Electron Charge-Mass Ratio

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Abstract

We attempt to calculate the charge-mass ratio for an electron by taking examining the trajectory of an electron within a uniform magnetic field that we generate using a pair of Helmholtz coils. We calculate this value by first finding the magnetic field contributions from everything but the coils. Then with this value we are we were able to calculate the ratio to be $(1.73 \pm .09) \times 10^{11} \frac{C}{kg}$ which is in agreement with the theoretical value of $1.76 \times 10^{-11} \frac{C}{kg}$ with only a 1.7% deviation.

1 Introduction

We can cause a particle of mass m and charge e to move in a closed circular orbit by placing it in the righht conditions. In these conditions, the particle is placed in an uniform magnetic field $\vec{B}$ and it is the magnetic force induced from this field when the particle moves perpendicular with velocity v to the field that provides the centripetal force required for circular motion. This force can be found to be $\vec{F} = e\vec{v} \times \vec{B}$. In the conditions for circular motion with a radius of r we find that the force becomes as follows

$$evB = m\frac{v^2}{r} \tag{1}$$

In this experiment, in order to get a particle to move with some velocity v, we will accelerate an electron through a potential difference V and get

$$\begin{aligned} eV &= \frac{1}{2}mv^2 \\ \Rightarrow v &= \sqrt{\frac{2eV}{m}} \end{aligned}$$

By substituting this induced velocity into equation (1) we find

$$\begin{aligned} evB &= m\frac{v^2}{r} \\ \Rightarrow \frac{eB}{mv} &= \frac{1}{r} \\ \frac{eB}{m\sqrt{\frac{2eV}{m}}} &= \frac{1}{r} \\ \Rightarrow \frac{1}{r} &= \sqrt{\frac{e}{2m}}\frac{B}{\sqrt{V}} \end{aligned} \tag{2}$$

In order to generate a uniform magnetic field for this experiment we will be using a pair of Helmholtz coils seperated by their radius R. Thus we find that the magnetic field B_c along the coils' axis is

$$B_c = \left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 nI}{R}$$

where $\mu_0 = 4\pi \times 10^{-7} N/A$, n is the number of turns in a coi, and I is the current supplied. However, what the electron in our experiment will actually experience in addition to the magnetic field from the Helmholtz coils is the field naturally generated from the surroundings the experiment is taken place in B_e. Hence, what the electron will be moving through a field $B = B_c + B_e$. Going back to equation (2) this means

$$\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{1}{\sqrt{V}}\left(\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 nI}{R} + B_e\right) \tag{3}$$

We can simplify this expression above by defining a few new variables

$$k = \frac{1}{\sqrt{2}}\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 n}{R} \qquad\qquad I_0 = \frac{B_e}{k}$$

and find that (3) simplifies to

$$\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{(I - I_0)}{\sqrt{V}}$$

2 Method

Our apparatus consist of a magnetic field produced by Helmholtz coils with current supplied by an $8\ V$ D.C. power supply. Between the coils is a glass bulb containing an electron gun and hydrogen gas. The anode which provides the potential difference is supplied by a D.C. power supply that ranges from 0 to $300\ V$. Figure 1 and 2 below show the circuit diagram of our set up and the actual physical set up in the lab.

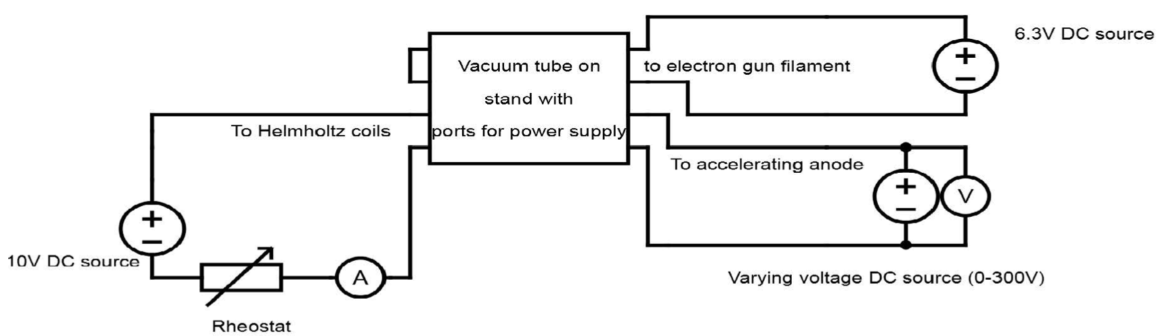


Figure 1: Circuit diagram of the lab set up taken from the lab manual

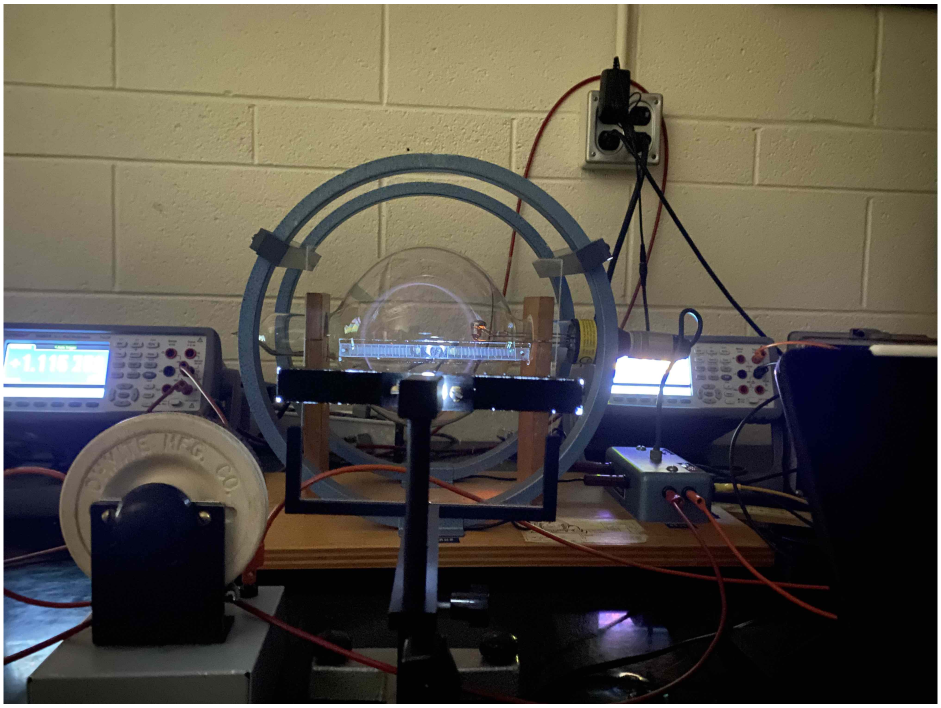


Figure 2: Photo of our physical lab set up

The electron beam curvature was obtained by two ways: the first from varying the current and the second from varying the accelerating potential. We first varied the current from about 1 to $2\ A$ at a constant voltage of $195 \pm 0.4\ V$. For each value of different value of current we observed a different curvature. If the beam did not did not follow a closed path we rotated the bulb to make sure the beam's trajectory was as desired.

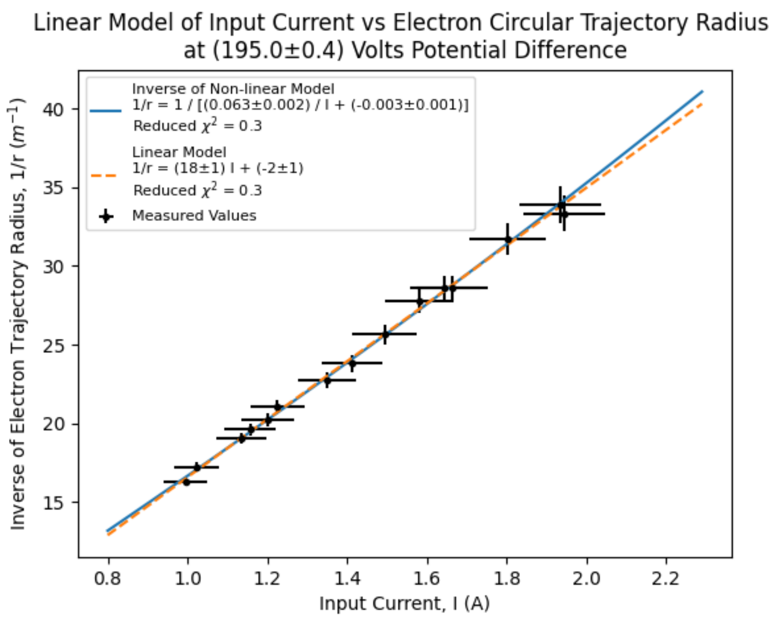
We then used the self-illuminated scale and plastic reflector to measure the radius of the closed circular path. We repeated this for 15 different current values. After we repeated the same process except with varying the voltage applied from about 80 to $300\ V$ at a constant current of $1.25 \pm 0.07\ A$.

3 Results and Discussion

The uncertainties that we have are on the measurements of voltage, current, and trajectory radius. The error on voltage and current were taken to be the claimed uncertainty of the manufactorers multimeters. For the voltimeter this value was $0.2\% + 0.06\ V$ and the ampimeter was $5\% + .006\ A$. The uncertainity o the radius was taken as $0.1\ cm$ as the last accurate significant figure.

For taking the measurements of the radius one can not simply directly place a ruler agaisnt the bulb to measure the trajectory. One must take into account the parallax effects. The self-illuminated scale and plastic reflector is used to help eliminate the parallax effect when measuring the radius. The image that appears on the plastic reflector screen is a mirror image that appears as though it is a depth behind the screen. By making that depth the same distance from the screen to electrons (as if the ruler is sitting at the same depth of the electron beam) then the parallax has been eliminated cause the ruler appears as though it moves along with the beam as one moves their head.

Below in figure 3 is plotted both the radius and inverse radius of the beam trajectory agaisnt the input current that produces the magnetic field. Plotted on top this data is the a linear model and a nonlinear model that's parameters are found using scipy's curve_fit() and uncertainities are found using the uncertainties above for the variance found with curve_fit().



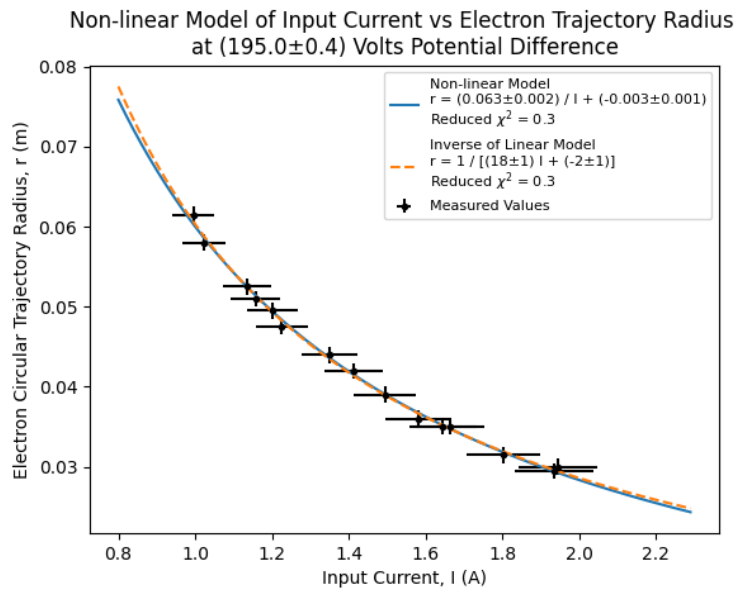


Figure 3: (top) Input current versus the inversus radius plotted with both the linear and the inverse of the nonlinear model
(bottom) input current versus radius plotted with the nonlinear model and inverse of the linear model

Examinig the reduced chi squares we find for both models and both sets of data we have $\chi_r^2 = 0.3 < 1$. This implication that the data is overfitted is not too suprising--if we examine the plots from figure 3 we see that the error bars are large and that the models appear to be fitting the error of the data. This is what is probably resulting in a 'small' reduced chi square value.

We may rearrange equation (3) in order to calculate the 'extra' magnetic field that is not produced by the coils. If we find the magnetic field from the coils B_c as a function of $\frac{1}{r}$ we may use the data where we hold voltage V constant and vary the current I and find the following relationship,

$$\sqrt{\frac{e}{2m}}\frac{1}{\sqrt{V}}(B_c + B_e) = \frac{1}{r}$$

$$B_c + B_e = \sqrt{\frac{2mV}{e}}\left(\frac{1}{r}\right)$$

$$B_c = \sqrt{\frac{2mV}{e}}\left(\frac{1}{r}\right) - B_e$$

Thus we can find that the 'extra' magnetic field is simply the vertical intercept of B_c plotted agaisnt $\frac{1}{r}$. Below is the plot of said relationship calculated from our data fitted with scipy's curve_fit()

and errors derived from curve_fit()'s variance

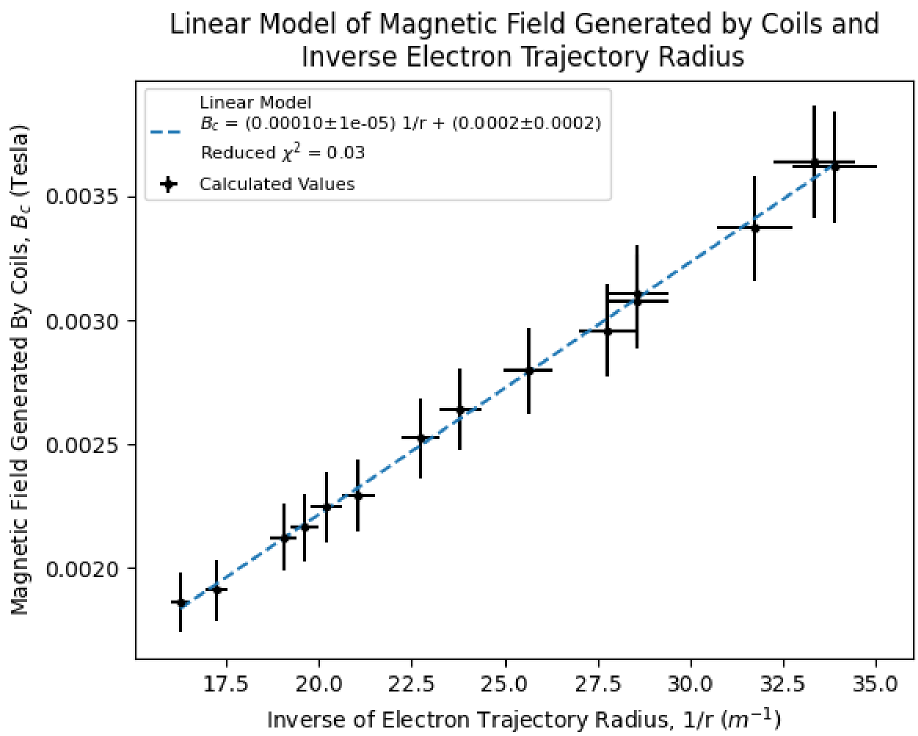


Figure 4: The magnetic field B_c induced from the Helmholtz coils vs. the inverse electron beam radius $\frac{1}{r}$ fitted with a linear model.

From figure 4 we find that the 'extra' magnetic field is

$$B_e = -.0002 \pm .0002\ T$$

The reduced chi square value for our linear fit was calculated to be $\chi_r^2 = 0.03 < 1$. This implies that our model is overfitting the data. Looking at our plot in figure 3 it appears as though the model is overfitting for the error bars and thus dropping our chi square value.

Using this the above value of B_e we may find the mass-charge ratio from the following equations

$$\frac{1}{r} = \sqrt{\frac{e}{m}} k \frac{I - I_0}{\sqrt{V}}$$

$$r = \sqrt{\frac{m}{e}} \frac{\sqrt{V}}{k(I - I_0)}$$

where $k = \frac{1}{\sqrt{2}}\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 n}{R}$ and $I_0 = \frac{B_e}{k}$. Thus when we hold the current constant as $I = 1.25 \pm 0.07\ A$ we have

$$\begin{aligned}
k(I - I_0) &= \frac{1}{\sqrt{2}}B_c - B_e \\
&= \frac{1}{\sqrt{2}}\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 n I}{R} - B_e \\
&= \frac{1}{\sqrt{2}}\left(\frac{4}{5}\right)^{3/2}\frac{\left(4\pi \times 10^{-7}\right)(130)(1.25 \pm 0.07)}{(.0625 \pm .0005)} - (-.0002 \pm .0002) \\
&= .0019 \pm .0002
\end{aligned}$$

Below in figure 5 is the plots for the varying voltage versus the trajectory radius. Similar to the above we have fitted to the data a linear model using curve_fit() with the uncertainty found using the same method as the above plots when using curve_fit(). From the parameters found from the fit, along with the equations above, we find that

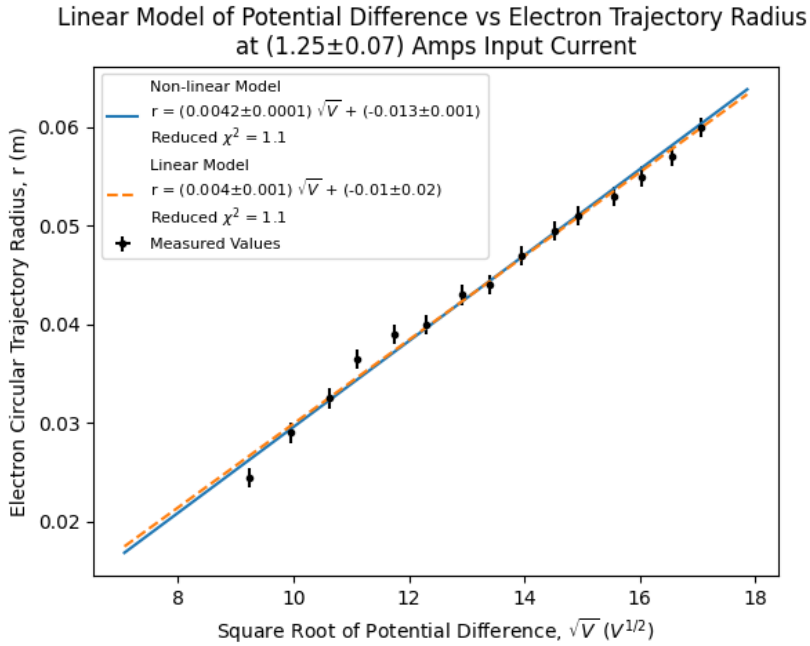
$$slope = \sqrt{\frac{m}{e}}\frac{1}{k(I - I_0)}$$

Thus rearranging to calculate the mass-charge ratio we get that

$$\frac{e}{m} = \left(\frac{1}{slope}\frac{1}{k(I - I_0)}\right)^2$$

Using our calculated values this means we get

$$\begin{aligned}
\frac{e}{m} &= \left(\frac{1}{.004 \pm .001}\frac{1}{.0019 \pm .0002}\right)^2 \\
&= (1.73 \pm .09) \times 10^{11}\frac{C}{kg}
\end{aligned}$$



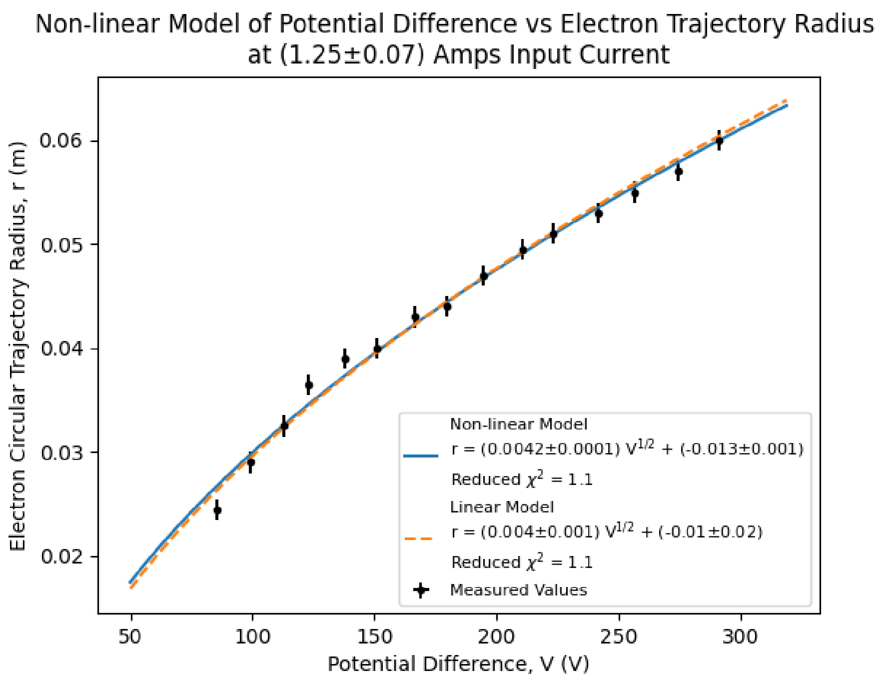


Figure 5: (top) square root of the potential versus the trajectory radius plotted with a linear model and the square of the nonlinear model
(bottom) potential difference versus the radius with the square root of the linear model and a nonlinear model

We note that the unertainities of the potential difference are very small and thus not visible in the plots of figure 5. When examining the calculated reduced chi square value we find that $\chi_r^2 = 1.1$. This chi square value implies that our models actually fit the data well and this is easily confirmed by examining the plots of figure 5.

We have the known values of the mass and charge of the electron: $9.11\times10^{-31}\ kg$ and $1.6\times10^{-19}\ C$ respectively. Using these values we find

$$\begin{aligned}\frac{e}{m} &= \frac{1.6\times10^{-19}}{9.11\times10^{-31}}\\ &= 1.76\times10^{-11}\frac{C}{kg}\end{aligned}$$

Comparing this theoretical value to the one we calculated we find that our value is indeed in agreement with the theoretical value. We have that our calculated value is only a 1.7% deviation from the theoretical value.

4 Conclusion

From our experiment we were able to calculate the mass-charge ratio for an electron to be

$$\frac{e}{m} = (1.73 \pm .09)\times10^{11}\frac{C}{kg}$$

This is in agreement with the theoretical value of $1.76\times10^{-11}\frac{C}{kg}$ with only a 1.7% deviation.

We note that the for this experiment we assume a constant magnetic field. However, this is not necessarilty the case. Around the edges (i.e. farther from the coil's axis) the magnetic field deviates from uniformity, thus we expect that electron beams with larger radius that traverse the non uniform magnetic field will not follow a perfectly circular closed loop path.

Similarly the 'extra' magnetic field component we calculated will be mainly from the earth, but there are still other changing components. For example, even the phones that we use that are in the labatory space contribute to the magnetic field. Thus we expect when the the 'extra' field component is technically not a constant as we have taken it to be in our calculaations. However, this contribution is appears to be minor and this seems to be support from the value of the ratio we were able to calculate.