Lab 02: Radioactive Decay

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Abstract

We tried to calculate the half-life of cesium and examine the random decay distribution of a fiesta plate glazed with uranium dioxide. For the cesium we found that a nonlinear exponential model fit the data best in comparison to a linear model and found a corresponding half life of $2.83 \pm 0.035\ min$. This value is not in agreement with the theoretical half life of cesium and overestimated by approximately 8.84%. For the random decay we found that the fiesta plate did indeed appear to follow a Poisson distribution and appeared to converge to a Guassian distribution. The analogous histograms for the background radiation also followed the expected distributions.

1 Introduction

Radioactive decay occurs when an unstable atomic nucleus loses energy. This loss of energy through some emission of a particle is what we refer to as radiation. The number of emissions over a period of time (and thus the intensity of the radiation) is dependent upon the number of atoms of the isotope and is associated with the death of an isotope. The intensity as a function of time, $I(t)$, thus follows an exponential as follows

$$I(t) = I_0 e^{-\frac{t}{\tau}}$$

Where I_0 is the initial intensity and τ is the mean lifetime of our isotope in observation. Another way we can represent the intensity is with the isotope's *half-life*. The half-life, $t_{1/2}$, is simply the amount of time it takes for the intensity to be half its original value.

$$I(t) = I_0 \frac{1}{2}^{\frac{t}{t_{1/2}}}$$

In the following report we will attempt to calculate the half-life of barium/cesium by both linear regression and nonlinear fit using scipy's `curve_fit()` for an exponential model. We know that Ba-137m decays by gamma emission and has a well known half-life of 2.6 minutes (156 seconds).

However, we must not forget that there also exists background radiation. Even without a radioactive sample present, our measuring apparatus, a Geiger counter, will still record counts. There exists background radiation in any space where we can actually conduct this experiment as there is radiation present from naturally occuring minerals to man-made elements. Thus our measured counts of the actual source of radiation, N_s, should be considered to be

$$N_s = N_T - N_b$$

Where N_T is the total counts when measuring the object of interest and N_b is the background radiation. Since background radiation is can't be measured simultaneously we make the assumption that it has constant statistics and will take N_b to simply be the mean.

Along with the fast decay of cesium, we will also be examining a much slower decay from a fiesta plate: plates that were glazed with low amounts of uranium dioxide. Uranium has an incredibly long half-life. Thus we will examine both the background and fiesta plate as if they are obeying random emission given by a Poisson distribution. Explicitly, we can find the likelihood of an event occurring

$$P_{\mu}(n) = e^{-\mu} \frac{\mu^n}{\Gamma(n+1)}$$

Where μ is the expected average number of events for a given time and n is the number of actual measured amounts of counts.

2 Method

The experiment and its set up was completely conducted by the instructor. Simply a geiger counter was put above the sample of interest. For cesium a measurement was made every 20 seconds while for the fiesta plate it was taken every 6 seconds.

The background was then also measured after each sample measurements were taken with the corresponding appropriate time intervals.

For analysis of the data, we used scipy's `curve_fit()` function in order to achieve the parameter values for the models of the data.

3 Results and analysis

3.1 Fast decay

Before plotting or modeling the data collected for cesium and the fiesta plate, the background radiation was calculated for each sample. The background count, N_b, was taken to be the average mean value and was directly subtracted from the sample count to get the approximate intrinsic sample count.

We know that $I(t) = I_0 e^{-\frac{t}{\tau}}$ and $I(t) = I_0 \frac{1}{2}^{\frac{t}{t_{1/2}}}$ and thus we can solve for $t_{1/2}$ as follows,

$$I_0 e^{-\frac{t}{\tau}} = I_0 \frac{1}{2}^{\frac{t}{t_{1/2}}}$$

$$e^{-\frac{t}{\tau}} = \frac{1}{2}^{\frac{t}{t_{1/2}}}$$

$$-\frac{t}{\tau} = ln(\frac{1}{2}^{\frac{t}{t_{1/2}}})$$

$$-\frac{t}{\tau} = \frac{t}{t_{1/2}} ln(\frac{1}{2})$$

$$-\tau ln(\frac{1}{2}) = t_{1/2} \tag{1}$$

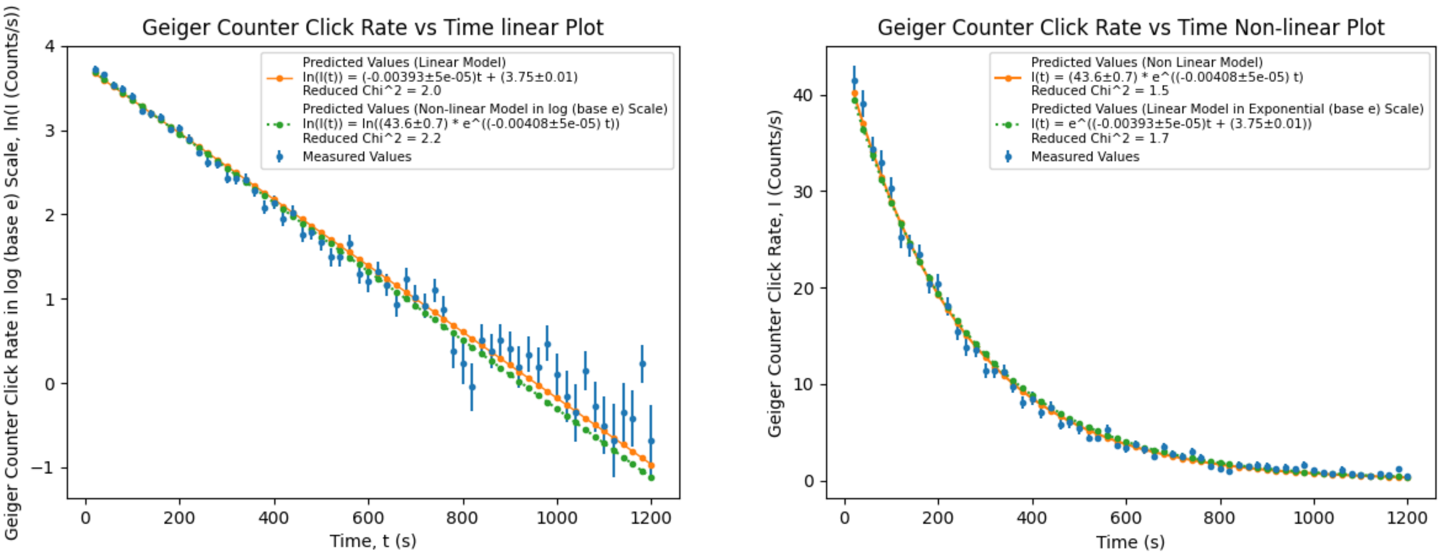


Figure 1: (left) Time v. counter plotted on log base$e$scale to fit linear model
(right) Time v. counter on a linear scaled axis to fit the nonlinear model

Doing the analysis with logarithms to get the linear regression fit we found that the following emission rate

$$I(t) = e^{(-0.00393\pm(5\cdot10^{-5}))t + (3.75\pm.01)}$$

$$= e^{3.75\pm.01} e^{-0.00393\pm(5\cdot10^{-5}))t}$$

So we have that the mean lifetime is $\tau_{lin} = \frac{1}{0.00393\pm(5\cdot10^{-5})}$ and thus by equation (1) we have

$$t_{1/2,\, lin} = -\frac{1}{0.00393\pm(5\cdot10^{-5})} ln(\frac{1}{2}) = 176.4 \pm 2.24\, sec$$

$$t_{1/2,\, lin} = 2.94 \pm 0.037\, min$$

For the nonlinear, using and exponential and scipy's `curve_fit()`, we found that

$$I(t) = (43.6 \pm 0.7)e^{(-0.00408\pm(5\cdot10^{-5}))t}$$

And thus we have that $\tau_{nonlin} = -\frac{1}{0.00408\pm(5\cdot10^{-5})}$ which we may then use to find

$$t_{1/2,\, nonlin} = -\frac{1}{0.00408\pm(5\cdot10^{-5})} ln(\frac{1}{2}) = 169.9 \pm 2.08\, sec$$

$$t_{1/2,\, nonlin} = 2.83 \pm 0.035\, min$$

Taking the nominal half-life of cesium to be $t_{1/2} = 2.6\, min$, we find that neither our linear or nonlinear half-lifes agree with this value: where the $t_{1/2,\, lin}$ and $t_{1/2,\, nonlin}$ overestimate the nominal value by 13.1% and 8.84%, respectively.

However, we find that the nonlinear exponential model comes closer to the nominal value compared to the linear model by about 4.23%. Simply examining the plots of figure 1 it is difficult to see whether one significantly models the data more efficiently than the other. The most significant visual deviation can be seen on the left plot of the log scaled axis. We see that the linear regression slope is less steep as it appears to be trying to accommodate the later data points with larger error bars.

The reduced chi square value for all our models appeared reasonable and implied decent fit. For the linear fit on the log scale was $\chi^2_{r,lin} = 2.0$while on the linearly scaled axis $\chi^2_{r,lin} = 1.7$. In either case we find the chi-square decently models the data, but models the linear scaled data slightly better. Similarly we find that for the nonlinear model we have $\chi^2_{r,\, nonlin} = 2.2$for the log scaled data and $\chi^2_{r,\, nonlin} = 1.5$ for the linearly scaled data. Both values again imply a decent fit of the data. We see that the linear regression does appear to model the data of the log scaled data slightly better while the nonlinear fit models the data of the linearly scaled data more efficiently.

We see that overall the reduced chi-square of the log scaled data is slightly higher than the linearly scaled. This is probably due to underestimation of the error variance and incomplete fit of the data that appear later in time. By examining the left plot we can see that our lines don't quite capture as much of the data as the error bars begin to grow and the measurements begin to deviate from a clear line. On the other hand, it's clear from examining the right graph that more of the data appears to be being captured by both our linear and nonlinear models--Hence a smaller reduced chi-square.

3.2 Slow decay/Background

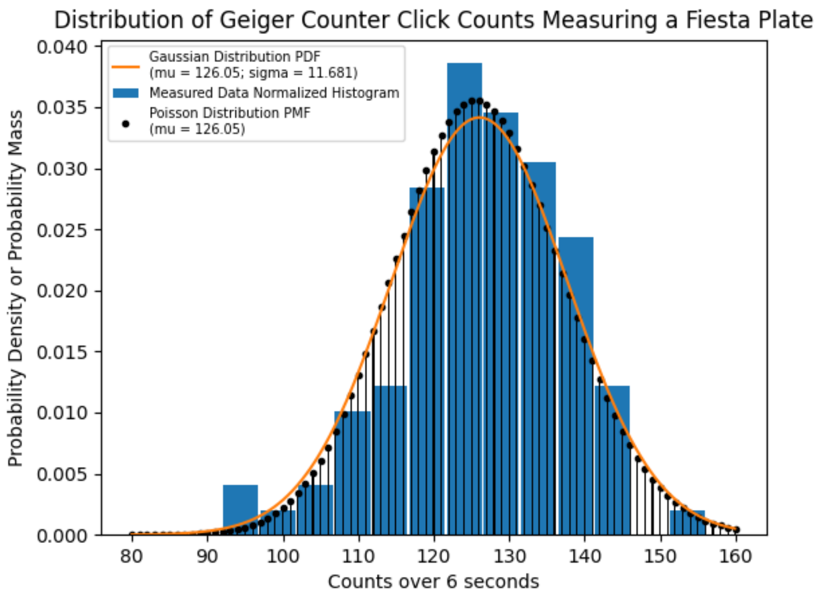
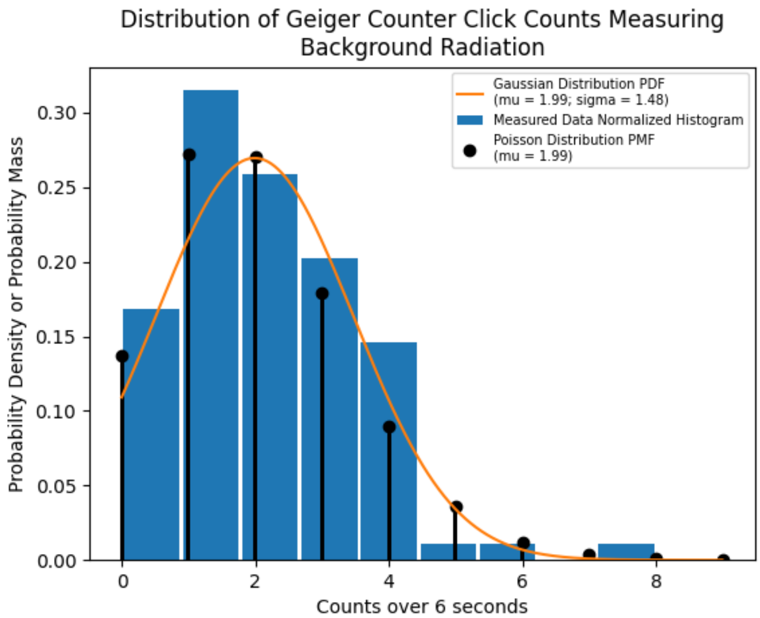


Figure 2: (Top) Histogram of the count of the Fiesta plate
(Bottom) Histogram of count of the background

Examining the histograms of Figure 2, we see that the Fiesta plate data appears to be centered around 126.05 while the background noise is centered around 1.99. Similarly, the fiesta plate has a larger spread than the background ($\sigma_{fiesta} = 11.681$while $\sigma_{noise} = 1.48$). This should be expected as the fiesta plate is still technically emitting radiation that exceeds what would be otherwise from the surrounding environment.

From examining the plots, as expected/in line by theoretical model, the Poisson distribution appears to model the data's distribution well. The Poisson distributions and the corresponding Guassian distribution appear to be converging toward one another. Similarly, for the analogous histogram for the background radiation we chose to plot the data in fewer bins as otherwise the distribution of the bins with any probability density had large gaps between them. Again, as expected, the background radiation also appears to follow the



expected Poisson distribution for a random decay and also appears to be converging to the gaussian distribution.

When comparing the two histograms and their corresponding modeled distributions, we find that for both the background and the fiesta plate the Poisson distribution peaks slightly higher and sooner than the Gaussian distribution.

4 Discussion and Conclusion

For the radioactive decay of cesium we used our linear and nonlinear model predicted a emission rate of $I(t) = e^{(-0.00393\pm(5\cdot10^{-5}))t + (3.75\pm.01)}$ and $I(t) = (43.6 \pm 0.7)e^{(-0.00408\pm(5\cdot10^{-5}))t}$ respectively. We found that neither model was able to produce the expected half-life of cesium--2.6 minutes. Our linear model suggested a half life of $t_{1/2,\, lin} = 2.94 \pm 0.037\, min$ while our nonlinear model suggested $t_{1/2,\, nonlin} = 2.83 \pm 0.035\, min$. Thus our nonlinear model came closer to predicting the theoretical value by about 4.23%.

Possible reasons for this deviation from the theoretical value are independent of temperature, concentration, etc. as half-life is not affected by any of these factors, but rather unique to the given isotope. Seeing as our results were an overestimation of the expected value possible sources of such an affect our factors such as background radiation. When we removed the background radiation, we removed the *average* background radiation. Suppose our average background radiation found was actually less than what was being contributed to the actual measurement. Such a situation would lead to an extended calculated half life as there would be more counts per sample modeled than what is intrinsic to the sample itself.

For the slow decay/fiesta plate we found that a Poisson distribution appeared to model the data fairly well. We also saw that the corresponding Guassin distribution indeed converged to the expected Poisson distributions. Thus implying that there was enough data measured to demonstrate the behaviour expected from the two distributions as dictated by the central limit theorem. Similarly the data collected for the background noise also appeared to follow a Poisson distribution well and converged to the Gaussian distribution.

We know the fiesta plate is emitting radiation and thus should possess a half-life. However the data appears to display a Poisson distribution that we would expect from random decay. This is also expected as we know the half-life is so long for the fiesta plate that for the time that we spent collecting the data we would not be able to measure any significant exponential decay. Thus, in the presence of the fiesta plate, we simply appear to be in particularly radioactive background noise. This appears to be supported by the data and histograms produced. As we see the background radiation distribution followed the expected Piosson distribution for random discrete events and the fiesta plate displayed the same correspondence to the distributions but simply shifting up the mean μ and extending σ.