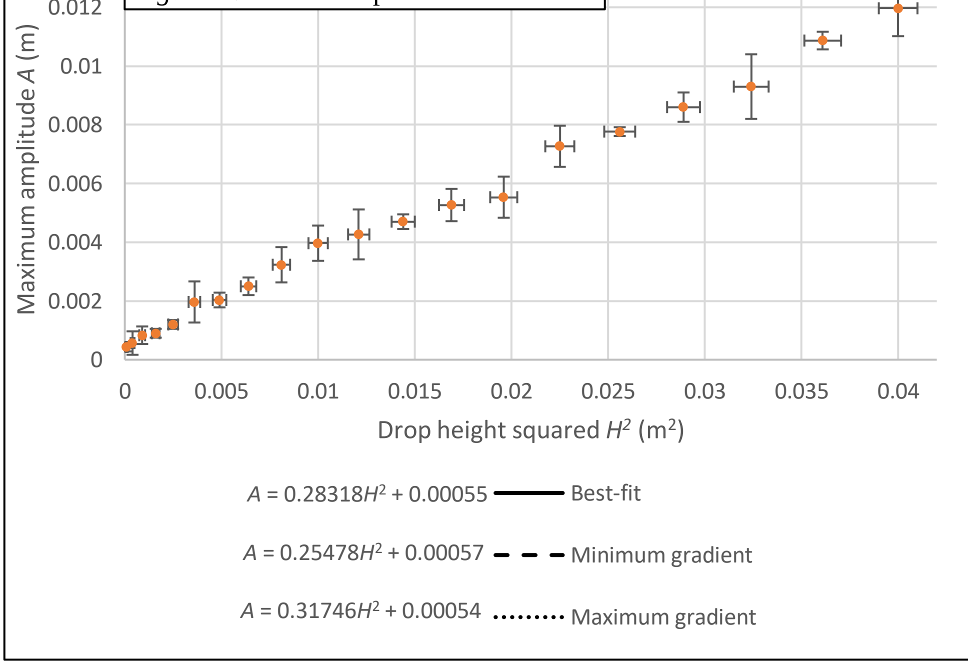


Figure-8 shows that the acceptable range of y-intercepts within the margin of error for the dataset on *Table-2* is $0.00057\text{ m} \geq A \geq 0.00055\text{ m}$. The origin is not included within this range, hence a systematic error is present.

Reviewing the experiment design, it is most likely that the identified systematic error is the result of incorrectly defining the independent variable *H*. Defining *H* as the vertical distance between the water surface and the bottom of the brass mass and expecting the trend line on *Figure-7* to pass through the origin makes the false assumption that the mass will possess zero potential energy at the water surface where $H = 0$. With reference to *Figure-9*, reaching the position $H = 0$ does not prevent the brass mass from descending further and becoming submerged in water, as the brass mass only stops moving once it reaches the bottom of the plastic tank where $H = -h_0$. This means that the brass mass still possesses potential energy by the amount of $E_p = m\,g\,h_0$ that can be transferred into kinetic energy and passed onto ripples at $H = 0$, which was unaccounted by the definition of *H*. To eliminate this systematic error, the independent variable for this experiment should be $H + h_0$ instead of *H*, measuring height from the bottom of the plastic tank instead of from the water surface. Thus, adjusting the linearized quadratic expression for the identified source of systematic error will have $(H + h_0)^2$ plotted on the *x*-axis, together with *A* plotted on the *y*-axis on *Figure-10*, where the calculated values for each data point are tabulated on *Table-3*.

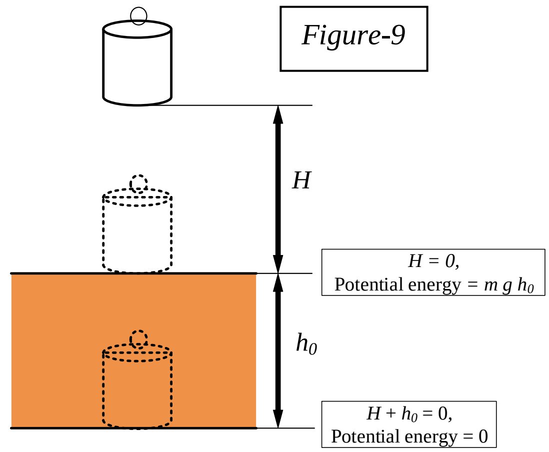


Figure-9

Table-3: Relationship between $(H + h_0)^2$ and *A*

Linear *x*-axis variable adjusted for systematic error		Maximum amplitude of ripples *A* (m) ± aggregate error
Average h_0 (m) ± aggregate error	$(H + h_0)^2$ (m²) ± propagated error	
0.040 ± 0.001	0.003 ± 0.001	0.0004 ± 0.0002
0.0403 ± 0.0007	0.0036 ± 0.0009	0.0006 ± 0.0004
0.0404 ± 0.0009	0.0050 ± 0.0008	0.0008 ± 0.0003
0.0397 ± 0.0007	0.0064 ± 0.0008	0.0009 ± 0.0002
0.040 ± 0.001	0.0081 ± 0.0008	0.0012 ± 0.0002
0.0403 ± 0.0008	0.0101 ± 0.0008	0.0020 ± 0.0007
0.040 ± 0.001	0.0122 ± 0.0009	0.0020 ± 0.0003
0.040 ± 0.001	0.0145 ± 0.0009	0.0025 ± 0.0003
0.0401 ± 0.0005	0.0169 ± 0.0009	0.0032 ± 0.0006
0.0403 ± 0.0008	0.020 ± 0.001	0.0040 ± 0.0006
0.0395 ± 0.0007	0.022 ± 0.001	0.0043 ± 0.0008
0.0406 ± 0.0007	0.026 ± 0.001	0.0047 ± 0.0002
0.0402 ± 0.0009	0.029 ± 0.001	0.0053 ± 0.0006
0.0401 ± 0.0009	0.032 ± 0.001	0.0055 ± 0.0007
0.040 ± 0.001	0.036 ± 0.001	0.0073 ± 0.0007
0.0400 ± 0.0007	0.040 ± 0.001	0.0078 ± 0.0002
0.0393 ± 0.0008	0.044 ± 0.001	0.0086 ± 0.0005
0.040 ± 0.001	0.048 ± 0.001	0.009 ± 0.001
0.040 ± 0.001	0.053 ± 0.001	0.0109 ± 0.0003
0.040 ± 0.001	0.058 ± 0.001	0.0120 ± 0.0009
0.0403 ± 0.0007	0.003 ± 0.001	0.0004 ± 0.0002

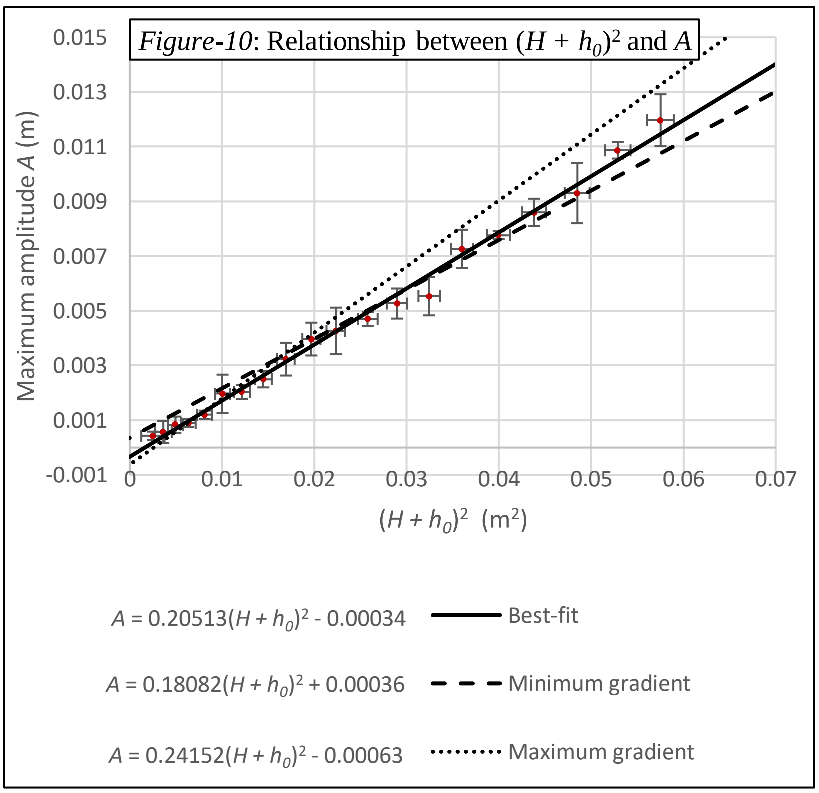


Figure-10: Relationship between $(H + h_0)^2$ and *A*

Figure-10 shows that the acceptable range of *y*-intercepts within the margin of error for the dataset on *Table-3* is $0.00036\text{ m} \geq A \geq -0.00063\text{ m}$. The origin falls well within this range, thus suggesting that the identified systematic error has been successfully eliminated. The linear line of best-fit agrees with all data points within their error bars, evidencing the validity of this model for the data set.

Conclusion

Adjusted for the identified source of systematic error, the results of this experiment as presented on *Figure-10* yields the mathematical relationship:

$$A = (0.205 \pm 0.03)(H + h_0)^2$$

$$\text{, where } h_0 = 0.040 \pm 0.001 \text{ m}$$

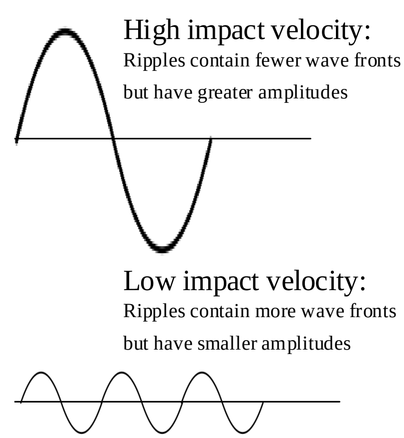
Thus, it can be concluded that within the tested domain of independent variable, the relationship between the height H from which a 200 gram brass mass is dropped and the maximum amplitude A of ripples created on its impact with a water body with a depth of h_0 before the mass reaches terminal velocity is:

$$A \propto (H + h_0)^2$$

This conclusion strongly disagrees with the hypothesis. The hypothesized relationship "$H \propto A^2$" predicts that the positive relationship between H and A will have a decreasing rate of change, but the concluded relationship "$A \propto (H + h_0)^2$" clearly shows that the positive relationship between H and A has an increasing rate of change.

The most reasonable explanation for this would be that ripples are more appropriately described as pulses rather than waves, where "pulses" refer to a packet of wave fronts that propagates through a medium as opposed to a continuous oscillation, which breaks the postulate of "$\Delta E \propto I$" implied in the hypothesis. (ΔE is the amount energy transferred between the falling mass and water in joules, and I is the intensity of the ripple as a measurement the rate of energy transferred per unit area in W m^{-2}). Although the amount of energy transferred from the falling mass into the body of water is indeed directly proportional to the height from which the mass is dropped, the postulate that "$\Delta E \propto I$" will only hold true if the window of time available for energy transfer remains constant, bearing in mind that intensity I measures the amount of energy transfer per unit area per unit time. Considering that the mass will accelerate toward the surface of water following the relation $v = \sqrt{2 \times 9.8H}$ * with respect to height H during free fall, where v is the impact velocity of the mass with the surface of water, the window of time available for energy transfer between the mass and water will become shorter as the impact velocity increases with drop height.

Figure-11: The qualitative behavior of ripples in relation to drop height.



As illustrated on *Figure-11*, it is a qualitative observation that masses at higher impact velocity due to being dropped from a greater height will cause ripples that contain fewer wave fronts but a greater amplitude. This is a direct consequence of the window of time available for energy transfer becoming shorter as the impact velocity increases, where this causes more volume of water to be displaced in a single wave front per unit time before the surface tension breaks to form the next wave front. While the ΔE increases proportionally to drop height, having fewer numbers of wave fronts in the pulse will cause each individual wave front to possess even more kinetic energy on average, hence explaining the increasing rate of change in the concluded correlation between A and $H + h_0$. Furthermore, the concluded relationship of $A \propto (H + h_0)^2$ also implies the relation $\Delta E \propto I^4$ for the domain of tested independent variable, given that $I \propto A^2$.

Evaluation

The validity of the conclusion reached by this experiment is limited by several factors. Firstly, this conclusion is only valid before the falling object reaches terminal velocity, after which the height from which the mass is dropped would make no difference on the impact velocity, and hence

* From $v^2 = u^2 + 2\,a\,s$,
where $s = H$, $a = g = 9.8$ m s^{-2}, $u = 0$ m s^{-1}

make no impact on the amplitude of ripples upon impact. Secondly, the dependent variable A was not measured across the domain $0 \geq H \geq -h_0$ due to the incorrect definition of the variable H. This signifies that the concluded mathematical relationship is based upon the unverified assumption that H and A will continue to follow the same observed behavior when the brass mass is partially submerged in water.

Reviewing the outcome of this investigation, it can be said that the designed experiment has overall produced meaningful and relatively precise data that enabled a mathematical conclusion to be formed after the systematic error has been eliminated. This is evidenced by how all data points plotted on *Figure-7*, *Figure-8*, and *Figure-10*, agreed with the best-fit trend lines within in the acceptable margin of error, while the relative uncertainty of the coefficient of $(H + h_0)^2$ in the concluded mathematical relationship only has a relative uncertainty of approximately 15%.

Besides the incorrectly defined independent variable H, there are still several identifiable sources of unreliability inherent to the setup and the method of data collection for this experiment. One major limitation in the method of data collection for this experiment lies with measuring h and h_0 as stain marks on folded paper stripes. The polarity of water will cause water molecules in the stained portions of the folded paper stripes to undergo capillary action along the fibers within paper, causing the stain mark to displace upwards spontaneously over time without contact with ripples, which makes the measurement of the amplitudes of ripples inaccurate. Empirically speaking, however, the effects of capillary activity only becomes pronounced after a fibrous material is partially soaked in water for an extended period of time. During the experiment, the folded paper stripes made out of unabsorbent Kraft paper only comes into contact with water for a short period of time, while the measurements for h and h_0 are made immediate after the paper stripes are removed from water, hence the impact of this source of unreliability on the validity of this experiment should miniscule, provided that unabsorbent Kraft paper is intentionally used to counter-act this effect. To completely eliminate this source of unreliability, the folded paper stripes as the measurement device for amplitude can be replaced by a slow-motion camera, where the amplitude of ripples can be evaluated by analyzing the slow-motion footage.

Reviewing *Figure-6*, another limitation of the experiment setup is that the friction between the nylon string and the metal loop can decelerate the brass mass's free fall. This can be resolved by using an iron mass that can be suspended and released into free-fall by using an electro-magnet.

Given that the controlled variable h_0 is also a proximate dependent variable measured in this experiment due to the difficulties to control h_0 at an accuracy consistent with the measuring instrument, the method of control for h_0 should be improved. One way to better control h_0 is to conduct the experiment in an entirely enclosed container, where water cannot leave experiment setup by any means, and hence the depth of water in the container will always remain constant.

The outcome of this experiment lends itself to several extensions to further investigate the amplitude of ripples caused by free-falling objects on water surfaces. The most meaningful extension to this investigation would be to measure the amplitude of ripples caused by the brass mass suspended at different displacements below the water surface, covering the domain $0 \geq H \geq -h_0$ that was unmeasured in this experiment and further validate the conclusion of this experiment. Another interesting extension to the current investigation would be to control the drop height and instead allow the variable h_0 to vary, where the relationship between the amplitude of ripples and the depth of water in the container is investigated, which will provide further insight into how the properties of the medium affects a transverse wave.

Work cited

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