Milikan experiment

Written by:
Andrew Zhang
M. L. A. Rehem

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Abstract

In this lab we conduct the Milikan oil-drop experiment. We were able to collect data from 50 oil droplets and from this data calculated the elementary charge using two methods. The first method depends only on the voltage required to suspend the oil droplet and the terminal velocity: using this method we found $Q = 1.11 \pm 0.04 \times 10^{-19}\ C$. The second method also depends upon the the terminal velocity of the droplet and also the voltage and velocity required to cause the droplet to ascend: using this method we found $Q = 0.91 \pm 0.5 \times 10^{-19}$. Thus we found that using either method we were unable to find the theoretical elementary charge and in fact consistently underestimated the value.

1 Introduction

The electric charge that is carried by a single photon or electron is refered to as the elementary charge, $e = 1.602 \times 10^{-19}\ C$. We can theortically calculate this value with Milikan experiment--though it should be noted that pratically is diffult and Milikan supposably 'fudged' his data.
The experiment is based on the way different forces will act upon an electrically charged particle moving through a homogeneous electric field. In this case, an electrically charged oil troplet is used and the electric field is generated by two plate capacitors. If the distance between the plates is d and the applied volatge V then we get an electric field $E = V/d$. Thus for a droplet with charge Q we find the follwing forced being acted upon the droplet:

- gravitational: $F_{grav} = m_{oil}g$
- Buoyance: $F_b = -m_{air}g$
- Electric: $F_E = QE$
- Stokes resistance (assuming laminar flow): $F_{drag} \approx 6\pi r\eta v$

where m_{air} is the mass of air displaced by the droplet, r is the radius of the droplet, η is the air viscosity, and v is the speed which the oil droplet is moving. By changing the applied voltage, and thus the strength of the electric field, we can put the oil droplet in one of three situations.

(1) the droplet is floating: $0 = g(m_{oil} - m_{air}) - Q\dfrac{V_{stop}}{d}$
(2) moving down with terminal velocity v_t: $0 = g(m_{oil} - m_{air}) - 6\pi r\eta v_t$
(3) moving up with acceleration: $ma = -g(m_{oil} - m_{air}) + QE - 6\pi r\eta v$

From this we can solve for Q with two methods. For method 1,

$$
\begin{aligned}
g(m_{oil} - m_{air}) - Q\frac{V_{stop}}{d} &= g(m_{oil} - m_{air}) - 6\pi r\eta v_t \\
Q &= \frac{d}{V_{stop}} 6\pi r\eta v_t
\end{aligned}
$$

and taking $r = \sqrt{\dfrac{9v_t\eta}{2g(\rho_o - \rho_a)}}$ then

$$
\begin{aligned}
Q &= \frac{d}{V_{stop}} 6\pi \sqrt{\frac{9v_t\eta}{2g(\rho_o - \rho_a)}}\eta v_t \\
&= \frac{1}{V_{stop}} d6\pi\eta v_t (v_t)^{1/2} \sqrt{\frac{9\eta}{2g(\rho_o - \rho_a)}}
\end{aligned}
$$

$$= 18d\pi\sqrt{\frac{\eta^3}{2g(\rho_o-\rho_a)}}\left(\frac{v_t^{3/2}}{V_{stop}}\right)$$

letting $\rho_o = 875.3\ \frac{kg}{m^3}$, $\rho_a = 1.204\ \frac{kg}{m^3}$, $\eta = 1.827\times10^{-5}\ Pa\cdot s$, and $d = 6.0\ mm$, then

$$Q = \left(2.0\times10^{-10}\ kg^{1/2}m^{1/2}\right)\left(\frac{v_t^{3/2}}{V_{stop}}\right) \tag{1}$$

For method 2, we can use situation (3) and assume the droplet is moving up with constant velocity v_2 and thus

$$\begin{aligned}
g(m_{oil}-m_{air})-6\pi r\eta v_t &= -g(m_{oil}-m_{air})+QE-6\pi r\eta v_2\\
0 &= -QE+18\pi\sqrt{\frac{\eta^3}{2g(\rho_o-\rho_a)}}\left(v_t^{3/2}+v_t^{1/2}v_2\right)\\
QE &= 18\pi\sqrt{\frac{\eta^3}{2g(\rho_o-\rho_a)}}(v_t+v_2)v_t^{1/2}\\
Q\frac{V_{up}}{d} &= 18\pi\sqrt{\frac{\eta^3}{2g(\rho_o-\rho_a)}}(v_t+v_2)v_t^{1/2}\\
Q &= 18d\pi\sqrt{\frac{\eta^3}{2g(\rho_o-\rho_a)}}(v_t+v_2)\frac{v_t^{1/2}}{V_{up}}
\end{aligned}$$

therefore, taking the same values we used in method 1, we have that

$$Q = \left(2.0\times10^{-10}\ kg^{1/2}m^{1/2}\right)(v_t+v_2)\frac{v_t^{1/2}}{V_{up}} \tag{2}$$

2 Material and method

2.1 Material

- Leybold - Heraeus apparatus
- LabVIEW software
- CCD Camera
- Spray nossel and oil
- Milikan chamber

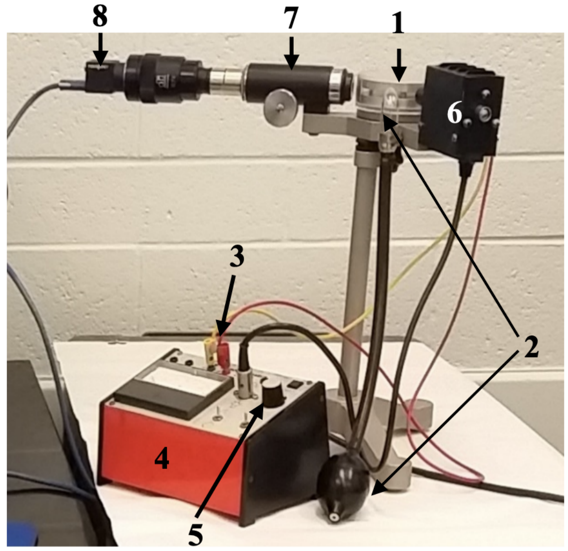


Figure 1: Ley-Heraeus apparatus figure taken from the pdf handout
1 – chamber (plate capacitor);
2 – oil atomizer with rubber bulb;
3 – socket pair for charging the
plate capacitor connected to DC
power supply 4 with adjusting
knob 5;
6 – light source;
7 – microscope connected to the
CCD camera 8

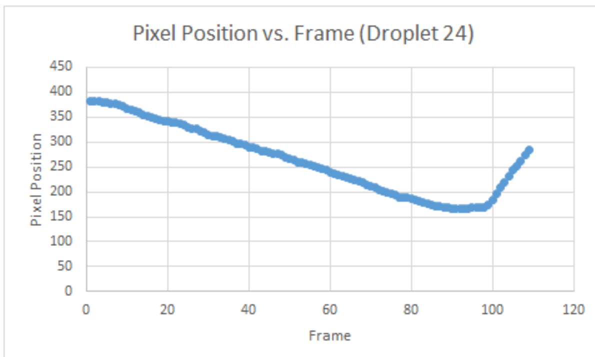
2.2 Method

1. We prepared 50 Excel sheets to collect data, one sheet for each droplet of oil
2. We turned off the lights in the room to perform the experiment in a dark room. With the 'gain' of the program and the voltage set on the max option we sprayed oil into the Milikan chamber.
3. We adjusted the camera till we were able to find oil droplets that we ascending. Once a suitable droplet was found we readjusted the voltage to so that it would remain approximatly suspended and readjusted and setting so that what we were left with was a black background with a bright white dot (the oil droplet).
3. We track the droplet by placing a green frame around it and adjust the voltage till the droplet remains with in the frame (suspended) and record the corresponding voltage.
4. We then begin the tracking/recording of the movement of the droplet. First we set the voltage to some greater value than its suspended voltage so that the droplet may begin ascending upwards. We record this voltage as the up voltage.
5. Before the droplet reaches the top of the chamber, we drop the voltage back down to the suspension

voltage so that it may come to rest.
6. Next we turn off the voltage and let the oil droplet fall to its terminal velocity. We should now have saved the data of the movement going up and falling with recorded suspension and up voltages.
7. With this data we calculate the average velocity going up and falling by examining the slope of the position vs. time plot using scipy's curvit_fit().
8. With our calculated velocities and the recorded voltages we may use equations (1) and (2) to then try
and find the elementary charge.
9. Repeat this till we had 50 sheets/oil droplets

3 Data and anaylsis

Below in figure 2 are example plots of the pixel position vs. frame graphs for two oil droplets. With these plots we convert to mm and seconds. The first downward slope corresponds with the upward velocity while the upward slope is the terminal velocity after turning off the voltage. Again, we used curve_fit() on different segments to find the slope, and thus velocity, of the droplet.



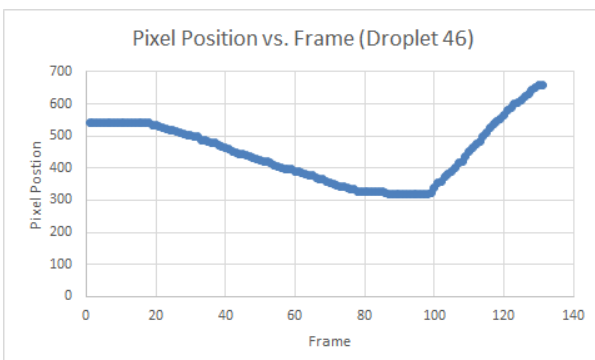


Figure 2: Pixel vs. frame plots for two oil droplets that were later converted to mm v. time

($540 \pm 1 \dfrac{px}{mm}$ and $10\ Hz$ frame rate)

Figure 3 below is the histogram of the calculated charges from our 50 droplets.

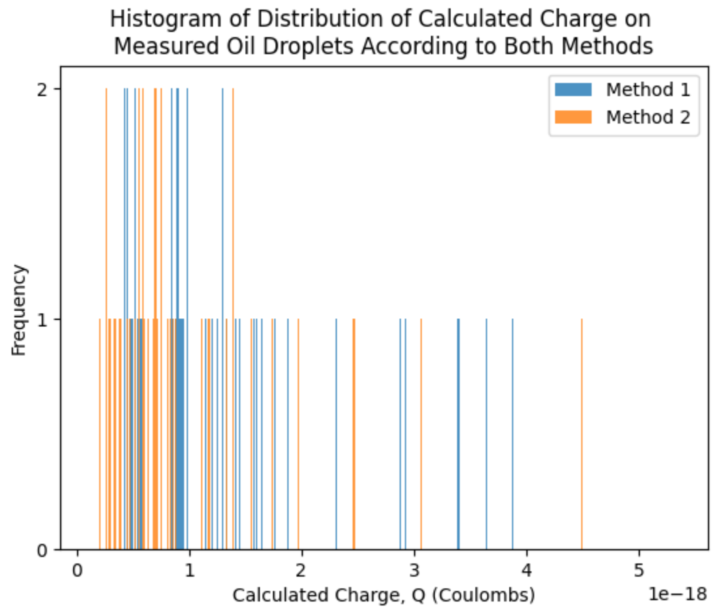


Figure 3: Histogram of calculated charge using method 1 (blue) and method 2 (orange)

The theoretical value of the elementary charge is $1.6 \times 10^{-19}\ C$. From the histogram above we find that the non of our values that we calculated are not in agreement with the known elementary charge--some even ranging to a whole order of magnitude off. However, we continue our anaylsis using a GCD finding algorithm, which its results are displayed below in figure 4.

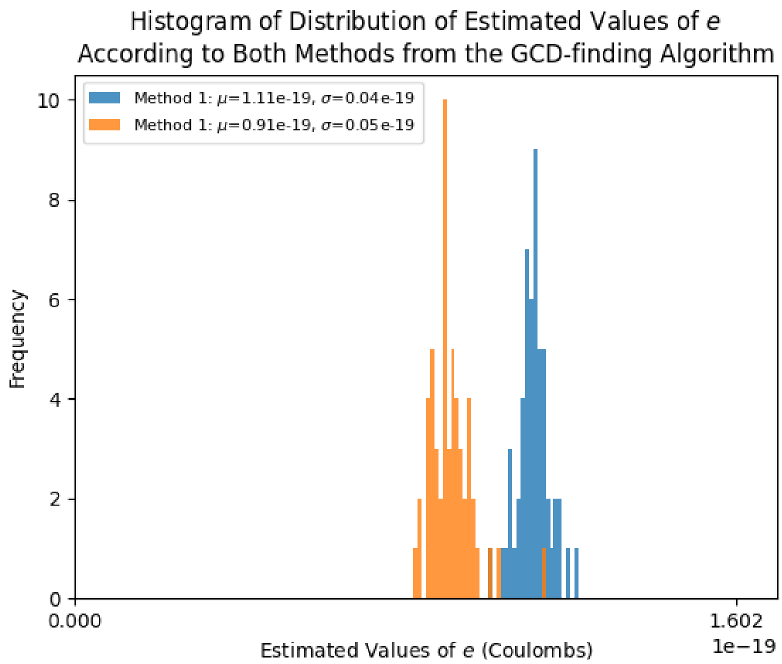


Figure 4: Histogram of calculated elementary charge using the GCD-finding algorithm

The algorithm takes every permutation of pairs of charges that we determined and calculates their approximate GCDs. This is done by multiplying each pair of charges by the accuracy_param and rounding them to the nearest integer, finding the GCD of this pair of integers, and then dividing this GCD by the accuracy_param again as the approximate GCD of the pair of charges. If the approximate GCD of a pair of charges is smaller than the determined uncertainty value of charges (error_threshold), the algorithm will consider the pair of charges to be of the same value and discard their approximate GCD, otherwise the approximate GCD is appended to an array, which is finally returned and plotted as a distribution on figure 4. We take our final calculate value of the elementary charge to be the mean of this distribution of approximate GCDs, and we take the error of our determined elementary charge to be the standard deviation of this distribution of approximate GCDs.

With this algorithm we found that using method 1 we were able to approximate the elementary charge to be $1.11 \pm 0.04 \times 10^{-19}\ C$. Where the uncertainity is simply taken to be the first standard deviation of the corresponding guassian distribution. This value deviates from the theoretical value by 30.6%. Using method 2 we find the elementary charge to be $0.91 \pm .05 \times 10^{-19}$. Method 2 deviated worse from the theoretical value with a 43.1% discrepency.

In either method we see that our calculted value is consistently an underestimation of the expected value. Examining equations (1) and (2) this could possibly be due to an underestimation of the terminal velocity (and/or the ascending velocity for method 2) and/or an overestimation of the suspension voltage.

4 Conclusion and Questions

Q1) Since we have that a droplet in falling through field-free space has a radius of $r = \sqrt{\frac{9v_t\eta}{2g(\rho_o - \rho_a)}}$, we will estimate the typical radius of the droplets in our experiment by taking the average terminal velocity from all 50 droplets collected. This value was $v_{t,\,ave} = 1.6 \times 10^{-4}\frac{m}{s}$ and thus if we take the other variables to be the values listed in section 1 we have

$$r \approx 1.2\ \mu m$$

Q2) We see that the buoyant force is $F_b = gm_{air} = g(V_{droplet}\rho_{air})$ where $V_{droplet}$ is the volume of the droplet displacing the air. We took the denisty of air to be $\rho_{air} = 1.204\frac{kg}{m^3}$, we also have that the density of the oil is $\rho_{oil} = 875.3\frac{kg}{m^3}$. The density of the air is only approx. 0.14% of the oil. Thus the buoyant force is also not a hugely significant force acting upon the droplet, but is still obviously present.

Q3) The experiment should work better for droplets with smaller radius for multiple reasons. One could argue that a droplet that is too large may not hold a single charge but multiple instead. This would cause an overestimation of the elementary charge. Seeing as our results greatly underestimated the theoretical value, it is unlikely that this is an issue we encountered.

Also if the droplet is too large then the voltage required to suspend the droplet becomes so large that it is either beyond the experiment equipment or that the accuracy of the results becomes poor at the instruments limits. When the droplets are smaller they fall more slowly as well and the tracking software is also better at keeping the droplet in frame and thus the accuracy of the position v. time is also improved.

Using method 1 we calculated the elementary charge to be $1.11 \pm 0.04 \times 10^{-19}\ C$ and using method 2 yielded $0.91 \pm .05 \times 10^{-19}\ C$. These estimated values deviated from the theoretical value of $1.6 \times 10^{-19}\ C$ by 30.6% and 43.2% respectively. Thus, neither method was able to successfully estimated the elementary charge (as not suprsing for a lab famous for failure). We do note that we are consistently understimating the theoretical value however.

One major difficulty in the experiment was being able to find a voltage that was able to properly suspend the droplet. Given enough time the droplet would often slowly drift away. However we wouldn't expect this to consistently understimate our calculated value unless we were consistently overestimating the suspension voltage.

What may be more likely is that either our tracking software tends to understimate the velocity of the droplet's movements. This wouldn't be too suprising as many there are many components that could effect the tracking ability. For example, the frame that is being tracked may pass other suspended particles that we were not able to eliminate with adjusting the background settings that cause could 'slow' the frame.