Lab 06: Electron Spin Resonance

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Abstract

In the experiment we attempt to calculate the gyromagnetic ratio for an isolated electron. From which we also then calculate for the Lande g factor. The experiment was set up using a pair Helmholtz coils and current to generate a magnetic field. In which we placed our sample of DPPH as the 'isolated' electron. Then we supplied combinations of current and frequency to find corresponding resonance. Using this data we constructed a linear fit which we then used to calculate $\gamma = (1.8 \pm .08) \times 10^{11} \frac{kg}{C}$ and $g = 2.0 \pm 0.09$. This calculation was found to be in agreement with the known value of $g_e = 2.00231$.

1 Introduction

Electrons have an inherit magnetic dipole moment $\vec{\mu}$ due to its spin angular momentum $\vec{S}$. This two
are connected by the gyromagnetic ratio γ by

$$\vec{\mu} = \gamma\vec{S}$$

Making simplifying assumtions that a electron is an uniform sphere with homogeneous charge would imply that $\gamma = \frac{e}{2m}$. However, this is not the case and instead we find that this assumtion is off by the Lande g factor g defined as follows

$$g = \frac{\gamma}{e/2m} > 1$$

In this lab we attempt to calculate the Lande g factor by finding the gyromagnetc ratio measurements on free electrons. We may do so my noticing the following physics. If we imagine putting an electron in a magnetc field that is in the z-direction B_z and noting that the spin momentum can only take the values $S_z = \pm\frac{\hbar}{2}$ then the electron will have only have two possible potential energies E in this field with energy difference ΔE as displayed below

$$E = \pm\frac{\hbar}{2}\gamma B_z$$
$$\Rightarrow \Delta E = \hbar\gamma B_z$$

Thus if we wished to 'flip' the spin of an electron (this is what electron spin resonance is) a possibility is we would either want to have the electron absorb or emitt a photon carrying this amount of energy. Recall that the energy of a photon is related to its frequency ν and with this relationship we see that $\nu = \frac{1}{2\pi}\gamma B_z$.
So by varying the current that induces the magnetic field and the frequency we may be able to find a combination that undergoes electron spin resonance.
With our previous equation for the frequency and recalling that the magnetic field from a set up of Helmholtz coils of radius R and n number of turns being supplied with a current I is

$$B = \left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 nI}{R}$$

where $\mu_0 = 4\pi \times 10^{-7}$ we find

$$\nu = \frac{1}{2\pi}\gamma\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 nI}{R} \tag{1}$$

Thus with known values of our apparatus we may find γ.

2 Material and methods

Our experiment's magentic field was set up with a Helmholtz coils, the diameter of which was measured and thus seperated by the radius, and AC current. The Basic unit held our free radical used (diphenylpicryl hydrazyl, DPPH) and controlled the strength and frequency of the photons.

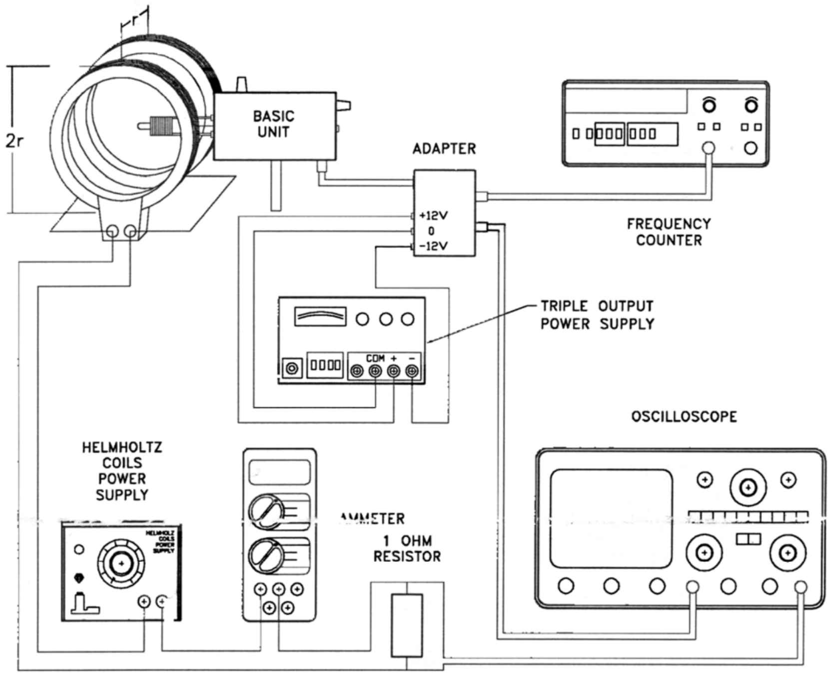

Figure 1: Experimental set up taken from lab handout

We aligned the axis of the copper coils to be perpendicular to the magnetic field. We started with the copper coils (which hold our sample of DPPH) with the most turns to the coil with the least turns.

For each coil, we adjusted the frequency to some set current so that the peaks we say on the oscilloscope would merge into one. We took the corresponding current at resonance to be the peak current going through the coils. We then recorded this current and the corresponding frequency. An example of the display is below.

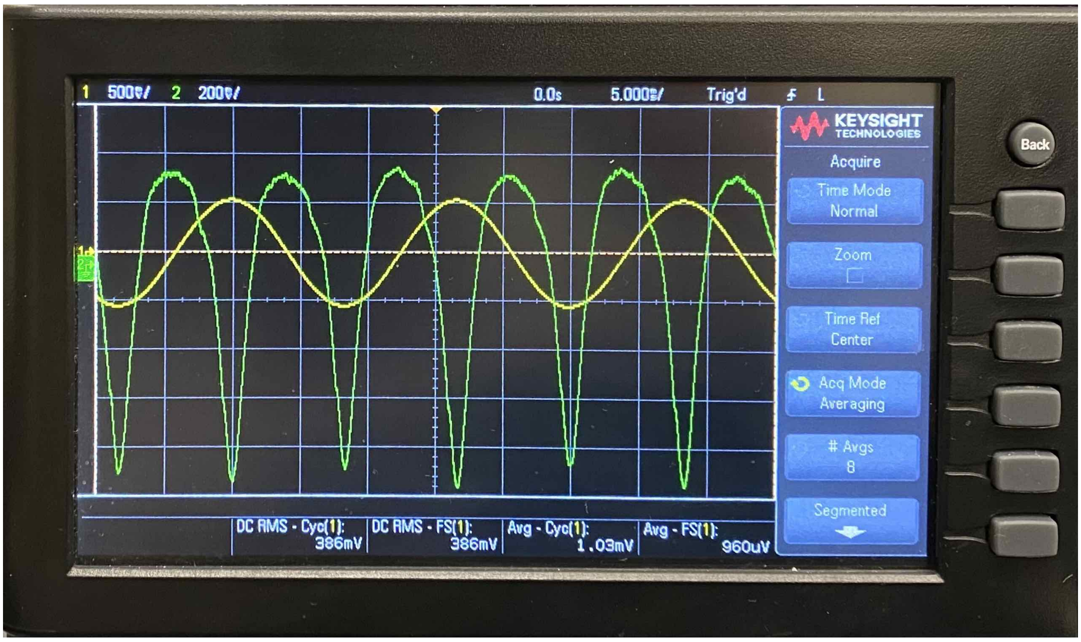

Figure 2: Oscilloscope display where the yellow trace is the supplied AC current going to the Helmholtz coils, which is proportional to the magnetic field. The green tace corresponds to the nvelope of the voltage across the RF oscillator, which dips sharply each time the B field passes through the resonance point.

The absorption of energy from the photon affects the permeability of the DPPH, which then in turn also affects the inductance of the coil and then finally the oscillations. This manifets an observable change in the current flowing through the oscillator. If the frequency does indeed satisfy equation (1) then we observe that somwhere between the maximum and minimum of the magnetic field then we see the resonance twice during each cycle.

3 Results and Disccusion

We measured the diameter of the coils to be $12.3 \pm 0.1\ cm$ where the uncertainty is simply taken to be one unit of the last significant digit. Thus we have a radius of $6.15 \pm 0.05\ cm$. The uncertainity of the current was taken to be the uncertainty the manufactor claimed for the specific device. For the multimeters used, Peterman 37XR, this uncertainty was 1.5% of the measurement $+\ 10$ counts of the most accurate digit. Lastly, the uncertainty of the measured frequency was taken to be $\pm\ 5\ kHz$. For the frequency we were unable to find the manufactors claimed uncertainty. Thus we took it to be $\pm\ 5\ kHz$ because, when conducting the experiment and recording the data, the frequency value displayed often fluctuated about this $5\ kHz$ from some central value which we took to be the measured frequency.

Below is the plot of of the resonance frequency plotted agaisnt current for the DPPH for each of the three copper coils. The a linear model was fitted using scipy's curve_fit() and the uncertainty taken from the corresponding variance found.

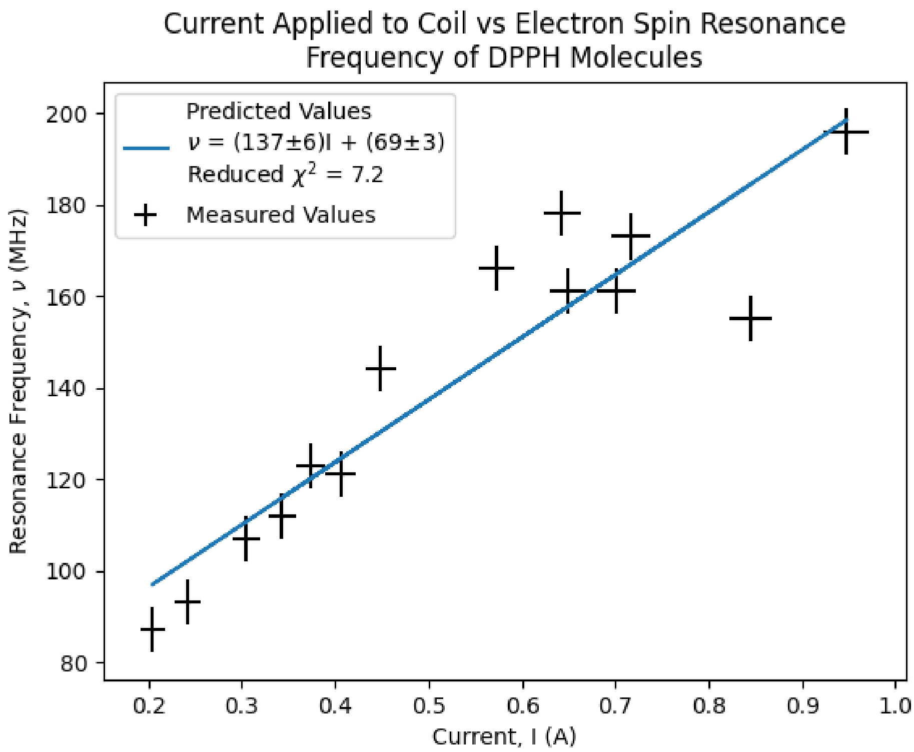

Figure 3: Resonance frequency (should be labeled as Megahertz) vs. current for DPPH for each of the three copper coils

Using the relationship described by equation (1). We may note that the slope, call it α, of our linear fit should correspond to γ as follows

$$\alpha = \frac{1}{2\pi}\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 n}{R}\gamma$$

$$\Rightarrow \alpha 2\pi\left(\frac{5}{4}\right)^{3/2}\frac{R}{\mu_0 n} = \gamma$$

Thus using $\mu_0 = 4\pi \times 10^{-7}\ N/A^2$ and $n = 320\ turns$ and our previous measured and calculated values of R and α we get that

$$\gamma = \left((1.37 \pm .06) \times 10^8\right)2\pi\left(\frac{5}{4}\right)^{3/2}\frac{.0615 \pm .0005}{\left(4\pi \times 10^{-7}\right)(320)}$$

$$= (1.8 \pm .08) \times 10^{11}\frac{kg}{C}$$

where the error here is calculated by combining in quadrature to find the relative error.

We find that our reduced chi squared value is $\chi_r^2 = 7.2 > 1$. This suggests that our linear model does not quite fit the data. Examining figure 3 we see that around $0.85\ A$ there appears to be a data point that has some possible significant deviation from the overall trend, though it is still ambiguous with the amount of data taken. Also the data points beyond about $0.5\ A$ tend to spread further around the the linear fit. All of which contribute to the increased χ_r^2.

With this we can now calculate the Lande g factor as follows

$$
\begin{aligned}
g &= \frac{\gamma}{e/2m} \\
&= \frac{(1.8 \pm .08) \times 10^{11}}{\left(1.6 \times 10^{-19}\right)/2\left(9.1 \times 10^{-31}\right)} \\
&= \frac{(1.8 \pm .08) \times 10^{11}}{8.8 \times 10^{10}} \\
&= 2.0 \pm 0.09
\end{aligned}
$$

The currently known Lande g factor for an isolated electron is $g_e = 2.00231...$ and we thus our calculated factor is in agreement with the currrent theoretical value known.

4 Conclusion

From our experiment we were able to model the data of resonance frequency versus current using a linear fit. From which we calculated the gyromagnetic ratio to be $\gamma = (1.8 \pm .08) \times 10^{11} \frac{kg}{C}$ and the Lande g factor to be $g = 2.0 \pm 0.09$. The reduced chi squared value was $\chi_r^2 = 7.2 > 1$. This suggestion of the poor fit is proabably due to the possible outlying value as well as the increase spread from the linear model from $0.5\ A$ onwards.

These later data points were taken using the plug-in copper coil G, which was the coil with the least number of turns, and finding corresponding currents and resonance frequencies from this coil was substantially more difficult and ambiguous than the other two.

In the end, when comparing our Lande g factor to the the known factor $g_e = 2.00231...$ we found that our value is in agreement with this accepted value.